Global Dual-Sourcing Strategy: Is It Effective in Mitigating Supply Disruption?

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Abstract

Most firms are still failing to think strategically and systematically about managing supply disruption risk and most of the supply chain management efforts are focused on reducing supply chain operation costs rather than managing disruption. Some innovative firms have taken steps to implement supply chain risk management (SCRM). Inventory management is part of SCRM because supply disruptions negatively affect the reliability of deliveries from suppliers and the costs associated with the ordering process. The complexity of existing inventory models makes it challenging to combine the management of the supply process and inventory in a single model due, for example, to the difficulty of including the characteristics of the disruption process in the supply chain network structure. Therefore, there is a need for a simple flexible model that can incorporate the key elements of supply disruption in an inventory model. This thesis presents a series of models that investigate the importance of information on disruption discovery and recovery for a firm's supply and inventory management. A simple two-echelon supply chain with one firm and two suppliers (i.e., referred to as the onshore and offshore suppliers) in a single product/component setting has been considered in this thesis for the purpose of experimental analyses. The sourcing decisions that the firm faces during periods of supply disruption are examined leading to an assessment of how information about the risk and length of disruption and recovery can be used to facilitate the firm's sourcing decisions and monitor the performance of stock control during the disruption. The first part of this thesis analyses basic ordering models (Model 1 and Model 2 respectively) without the risk of supply disruption and with the risk of supply disruption. The second part analyses the value of supply disruption information, using a model with advance information on the length of disruption (Model 3) and a model with learning about the length of disruption (Model 4). The third part explores a quantitative recovery model and the analyses in this part consider of three models. Model 5 assumes a basic phased recovery model, Model 6 assumes advance information about the phased recovery process and Model 7 assumes learning about the phased recovery process. The last part of this thesis investigates the order pressure scenario that exists in the firm's supply chain. Under this scenario, disruption to one part of the supply chain network increases demand on the remainder resulting in a lower service levels than normal. This scenario is applied to all the previous models apart from Model 1. The models in this thesis are examined under finite and infinite planning horizons and with constant and stochastic demand. The objective of the models is to minimise the expected inventory cost and optimise the order quantity from the suppliers given the different assumptions with respect to the length of supply disruption and information about the recovery process. The models have been developed using the discrete time Markov decision process (DTMDP) technique and implemented using the Java programming language. The findings of this thesis could be used to help a firm that is facing the risk supply disruption to develop its SCRM program. The findings highlight the importance of considering quantitative measures of the disruption and recovery processes, something which is still not popular within SCRM in some organisations.

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Notations

N - The onshore supplier.

F - The offshore supplier.

sp - Set of the suppliers, $sp = \{N, F\}$.

u - The state of the offshore supplier is up.

w - The state of the offshore supplier is down.

a - Set of the states of the offshore supplier, $a = \{u, w\}$.

t - time period in the firm's planning horizon.

j - The number of the disruption periods.

W - The amount of j disruption periods in the disruption process.

R - The amount of j disruption periods in the recovery process.

Q - The amount of t periods in the firm's planning horizon.

M - The amount of t used as a warm up in the simulation model.

n - Number of replication in the simulation model.

D_t - Demand in period t.

d_t - Expected demand in period t.

λ - The demand rate.

K - The value at which the demand probability distribution is truncated or the maximum value of demand to be considered in the analyses.

$P(D_t = d_t)$ - The probability that the demand in period t is d_t.

$P(d_t) \sim Pois(\lambda, K)$ - The truncated Poisson distribution.

$E(d_t)$ - The total expected demand value per period.

sh_t - The value of shortage in t period.

$E(sh_t)$ - The expected shortage value per period.

i - Inventory level.

I - Maximum firm's storage capacity.

L^{sp} - Lead-time of supplier sp.

q_t^{sp} - Quantity of raw/semi materials supplied by supplier sp at t period.

c^{sp} - Fixed ordering cost of supplier sp per order.

v^{sp} - Variable ordering cost of supplier s per item.

h - Holding cost per item per time unit.

x - Holding cost per unit of capital tied up in inventory per year (i.e., used to buy current inventory), $0 > x > 1$

m - Penalty cost per item of lost sales.

$HOLD$ - Total of the holding cost.

$ORDER$ - Total of the fixed and variable costs.

$PNLTY$ - Total of the penalty cost.

T - Set of decision epochs for the Discrete Markov Decision Process (DMDP) model.

y - States for the DMDP model.

Y - State Space for the DMDP model, $y \in Y$.

b - Actions for the DMDP model.

$B(y)$ - Set of admissible actions, b, when the process was in state y for the DMDP model.

$p_{yz}(b)$ - The probability of making a transition from state y to state z in one period when action b has been chosen or refers as a transition probability for the DMDP model.

X - Transition matrix for the DMDP model.

$C_y^t(b)$ - The inventory cost during period t when action b has been chosen when the process was in state y or refers as a one-step cost under the DMDP model.

Δ_t - Decision rules at every decision epoch, t, under the DMDP model.

π - The ordering policy under the DMDP model.

Δ_t^* - The decision rules consisting of the actions which minimise the optimality equation for each state at decision epoch, t, under the DMDP model.

π^* - The optimal ordering policy that prescribes an action in every state $y \in Y$ and at every decision epoch, t, under the DMDP model.

$V_t^y(\pi)$ - The expected cost of inventory policy π when there are t periods to go and the process is in state y, as a result of the chosen π.

V_t^y - The minimum expected total inventory cost with t periods to go when the process is in state y under the optimality equation.

g - The minimum long-run average cost under the infinite-horizon DMDP model.

H_t - Maximum difference in the minimum expected cost for any state between iterations t and $t-1$.

J_t - Minimum difference in the minimum expected cost for any state between iterations t and $t-1$.

ε - The tolerance value to detect convergence after t-iterations under the infinite-horizon DMDP model.

$\overline{p}_{y,z}(b)$ - The perturbed one-step transition probability under the perturbed DMDP model.

τ - The constant value for the perturbed one-step transition probability under the perturbed DMDP model.

r - The $Uniform$ (0,1) pseudo-random number.

$P(o)$ - The discrete probability pseudo-random number distribution for $o = 0, 1, \ldots, O$.

P_2 - The average fill rate.

I_A - The average inventory level.

s - Reorder point in the optimal ordering policy.

S - Order-up-to level in the optimal ordering policy.

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1. Introduction

1.1 Need for the Research

Today, in a supply chain network, most suppliers for firms are located in different regions and countries such as low-cost countries which are China, India, Sri Lanka, Vietnam and Eastern Europe. These types of low-cost countries have been favourable places for companies like General Motor, General Electric, Home Depot and Avon. We call this type of supply chain network a *global supply chain*. The objective of a global supply chain is to have an efficient supply management system which can secure the flow of materials and enjoy a low cost of product. This global outsourcing strategy can deliver extremely powerful business practices, allowing firms and manufacturers to significantly reduce costs, increase profits and to focus efforts on their core competencies. In turn, this has led to a shift in the strategy adopted to achieve their competitive targets towards long-term partnerships with fewer and cheaper suppliers as the lower cost is often the main decision criterion for global sourcing (Handfield et al., 2007).

Nevertheless, nothing is perfect. Sting and Huchzermeier (2010) identify two potential drawbacks for those firms who offshore their sourcing activities. Firstly, the distance to supply source, either geographically, organisationally or culturally, can affect supply flexibility (i.e., long delivery time and sourcing flexibility). Secondly, delivery performance and product quality have been prolonged issues among supply chain managers due to low reliability and poor quality of product. Other than these two main issues in global supply chain, the supply process is also becoming more complicated with the risk of supply disruption which the decision to offshore the supply usually under-estimates this risk (Chopra and Sodhi, 2014; Hendricks et al., 2009; Handfield et al., 2007). The increasing tendency of the firms/manufacturers to outsource processes to global suppliers will increase the risk of exposure to a supply chain disruption. One executive interviewed by Handfield et al. (2007)

from a major electronic company noted:

> " We have successfully outsourced production of our products to China. Unfortunately, we now recognise that we do not have the processes in place to manage risk associated with this supply chain effectively!"

The above statement shows that some of the firms that have implemented the global souring policy still do not have mitigation strategies in the event of the disruption at their offshore suppliers. Most firms are still failing to think strategically and systematically about managing supply risk, where most of the supply chain management efforts have focused on reducing supply chain operation costs instead of managing disruption (Chopra and Sodhi, 2014; Sheffi, 2005; Kilgore, 2003) due to the fact that disruptions are infrequent and hard to predict (Hendricks and Singhal, 2005).

There are quite a number of factors that can cause supply disruptions such as natural disasters (i.e., earthquake and flood), man-made disasters (i.e., fire and war), world financial crisis and bankruptcy. Examples of such catastrophic disruption events related to supply disruptions that have occurred in the past are: (1) in 2007, delivery of Boeing's 787 Dreamer airplane was delayed due to late delivery of two critical parts by their base supplier, Advance Integration Technology (AIT), resulting a cash hit of $2.5 billion in penalties to customers and in keeping the supplier alive in the business market due to serious cash flow disruption (Greising and Johnsson, 2007), (2) in 2002, the US West Coast port strike by labour union halted all material flow through 29 ports and left hundreds of cargo-bearing ships stuck at sea during the 10-day ports closure, resulting in an estimated cost to the economy of $20 billion (Schmitt et al., 2015) and (3) in 2000, a fire at the supplier semiconductor plant for Nokia and Ericsson caused both firms an anticipated one week delay in shipment, resulting in Ericsson reporting long-term losses of $2.34 billion and eventually withdrawing from cell phone market, while Nokia increased their market share by 3% in the first 6 months after the fire because of their proactive action to mitigate the incident (Schmitt et al., 2015, 2010).

The examples show that such catastrophic events can have a negative impact to the business and financial operations. The study conducted by Hendricks and Singhal (2005) reveals that firms who suffer from supply chain disruption experience about 30% lower stock returns than their benchmarks and Rice and Caniato (2003) present results from their research that estimates a $50-100 million cost impact for each day a supply network is disrupted. The adverse impact of disruption has led some firms to realise that Business Continuity Process (BCP) mechanism is not enough to mitigate the impact of disruption due to events such as 9/11 attacks in 2001, hurricane Katrina in 2005 and radioactive water leak caused by earthquake at Fukushima nuclear plant in 2012. Therefore, in managing disruptive supply events, the firms recognise that they need to plan for strategies to facilitate the mitigation process of supply disruption in their supply chain networks.

In the paragraphs above, we have described the motivation behind this research by placing the study in a business and an organisational behaviour context. Next, in this introductory chapter, we will present in section 1.2 some terms and definitions to facilitate the readers understanding of this research followed by an overview of the research area in section 1.3. The scope and the importance of the research are explained in section 1.4 and section 1.5 respectively. The aims and objectives of the PhD thesis are given in section 1.6 and finally, the structure of the thesis is outlined in section 1.7.

1.2 Definitions

For a better understanding, we present some terms and definitions which will be used throughout this thesis. The accuracy of understanding of these terms is important to avoid any ambiguity and misinterpretation for readers while reading this thesis. The terms and their definitions are as follows:

a. *Supply Chain Risk Management* or SCRM is the practice of managing the risk of any factor or event that can materially disrupt a supply chain by integrating the chain nodes in the risk planning strategy.

b. *Business Continuity Plan* or BCP is a strategy that involves defining risks, determining how those risk will give a negative effect to the operations, designing procedures to mitigates those risks, testing and reviewing those procedures periodically to ensure its effectiveness.

c. *Disruption* is any factor/event that can interrupt any procedure, process or movement.

d. *Supply Management* is a practice of managing products or services needed to operate a business.

e. *Supplier* is someone whose businesses to supply products or service.

f. *Dual-supplier* is a group of two suppliers both of whom are capable of supplying a firm with a product or service.

g. *Supply Disruption* is any factor/event that can interrupt the process of supply for regular business operations.

h. *Supply Risk* is defined as the probability of an incident associated with inbound supply from individual supplier failures or the supply market occurring, in which its outcomes result in the inability of the purchasing firm to meet customer demand or cause threats to customer life and safety (Zsidisin, 2003a).

i. *Inventory Management* is a practice of managing stocks including the process of viewing and controlling the inventory level and ordering.

j. *Ordering Policy* is a rule that determines when to order inventory and how much inventory to order.

1.3 Overview of Problem Statement

The key task in any sourcing strategy is to specify strategic supply allocations and ordering policies (Allon and Van Mieghem, 2010). Theses tasks are vital for any firm that has chosen to multi-source globally since this strategy will affect the inventory costs and supplier management. The firm may need to split its orders and award parts of the order to different suppliers. This will, for example, be the case when the firm faces a trade-off between lower component costs and shorter delivery times. Let us suppose that such a firm has two different suppliers and that the firm operates a dual-supply strategy. The first supplier (who we shall refer to as the 'onshore supplier') offers comparatively higher component costs and shorter delivery times. The second supplier (known in this case as the 'offshore supplier') offers lower component costs and longer delivery times. This situation is an example of *non-identical suppliers*. The sourcing decisions in this case will involve a trade-off between component costs and delivery times. We summarise the differences between these two suppliers by the fixed cost, the unit cost, the lead-times and the status of suppliers in table 1.1.

Table 1.1. The differences between the offshore and onshore suppliers

Factor	**Offshore supplier**	**Onshore supplier**
Fixed cost	high	low
Unit cost	low (unit)	high (unit)
Lead-time	long	short
Supplier status	subject to disruption	always reliable

From table 1.1, the onshore supplier will typically be a local supplier that is facing high operating costs and/or has significant pricing power over the local market. Even though the offshore supplier can offer the lowest ordering cost, this type of supplier is usually exposed to disruption risk. For example, the supplier tends to have a long order delivery (i.e., a long geographical distance from the firm) and to be located at a location that is prone to disruption (e.g., floods in China and Bangladesh). Therefore, there is a trade-off between longer lead-times and supply disruption risk from the offshore supplier and higher costs from the onshore supplier.

Theoretically, the strategy to diversify the component order quantities from more than one supplier is to help the firm to deal with the risk of supply disruption and the assumption is, if the one supplier (e.g., the offshore supplier) faces any disruption, then there is a backup supplier (e.g., the onshore supplier) who can step in and make up the difference at short notice. We would say, the firm assumes that the offshore supplier operation is totally independent to the onshore supplier operation, but this assumption does not hold in all cases. These suppliers might be linked through their industries, suppliers, customers, national economic and global trade regulations (An et al., 2009). In reality, the event that causes the disruption is likely to affect the operation of onshore and offshore suppliers at the same time. Therefore, for the supply disruption mitigation strategies, the firm should also take into consideration the dependency of operation between the offshore and onshore suppliers.

Other than the problem of supplier dependency in managing disruptions, the firm should take into consideration information about the disruptions. Such information is important for the firm. With wrong information about the disruptions, the effort of the firm to restore its capacity during the disruptive event can be ineffective and turn out to be wasted. Handfield et al. (2007) reports that firms with a high exposure to global supply chain risk invest more in improved inventory and capacity visibility systems to detect disruptions. From the moment the disruption occurs, the speed at which the firm recognises and responds to disruption effectively determines how well the problem is contained and how costly the disruption will be to the firm (Handfield et al., 2007). It is thus convenient to distinguish between the ability of the firm to discover the disruption that has occurred and the ability of the firm to recover from the disruptions.

On the one hand, the ability to discover the disruption depends on how much information the firm has about the disruption and the status of suppliers who experience the disruption. The firm might have an extensive database on the length, frequency and severity of past disruptions as well as the identities of the suppliers involved. In addition, some of the affected suppliers might be willing to provide timely and accurate information about the expected length and severity of disruption. On the other hand, recovery from the disruption will depend on the quality of information (i.e., accurate and timely information) obtained during the disruption discovery process. These types of information will be a part of the criteria for accessing the effectiveness of the firm's disruption mitigation strategies. If the information gained from the suppliers and the database about past disruptions were wrong and inaccurate, then, the firm only develops futile recovery strategies.

Given all these challenges, it is important for the firms to decide how they could best devise and implement a dual-sourcing strategy that avoids overspending on the onshore supplier and/or underestimating the supply disruption risk for the offshore supplier. For a summary of our research problem, we focus on the value of disruption information to the discovery and recovery processes and how this information can help the firm to improve the mitigation strategies that are already in place. Then, based on the dual-sourcing strategy, we are interested to study how the supplier dependency affects the firm's inventory management.

1.4 Scope of Research

This research falls under the topic of supply chain risk management focusing on the operational of supply process and the engineering of inventory policies. First, we examine the firm's ordering policies with non-identical suppliers and re-design the policies according to several scenarios for supply disruption information in the firm's supply chain network. Then, we explore the scenario of suppliers' operation dependency based on the supply disruption information.

1.5 Importance of Research

In this thesis we propose several inventory policies based on different types of supply disruption information and supplier dependency conditions that capture correlation between the supply disruption and the optimal ordering policies. There are gaps in existing studies under the operational research/ management science (OR/MS) area when modelling the inventory policy under the dual-source option.

From the modelling perspective, firstly, inventory policies based on the outcome of this research analyses will enable a new operational insight on the supply dependency scenario in the event of supply disruption. All the previous OR/MS studies have assumed that the suppliers are operationally independent and any changes in the supply condition at any individual supplier will not affect other suppliers in the supply chain network. In contrast, the models proposed in this thesis consider the condition where there is a probability that the suppliers are operationally dependent. Secondly, we generate additional insights in the study of inventory model with non-identical suppliers. In this thesis as opposed to existing studies, an additional element of disruption has been added to the inventory model where we look at the parameter of the length of disruption under the non-identical suppliers settings.

From the real world perspective, we propose realistic and simple inventory policies under several supply disruption conditions that can be considered by the manufacturers and practitioners in SCRM and BCP as a benchmark/references or mitigation strategy plan for future disruptive supply events.

1.6 Aims & Objectives

The aim of this thesis is to investigate how effective the non-identical dual-source option is to the management of supply chain networks that subject to supply disruption by addressing the issues of supply disruption information and supplier dependency. More specifically, we aim to investigate how the supply disruption should be taken into account when designing the dual-source inventory policies and how this disruption can affect the operational status of the suppliers. We compare the minimum cost and optimal order quantity of the inventory policies without disruptions with the policies with disruptions. We intend to develop a simple form of inventory policies with the objectives:

a. To identify optimal inventory policies for a firm that operates with non-identical dual suppliers.

b. To investigate how to redesign the existing optimal policies according to several supply disruption conditions.

c. To examine the scenario of supplier dependency in the operational decisions of supply chain according to several supply disruption conditions.

1.7 Structure of Thesis

The structure of this thesis is as follows. In Chapters 2 and 3, we review the relevant literature and methodology respectively. Six models are proposed in Chapters 4-7. The modelling problems, formulations and results will be discussed in detail within each chapter and in Chapter 8 we present the conclusion of this thesis.

We begin our investigation in Chapter 4 with preliminary studies of simple models, *a model without supply disruption* (the first model) and *a model with supply disruption* (the second model). In the first model, the onshore and offshore suppliers are assumed to operate under the routine operation without disruptions. A discrete time periodic review inventory model is used and non-identical supplier condition is a challenge stage in the process of the inventory policy development, hence several parameters in the model have been tested in various conditions such as constant and uncertain demand and finite and infinite production horizon planning to give a rich insight on the management of inventory with non-identical suppliers in supply chain networks. A supply disruption parameter is introduced in the second model where the inventory model setting is still similar to the first model. The assumption made on the supply disruption is the supplier is either at the *up* or *down* states.

In Chapter 5, the third and fourth models extend the second where the disruption parameter is now represented by the parameter of the length of disruption. Assumptions that have been made for the third and fourth models are *lengths of disruptions differ but the length is known at the start of the disruption* and *lengths of disruption is unknown but has a known probability distribution*, respectively. The parameter of the disruption length is used to represent the disruption information which will allow the firm to discover the disruption in its supply chain network.

In Chapter 6, the fifth, sixth and seventh models are presented. These models address the process of phased recovery. The fifth model is modelled with an assumption that the firm's supplier requires several recovery phases before it can resume the production as per routine operation. The sixth and seventh models extend the fifth with additional information available on the length of each phase of the process, in a similar way to the third and the fourth in Chapter 5. The sixth model is analysed with an assumption that advance information of the length of a phase of the recovery is available at the beginning of each phase. More specifically, *the firm knows the distribution of the overall length of disruption*. The sixth model extends the fifth in the same way as the third model extends the second. The seventh model uses a modification of the sixth which has more limited information about disruptions at the offshore supplier. More specifically, *information on the probability distribution of the duration of each phase of the recovery plan is available for the firm, but there is no advance information how long any phase will last.*

Chapter 7 brings the issue of supplier dependency in the firm's supply chain network in the event of supply disruption. We introduce the final eight and ninth models. Two assumptions have been made for these model; *there is no pressure in the chain of supply network* and *there is a pressure in the chain*.

Chapter 8 presents the conclusion of this research including the summaries of the modelling and practical contributions in this thesis. The limitations associated with our research and opportunities for future work in this area will also be discussed in this chapter.

1.8 Conclusion

The management of supply disruption for the firms who operate with non-identical suppliers is a very important problem from an academic and a practitioner perspective. From the academic perceptive, there is great potential for improvement in designing models in this area. A number of research projects have been conducted in the area of inventory modelling with supply disruptions issues. Most models of inventory management with non-identical suppliers have neglected the adverse impact of supply disruption and a possible existence of supplier dependency if the disruption has occurred. The purposes of this research are to investigate the effectiveness of non-identical dual suppliers in managing disruptive supply events and to explore the possibility of correlation within the suppliers' operation. From a practitioner perceptive, this research can provide some alternative views of day-to-day operation in managing supply disruption that may be of value to the industry.

2. Literature Review

2.1 Introduction

Today, the management of global supply chain has become more complicated with an increase in supply disruptions risks, in which the later have been recognised as a part of main reasons for losses, financially and operationally in supply chain networks (Chopra and Sodhi, 2014; Handfield et al., 2007; Hendricks and Singhal, 2005; Debra et al., 2005). The adverse effects of supply disruptions have caused an emergence in research in the area of operational research/management science (OR/MS), especially under the Supply Chain Risk Management (SCRM) study. For this reason, it is believed that there is still potential research that can be done in this area, particularly in the context of “inventory model that is subject to supply disruptions”. This chapter presents a review of the literature on supply disruptions in global supply chain networks in general, and focuses on the modelling of inventory policies that is subject to supply disruptions.

Supply disruptions in global supply chain networks are a significant issue in the management of inventory as they can negatively affect the supply source reliability and the costs associated with the ordering supply process. Existing research on inventory models related to the topic of supply disruptions have been widely studied in various forms of supply chain network structures (i.e., number of echelons and flow of materials) and inventory system spectrum (i.e., finite and infinite production horizon planning and, constant and stochastic demand). Nonetheless, there are still a number of limitations within these existing inventory models in the OR/MS literatures due to several factors, such as the difficulties in defining the disruption process parameter in supply chain network structure. Therefore, the intent of this chapter is to find the gaps in the studies and discuss these limitations.

This chapter is organised into three parts. The first part presents related literatures in the SCRM from a managerial perspective that includes a description on how to discover and recover from supply chain disruptions in section 2.2, ways to mitigate supply chain disruption in section 2.3 and an overview on the supply risk in section 2.4. In the second part of this chapter, which is section 2.5, some related quantitative studies are presented that includes an overview of supply uncertainty in inventory model in section 2.5.1, a development of disruption processes design in the inventory models in section 2.5.2 and a modelling of mitigation strategies in inventory models that are subject to disruptions in section 2.5.3. Finally, in the third part of this chapter, a summary of the literature and research gaps in earlier studies are outlined in section 2.6 and an overview of the research model framework is presented in section 2.7 .

2.2 Supply Chain Disruption Discovery and Recovery

Handfield et al. (2007) defines disruptions in supply chain as major breakdowns in the production or distribution nodes in a supply chain. Supply chain disruptions also can be classified as unplanned and unanticipated events that disrupt the normal flow of goods and materials within a supply chain (Kleindorfer and Saad, 2005). Disruptions are a pervasive property and are also hardly avoidable and unpredictable (Yu and Qi, 2004). They can occur anywhere along the supply chain; at the inbound suppliers side, during manufacturing, inside the firm's facilities or at the outbound or demand side. Given the complexity and dynamic nature of supply chains, there are needs for firms to be able to recover from disruptions efficiently, to avoid huge losses in profit, to survive in the business world and, more importantly, to maintain the firms' reputation towards their customers (Chopra and Sodhi, 2014; Handfield et al., 2007; Craighead et al., 2007). Quick response to the disruptions are significantly important as one of the key aspects to reduce the impact of disruptions. Therefore, understanding the process of disruptions are crucial in SCRM. Handfield et al. (2007) propose two critical components to understand the process of disruptions in a risk planning, which are disruptions discovery and recovery.

Disruption discovery can be defined as a process to identify the occurrence of the disruptions in the firm's supply chain network (Handfield et al., 2007). This discovery disruption process is usually a flexible process which includes various steps and ways to discover the disruptions, depending on the planned mitigation strategies in the SCRM before the occurrence of the disruptions (Handfield et al., 2007). In most of the firms' SCRM, disruption discovery is an on-going process, involving a team of crisis action plan that is equipped with some computerised systems to monitor risks of any in-coming disruption at any point of the firm's supply chain nodes and at all times. Disruption recovery is also a flexible process, depending on mitigation strategies planned after the disruptions have occurred, which aids the firms in recovering from the disruptions (Hishamuddin et al., 2014; Hishamuddin, 2013; Losada et al., 2010; Handfield et al., 2007). The objective of recovery is to bring back the operation to a complete or near normal status of regular operations.

Normally, most firms that have the SCRM program in place will have a systematic process to identify in-coming disruptions into its supply chain network and plans and strategies to recover from the disruptions. Description on systematic process and strategies of disruption mitigation plan in the SCRM will be explained in detail in section 2.3. In this section, the discussion will be focusing on the process of disruption discovery and recovery in the firms' supply chain. In what follows, the scenario of disruptions discovery and recovery when dealing with disruption events especially for firms that the sources from offshore suppliers is described.

2.2.1 A Concept of Discovery and Recovery

There are two critical components in a risk planning strategy which are *the ability to discover that a disruption has occurred* and *the ability to establish plans to effectively recover from the disruption* (Handfield et al., 2007). If we look at the event of disruptions in the firm's supply chain network, from the moment the firm has discovered the disruptions, other than detecting the sources of disruptions, it is important for the firm to recognise recovery speed when responding to the disruptions. The identification of the recovery speed is dependent on the information gained from the sources of disruptions and the actions taken to recover from the disruption. Fast and efficient recovery will result in reducing the loss of disaster and speeding up the recovery process. In addition, the assessment from recovery provides vital information as to when and to what degree the firm can be restored to a normal status. Each and every firm that faces disruption is required to and will go through a certain recovery phase after the discovery disruption stage. There is a specified duration allocated for a recovery process and this recovery process will usually be conducted in a phase approach (Hishamuddin, 2013; Chen et al., 2009; Allen and Toder, 2004). From the disruption recovery managerial perspective, there are various ways to identify the recovery phases. Chen et al. (2009) propose four recovery phases which include information acquisition, effective response, focusing and dealing and fast recovery. Allen and Toder (2004) also propose four recovery phases which are pre-disaster, post-incident, re-building and ongoing recovery.

In what following, for a better understanding, we bring some analogy which is based on study by Handfield et al. (2007) in discussing the importance to discover and recover from the disruptions. This analogy is based on the supply process in a simple supply chain between a firm and two suppliers, as illustrated in figure 2.1. From figure 2.1, the firm's production process is represented by two states of the firm's operation, namely, routine and crisis operations, and the supply process is represented by the blue and red lines. During the routine operation, the firm receives the supply on a routine basis from the suppliers. At this stage, the firm's suppliers are always reliable and capable of delivering orders on a timely basis. The blue line shows the supplies made by this firm. However, during the crisis operation (i.e., when the disruption has occurred), the firm presumably never receive the supply from the suppliers which refer to the red line.

There are two disruption discovery recovery events namely event A and event B. We present event A with a curve of blue area and event B with a curve of black dash line. We assume that event A represents the disruptive event at supplier 1 and event B at supplier 2. The firm has an information on the supply disruption for event A, but not for event B.

When the disruption occurs at the supplier's side, the gained information has speed up the discovery process in event A, thus, the discovery process in event A is faster than event B. Since discovery process in the event A is faster than event B, the recovery process at supplier 1 is most probably faster than the recovery at supplier 2 and eventually will reduce the impact of disruption of the firm.

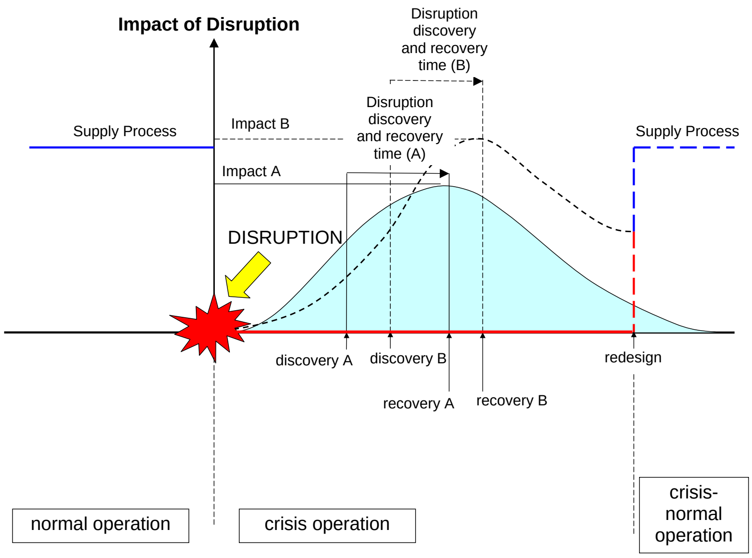


Figure 2.1. Comparison of Disruption Discovery and Recovery Events of the event A and the event B (Modification from Handfield et. al. (2007))

To exemplify the above situation, we refer the event A and the event B to a real case in 2000; a fire at Phillips semiconductor plant in Albuquerque, New Mexico, a semiconductor supplier for companies of Nokia and Ericsson (Schmitt et al., 2015, 2010). Based on the information from the supplier, there were an anticipated one week delay in shipment. Nokia was not satisfied with the information and has sent a team to investigate the disruptive event at the supplier's plant. The team has discovered that the disruption was even far worst than they expected. Therefore, the management of Nokia decided to order the part from another supplier as another mitigation strategy in the recovery process. On the contrary, Ericsson simply do nothing. Eventually, the delay was longer than expected, resulting in Ericsson reported a long-term losses of $2.34 billion and finally withdrew from cell phone market, while Nokia increased their market share by 3% in the first 6 months after the fire (Schmitt et al., 2015).

Based on this "Nokia and Ericsson" real story, depending on the quality of the supply disruption information and how the firm has wisely utilised the information to mitigate the disruption, there will be different outcomes, as a result of different strategies. Because of Nokia proactive action to mitigate the incident, they have speed-up the recovery process as opposed to the *wait-and-see* action by the Ericsson, which has caused this company to withdraw from the market. We believe that disruption information is one of the main elements to discover the disruptions and measuring the recovery process can be useful for a firm in planning the mitigation strategic decisions. Realising the importance of understanding the disruptions discovery and recovery process, these two elements will be the main focus in developing the research framework of this thesis.

2.3 Managing Supply Chain Disruption

Even though disruptions in supply chain are inevitable, we can hedge against them by designing strategies to detect the disruption risks and reduce their negative consequences (Chopra and Sodhi, 2014; Kleindorfer and Saad, 2005). The SCRM becomes popular among the top management after they discovered that the conditions of the worst state of several disasters (i.e., Hurricane Katrina and 11/9) and the rampant increase in commodity prices (Handfield et al., 2007). The agenda of SCRM is typically concerned with maintaining businesses continuity with the objectives of to minimise the disruption risks and reduce the likelihood of disruptions, which is similar to the Business Continuity Plan (BCP) agenda. However, the integration within the supply chain nodes when responding to the disruptions is an advantage of using SCRM strategy. There are three main results that can be accomplished with SCRM which are an achievement of high level information sharing on the disruptions, co-operation organisational relationships during the disruptive events and effective business processes in the disruption recovery process (Sodhi and Tang, 2009; Handfield et al., 2007).

The common way in conducting studies under the SCRM area are based on the topology of supply chain network which refer to the sources of risks that can trigger the disruptions in the network as shown in figure 2.2.

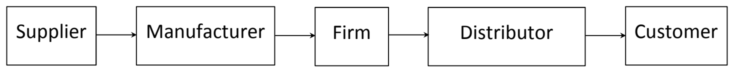


Figure 2.2. A simple supply chain topology

Based on figure 2.2, the source of risks have been studied either at the single node or within two or more nodes or at all nodes of the chain (Wagner and Bode, 2008; Chopra and Sodhi, 2004). Some examples of risks that have been classified at each node are demand risk at the customer node, supply risk at the supplier node and process, production and quality risks at the manufacturer nodes. Nonetheless, some of studies in the areas of organisational behaviour and management of the supply chain are also have investigated the risks in supply chain as a whole as being studied by Trkman and McCormack (2009). According to Peidro et al. (2009), each risk can have very different implications on supply chain system. Operationally, in fact, the strategy to deal with risk at one node is in many cases, very different from the other nodes. Therefore, the risks at each node in the supply chain topology should be treated independently. For example, a strategy to deal with a demand risk is totally different from a strategy to deal with a supply risk, unless the objective of the study is to emphasis on the integration between these two risks. In addition, the study on integrating demand and supply uncertainty in several studies proved to be too complex in some systems such as inventory policies. Bush and Cooper (1988) and Buxey (1993) discovered that firms that are experiencing these conditions do not have any formal strategic mechanism to mitigate them simultaneously. Nevertheless, we do believe that research on one node in supply chain network is still significant and has a potential development.

There is a rich body of literature on how to reduce the disruption impacts theoretically and operationally. The process in the SCRM in mitigating the disruptions includes measurement of the risks of critical nodes in the network, identification of risk reduction mechanisms for high-risk nodes, investment on the improvement of the visibility systems and re-engineering major supply chain design (Handfield et al., 2007; Craighead et al., 2007; Kleindorfer and Saad, 2005). Sheffi (2005) recommends four steps to mitigate the disruptions in the SCRM which are creating awareness, prevention, response management and consequence management. Feng et al. (2013) proposed a framework for disruption risks in supply chain which includes disruption risks identification, disruption risk assessment, disruption risks management decisions and implementation and disruption risks monitoring. For an extensive reading on the perspective of SCRM, we refer to Chopra and Sodhi (2014), Sodhi et al. (2012), Handfield et al. (2007) and Craighead et al. (2007).

In this research, we will focus on the issue at the supply node that addresses the risk of supply disruptions in the inventory system. Looking at the real worst effect of supply disruptions that have been explained in Chapter 1, it has motivated us to study the topic of supply uncertainty in the area of inventory management. In the following, ways and strategies to mitigate disruptions are presented.

2.3.1 Disruption Mitigation Strategies

Firms that exposed to the supply disruptions should incorporate corporate level strategies and specific risk management operational practices to create an environment that is resilient to any potential supply disruption. According to Sheffi (2005), supply chain resiliency is a condition of the disrupted organisations or processes to have an ability to recover from the disruption or to return to normal level of supply chain's operation. Contingency strategy or reactive strategy are needed partially or completely in the event of the disruptions (Shao and Dong, 2012). Those strategies can either be implemented only in the event of disruption occurs (i.e., reactive strategy) or some actions are taken in advance of a disruption (i.e., contingency strategy).

The resiliency of supply chain against supply disruptions can be built through redundancy and flexibility in supply disruption mitigation strategies (Xu et al., 2015; Zsidisin and

Wagner, 2010; Handfield et al., 2007; Sheffi, 2005; Christopher and Peck, 2004). The strategy of *redundancy* is a reactive plan to reduce or mitigate the adverse impact of supply disruption through the supply source contingency (Shao and Dong, 2012; Zsidisin and Wagner, 2010). On the other hand, the strategy of *flexibility* is a proactive plan or we can call it a contingency tactics that involves activities of organisational and inter-organisational development to detect disruption threats in supply chain and react to them quickly (Xu et al., 2015; Tang and Tomlin, 2008; Handfield et al., 2007; Sheffi and Rice, 2005). In what to follows, we present the related literatures discussing the reactive and proactive plans.

Reactive plan

Reactive plan are immediate actions taken once the disruptions have occurred. Normally, the actions are based on the well-planned mitigation strategies in the SCRM procedures that includes the planning both in the inventory management and supply management. Examples of the reactive plan at the operational level are having safety stock (Shao and Dong, 2012; Sodhi et al., 2012; Snyder et al., 2010; Vakharia and Yenipazarli, 2009; Tomlin, 2009) and operating at an alternative (low or high) capacity utilisation rate. Safety stock is the extra stock held in the inventory to protect against any possible supply disruption. Example at the strategic level is to use a backup sourcing (Su and Liu, 2015; Sawik, 2014b; Yan et al., 2014; Fang et al., 2013; Sodhi et al., 2012; Vakharia and Yenipazarli, 2009; Craighead et al., 2007) where it can be either source from any dedicated suppliers that are available (i.e., there is a contract between the buying firms and the suppliers) or the third-party source (i.e., suppliers that do not have contract with the firms)(Snyder, 2014; Tomlin, 2006). Other reactive plan is an acceptance where the firms will do nothing after deciding that the high cost of recovering from the disruption (Snyder, 2014; Tang and Tomlin, 2008; Tomlin, 2006).

Proactive plan

Proactive plan refers to organisational actions designed to minimise the impact of the disruptions which the firm takes some actions before the occurrence of the disruptions. Proactive plan which considers as a flexibility strategy can be implemented in five aspects which are control system, supply and procurement, distribution and customer-side activities, conversion and corporate culture (Vakharia and Yenipazarli, 2009; Sheffi and Rice, 2005). There is a cost of action incurred in this proactive plan regardless of whether a disruption occurs. Several operational ways to mitigate disruptions with the proactive plan have been well discussed in the literatures as follows:

Supply diversification

Supply diversification refers to firms having more than one suppliers and use this multiple supply sources on an ongoing basis (Gurnani et al., 2014; Whitney et al., 2014; Schmitt and Tomlin, 2012). The objective of this strategy is to avoid the reliance on a single supplier (Sheffi and Rice, 2005). If a primary supplier is unable to deliver a timely order (due to disruptions), then the firms always can divert the order to secondary suppliers or third party sources for a backup supply. We can see the implementation of supplier diversification from the topology of firms' sourcing in supply chain. In this situation, firms can either have two or multiple supplier (Su and Liu, 2015; Sawik, 2014b; Tang et al., 2014; Fang et al., 2013; Allon and Van Mieghem, 2010; Burke et al., 2009; Handfield et al., 2007; Bozarth et al., 1998). The process of supplier diversification includes the selection of potential suppliers from a set of candidates and the allocation of order among the selected suppliers (Sawik, 2014a, 2013; Zhou et al., 2011; Zeng et al., 2005).Usually, costs, lead-times and reliability of those suppliers are the main criteria in selecting the suppliers as a a backup supply (Boute and Van Mieghem, 2014; Gurnani et al., 2014; Tomlin, 2006).

Capacity flexibility

Capacity flexibility is a way for a firm to have reliable suppliers that can offer additional quantity to order prior to or after disruptions (Xu et al., 2015; Hu et al., 2013). This strategy is closely related to the supply diversification strategy. If the risk of disruptions at an unreliable supplier is too high, the reliable suppliers are preferable to place the order. One would expect the reliable suppliers will always have an extra capacity beyond their normal capacity. At some point, the reliable suppliers will have a limited space capacity (Schmitt and Tomlin, 2012). Therefore, a firm will offer some incentive to the reliable suppliers to rebuild capacity as an investment for capacity restoration effort during the disruptions crisis (Xu et al., 2015; Hu et al., 2013; Kim et al., 2010). Most research on incentive investment for capacity restoration have been conducted in supplier-buyer contract and procurement study (i.e., Su and Liu (2015); Xu et al. (2015); Sting and Huchzermeier (2010)).

Information visibility

Information visibility is a way of an organisational knowledge of external threats that come from the supply risks. This strategy is important for the firms to have a capability to sense and respond quickly to supply disruption threats (Zsidisin and Wagner, 2010). The activities included in this strategy are development of supply risk database, specific system for the firms to monitor the suppliers' performance and platform for information sharing between buying firms and suppliers (Atasoy et al., 2012). Zsidisin and Wagner (2010) revealed that the biggest benefit to create resiliency with visibility technique appears to come from the risk of offshoring the supply. A continuous engagement between firms and suppliers in exchanging information can uncover risks that may exist in the process of supply delivery from far-off locations, while creating a database on suppliers' risks. According to Handfield et al. (2007), some firms invested in the supply monitoring system like Vendor Managed Inventory (VMI) where this system allows the supplier to monitor the firm's inventory and at the same time the firm is able to monitor the status of the supplier. Other system is a warning system that have been studied to detect an in-coming disruptions in the supply chain network (Fei and Wang, 2008; Qinghua et al., 2008). Having a good quality information is also important to a firm. A conceptual foundation of an accurate assumption for improving the effect of information possession in most analytical models is based on full- or partial-information type of data (Chen, 2014; Zhao et al., 2014; Chen et al., 2010).

Agility

Agility is a way to react quickly to change in business processes due to disruptions. It can minimise the uncertainty of a negative impacts from the disruptions with a power of surplus value of monetary allocation (Mercier et al., 2010). This strategy is related to the strategies in reactive plans, supply diversification and information visibility. It could be a strategy to avoid or minimise the disruption impacts depending on the degree of importance of the exposed area, but the additional costs planning in hand will be a main criterion in the decision making process (Mercier et al., 2010). For example, “Should I buy component A from supplier Y if supplier X cannot deliver the order?” or “How much inventory should I keep for component B if my supplier can not deliver in a timely order?”. To speed up the recovery disruptions process, response time and magnitude play an important role that includes detection time, coordination time and response magnitude (Tomlin et al., 2012; Tomlin, 2006). According to Wang et al. (2010), “*the shorter the response time (how long) and the higher the response magnitude (how much), then the more protection the supply chain will be*”.

2.3.2 Conclusion

In this section we discussed several ways in mitigating the supply chain disruptions that cover the reactive and proactive plans. Based on these plans, with organisational mitigation procedures and strategies, the firm’s SCRM program will become more relevant when dealing with any type of risks that exist in the supply chain network. From a discussion in the previous section, we chose to investigate the risk at supplier’s side. In what follows, therefore, we will discuss about the risks in the supply process at the supplier node. Consequently, what to follow is the discussion on risks in the supply process at the supplier's node.

2.4 Supply Risk

This section describes various definitions of supply risk and the sources of the supply risk in the supply chain network. The descriptions as follows.

2.4.1 Definitions

Zsidisin (2003a) defines supply risk as a probability of an incident associated with inbound supply from individual supplier failures or the supply market occurring, resulting in the inability of the purchasing firm to meet customer demand or cause threats to customer life and safety. Another definition of supply risk is the probability of supply being affected because of problems at the supplier's end (Sarkar and Mohapatra, 2009). According to Zsidisin (2003a), supply risk can be classified as inbound and outbound. Inbound supply risk is identified as the potential occurrence of an incident associated with inbound supply from individual supplier failures or the supply market, resulting in the inability of the purchasing firm to meet customer demand (Wu et al., 2006). In contrast, outbound supply risk is associated with the disruptions beyond the suppliers activities such as market/price fluctuation, inflation and change in custom and immigration regulation (Zsidisin, 2003a).

2.4.2 Sources of the supply risks

Zsidisin and Wagner (2010) and Wagner and Bode (2006) classified the source of supply risk based on *"the result of a supply chain disruption that emerged from the supply-side risk source"*. The sources of supply risk comprise numerous events that affect the continuity of supply and resulting in operational, temporal or permanent termination of supplier-buyer relationships and, financially, in the increase (and decrease) of inventory costs-related (and profits). Handfield et al. (2007) provide a comprehensive list of different forms of supply risks that can exist in different environments which includes price, competitor, capacity, supply, technology, political and economic of a country, and regulatory policy. For an extensive reading on supply risk sources and definition, we refer to Zsidisin (2003a,b).

Example of supply risk threats in general are, financial stability of the supplier and the consequence of insolvency or bankruptcy (Wagner and Bode, 2006), capacity constraints and poor logistics performance due to unresolved problems in supplier's management (Sarkar and Mohapatra, 2009), changes in regulatory and trade policies at the country where the suppliers' plants are located (Ahmadi-Javid and Seddighi, 2013), inability of suppliers to adapt to technological change or to design product to satisfy customer requirements (Zsidisin et al., 2004) and a quality of product that does not follow the product specification delivered by the suppliers (Zhou and Johnson, 2014; Ellis et al., 2010). All these example of threats can be categorised in supply type risk of supplier, supply market and extended supply chain (Zsidisin and Wagner, 2010; Zsidisin, 2003a).

The first source of supply risk is the risk from the *supplier* which considers any negative event happened at or arisen from the current supplier portfolio of the buying firms (Zsidisin and Wagner, 2010; Tomlin, 2006). The negative events mainly on the interaction between the firms and the suppliers that includes the issue on the information sharing between these two organisation (i.e., machine breakdown and transportation problems), the suppliers' financial stability (i.e., bankruptcy) and powerfulness of suppliers' management (i.e., suppliers' inventory management). Second source of supply risk comes from the *supply market* which considers the issues beyond the scope of a single supplier or buyer-supplier relationship (Zsidisin and Wagner, 2010) which includes the environment of supply chain structure where that firms and suppliers are compete in (i.e., oligopolies and monopolies), the number of potential suppliers and the availability of suppliers' capacity (i.e., the suppliers have a constraint in the amount of order that they can deliver). The third source of risk is the risk from *extended supply chain* related to the structure of global supply chain. This type of risk exists when the buying firms get its sources/services far from their locations. A long distance of lead-time and a long pipeline in the supply chain can create uncertainty in the sourcing process (i.e., the suppliers fail to deliver a timely order)(Schmitt and Tomlin, 2012).

2.4.3 Conclusion

In this section, we discussed in detail the supply risk covering the definition, the sources and the impacts of this risk in various products and services sectors. The studies about supply risk in the SCRM have been studied in various research topics such as transportation/vehicle routing (Hosseini et al., 2014; Cai and Zhou, 2014; Ahmadi-Javid and Seddighi, 2013; Rosic, 2011; Wilson, 2007), procurement and contract (Tang and Kouvelis, 2014; Wakolbinger and Cruz, 2011; Kim et al., 2010) and disaster management/plant allocation (Balcik and Beamon, 2008). Other than these areas, supply risk issue also has been studied in the area of inventory management. In what to follows, we present the literatures that relate to inventory management and supply risk.

2.5 Supply Risks in the Inventory Model

Effective inventory management systems help to ensure the exact numbers of supplies/ products are available in the right place and at the right time. Basic inventory management aims to minimise the cost associated with maintaining inventory and meeting customer demand. The crucial decisions involved in managing inventory systems based on when should an order be placed for a product and how large should each order be (Silver et al., 1998). Information that may influence these decisions include inventory cost, the lead-time (i.e., time between order placement and order delivery), single item or multi-item system and single or multi-location supply chain structure. The outputs and structures of decision-making models in the inventory management can be classified in various dimension such as replenishment systems (e.g., EOQ, EPQ and (s,Q) system), costs structure (e.g., fixed, variable, holding and penalty costs), decision variables (e.g., service level, reorder point, order quantity and order-up-to level), demand utilisation (e.g., backorder or lost-sales) and a production horizon plan (e.g., periodic or continuous review) (Snyder et al., 2010; Silver et al., 1998).

Most studies on inventory models under the topic of supply chain management have been conducted with an assumption that the inventory system is operated in a stationary environment. In fact, from an economic perspective, inventory systems are sensitive to the fluctuation in the business markets (i.e., financial, social and political) which can affect demand, supply and all costs (Arifoglu and Özekici, 2011). For example, during supply disruptions, most probably there will be a stockout/overstock in inventory that can cause a high penalty or holding costs, lost-sale for unfilled demand and definitely there will be no replenishment orders if suppliers are “down”. Therefore, the firms and suppliers must have the capability to access or to provide the required timely information/knowledge on the status of inventory for better mitigation decision support in the event of supply disruptions (Micheli et al., 2014; Zhang et al., 2011).

Inventory management is one of the key aspect of the SCRM (Caballini and Revetria, 2008). Like other inventory models, the objective of models with disruption is to find an optimal replenishment policy. Disruption mitigation strategies that will be implemented in the inventory management must be deployed pro-actively, since most models are assumed replenishment orders cannot be placed during a disruption. In the existing inventory related literatures, some research have studied ways to model the strategies in mitigating the supply disruptions. The common ways for researchers/ practitioners in modelling these strategies are by defining how to measure the disruption processes and making plans to re-engineer the optimal policies based on the measurement of the disruption processes.

In what to follows, literature on supply uncertainty in inventory models is presented in section 2.5.1. Consequently, related studies on how to measure the supply disruption processes are presented in section 2.5.2. Finally, related literatures on supply disruption mitigation strategies in inventory model are discussed in section 2.5.3.

2.5.1 Form of Supply Uncertainty

Most existing studies in modelling inventory model have categorised supply randomness in two forms of supply uncertainty which are *yield uncertainty* and *supply disruption*. Yield uncertainty is a continuous-event uncertainty. Usually, research on the yield uncertainty models a form of supply uncertainty where quantity produced and received differs from the quantity ordered in a random way. The disruption caused by the yield uncertainty in business operation usually occurs more frequently or at all times. The other category of supply disruption is a discrete-event uncertainty. The supply disruption is a unique event and typically it occurs rarely and tend to be temporary (Tomlin, 2006). Generally, the impacts of supply disruption can cause a massive change to the business operation compared to the yield uncertainty which is less severe (Schmitt and Snyder, 2012). If we form these two supply uncertainties as a probability distributions based on the uncertain event, the yield uncertainty distribution is a Binomial distribution experiment with a probability of supply mode status is either Up or Down in n-trials, while supply disruption is a Bernoulli distribution experiment which can refer to as one of n-trials in Binomial distribution experiment.

Yield Uncertainty Studies

Studies on the yield uncertainty problems in inventory models are believed to have been investigated after 1950s, with an observation of the quantity received is differ from the quantity ordered (Karlin1958). After late 1970s, there are quite a number of studies in inventory modelling that includes the randomness in supply (Arifoglu and Özekici, 2011). For further discussion and extensive review on the topic of yield uncertainty, we refer to Grosfeld-Nir and Gerchak (2004) and Yano and Lee (1995). In what to follow, other studies that related to yield uncertainty problems are presented. Study by Parlar et al. (1995) in the continuous review (Q,r) system showed that a base-stock policy is optimal under Markovian supply availability with random demand and lead time. Gupta (1996) examines a (Q,r) system with Poisson-distributed demand where a supplier is subject to disruptions and lost sales. He reveals the lead times can be the dominant factor in setting optimal values. Parlar (1997) demonstrates the increase in cost from using solutions that ignore disruptions in (Q,r) system with disrupted supply, random demand and random lead times. With (s, S) inventory policy setting, Arreola-Risa and DeCroix (1998) consider a stochastic inventory system in which supply could be randomly disrupted and the disruptions lasts a random period. They assume all unmet demand is 'partial backorders' where some customer orders may wait as backorders while other orders become lost sales. They proposed a modified (s, S) inventory policy where when the inventory level is at or below s and the supply is available, procure the necessary amount to bring the inventory level up to S. Later literatures studied about yield uncertainty are Gong et al. (2014) and Schmitt and Snyder (2012).

Supply Disruption Studies

Studies on supply disruption problem in the inventory models started later. The study by Parlar and Berkin (1991) is believed to be among the earliest studies that incorporate supply disruptions into the inventory model. The paper analysed the supply uncertainty in an EOQ model and studies the pattern of supply availability during an interval of a random length (i.e., supply either available or unavailable). They used renewal reward theorem and construct an average cost objective function and find the optimal value of the order quantity. The model assumed decision maker knows the availability status of supplier at anytime and retailer follows a zero-inventory ordering policy. Berk and Arreola-Risa (1994) noticed an incorrect cost function in Parlar and Berkin (1991) model and provided a correct one. Parlar and Perry (1996) presented multi-supplier case and relax Parlar and Berkin (1991)model's assumptions. Their model deals with the case where decision maker is not aware of ON-OFF status of supply and allow non-zero reorder points. They have also included fixed cost and both deterministic and random yield of supplier in the model. Snyder (2014) also extended Parlar and Berkin (1991) model and introduced approximation to the model where it can be solved in closed form while Qi et al. (2004) extended the model to include disruptions at both supplier and retailer. Both aforementioned studies considered multi-supplier scenario.

Gürler and Parlar (1997) considered a deterministic demand with two suppliers where both of them could experience disruptions with ON and OFF periods status. The ON periods have an Erlang distribution while the OFF periods follow a general distribution. But, Li et al. (2004) modeled these two suppliers in a continuous demand setup. They investigated supply disruption where the availability of supply is modelled as a renewal process and general distributions are used to model the durations of availability periods. Papers related to newsboy model; Güllü et al. (1997) relate the optimal base-stock level to the newsboy problem by examining dynamic deterministic demand over infinite-horizon and non-stationary disruption probabilities and Dada et al. (2007) extended the stochastic-demand newsboy model to include three unreliable suppliers. Mohebbi and Hao (2008a, 2006) and Mohebbi (2003) studied random lead-time problems in the event of supply disruptions.

Yield Uncertainty and Supply Disruption Studies

There are several studies that have attempted to combine yield uncertainty and supply disruption. Chopra et al. (2007) analysed the costs involved in bundling the variance from these two different sources in a single-period setting. They emphasised that it is important to identify and analyse the types of stochasticity in supply correctly. Schmitt et al. (2015) consider a system with both yield uncertainty and supply disruptions and extend the analysis to an infinite-horizon setting. They demonstrate the importance of considering the long-term impact of disruptions through the multiple-period analysis. Xanthopoulos et al. (2012) also consider yield uncertainty and supply disruption in their studies. They studied a single-period model under the conditions of two suppliers are unreliable and used convex optimisation technique to analyse the model. Assumptions that have been made in the event of a disruption are the following. Only a proportion of the total batch quantity ordered is delivered on time (within the selling period). Demand is positive stochastic random variable with probability density function and surplus stock at the end of time period is sold by a discount price or sold to secondary market at a salvage price.

Wang and Tomlin (2006) did not choose any of these supply disruption types. They consider their paper to be classified in stochastic lead-time category. Stochastic lead time category or stochastically-proportional Bernoulli random-yield model for supply failure is appropriate when decision maker's perspective is less sharp than the distinction between disruptions and random yield. Typically, stochastic leads times have been studied in recurring demand setting rather than in a newsvendor setting. For this, we refer to Zipkin (2000) for a comprehensive discussion of stochastic lead times in the context of recurring demand under the inventory model study.

To conclude this section, we have discussed several types of supply uncertainty which are yield uncertainty and supply disruption. Therefore, the focus of this thesis will be on the supply disruption category due to the fact that this category is a rare event with low probability of occurrence but has high negative impacts to the firms' supply chain network. In the study of supply disruption, the common way to measure the disruption processes is by looking at the disruption parameter measurement. Therefore in the next section, we present related literatures that study the supply disruption problems and describe how the disruption processes have been measured in these studies.

2.5.2 Measuring Supply Disruption Processes

In most related inventory-supply disruptions studies, common ways to model the disruptions are by looking at the disruption process spectrum and the outcome of the disruption events. Theoretically, Zsidisin and Wagner (2010) proposed on how to measure the disruption by looking at the inventory operational performance such as capacity utilisation, order fill rate and the costs associated with the disruptions (e.g., stock price valuation and additional inventory backup cost). As oppose to the study by Vakharia and Yenipazarli (2009), they evaluated risk based on the disruption process while looking at the outcome of risks. The suggested measurement of disruption processes from the literatures are summarised in figure 2.3.

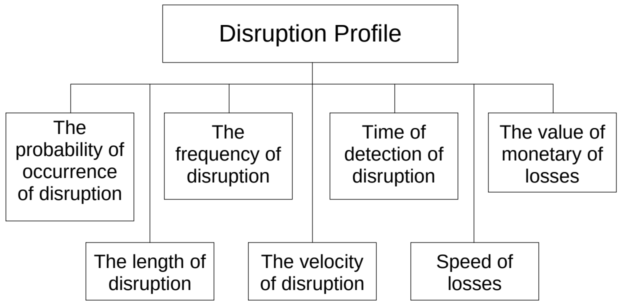


Figure 2.3. The measurement of disruption processes

Figure 2.3 shows the measurement of the disruption processes include that the probability of the occurrence of disruption, time of detection of disruption, the frequency of the disruption, the velocity of disruption and the length of disruption. Sometimes, the supply risks can also be measured by the monetary amount of losses and the speed of losses. In the OR/MS literatures, most studies discussed the sources and characteristic of disruptions in a wide range, so the supply disruptions processes are easy to model. Most researchers chose to combine the expected disruption length and the frequency of disruption; *infrequent-but long* spectrum and *frequent-but short* spectrum. For example, Schmitt (2008) and Tomlin (2006) considered two types of disruption outcomes which are the frequency and duration of disruptions to measure the performance of the suppliers and the impacts of the disruptions.

Disruptions occur usually under an uncertain and volatile environment and decision making under uncertain conditions require a way of identifying a component that can represents the stochastic parameters (Su and Liu, 2015). According to Lee and M.Chang (2007), the stochastic process is a common way when designing the measurement of supply disruption processes in some disruption-inventory models. Example of this are Tomlin (2006) and Saghafian and Van Oyen (2012). Tomlin (2006) chooses the expected disruption length to design its supply disruption parameter. He measured supplier U reliability by the percentage of time that supplier U is up, i.e $\pi(0)$. A given percentage uptime can result from frequent but short disruptions or from rare but long disruptions. He made an assumption which a disruption length is the sum of a constant and a geometric random variable. The assumption made is, a disruption lasts for a minimum of M periods, after which there is a constant probability λ_{du} of the disruption ending in each period. Saghafian and Van Oyen (2012) modeled the supplier's threat level based on the current risk at the supplier side as a Discrete Time Markov Chain (DTMC). Their model followed an example of the S&P credit risk rating system with states of an analogous system that commonly used (i.e., $\{1 = AAA, 2 = AA, 3 = A, 4 = BBB, 5 = B/BB, 6 = CCC/CC/C\} \cup \{0 = Default\}$) where their Markov model is a step forward from independent and identically distributed (i.i.d) Bernoulli disruptions. Snyder et al. (2010) make the assumptions on the disruption process which are exponentially distributed in the continuous-time case or geometrically distributed in the discrete-time case.

Another common way to translate the supply disruption processes is based on the availability of the suppliers. Usually, the disruption processes at the supplier have been designed as an availability of supplier with two-state Markov chain where the supplier is either is at Up or Down state (Silbermayr and Minner, 2014; Snyder et al., 2010; Mohebbi and Hao, 2008a,b, 2006; Tomlin, 2006; Mohebbi, 2003). The up and down states may have the same steady-state probability of being disrupted, but they may have very different disruption profile too. There are two assumptions made in these studies which are that the disruption process is exponentially distributed in the continuous-time case and geometrically distributed in the discrete-time case.

Other way to design the measurement of supply disruption processes is by using a timeline framework. Sodhi and Tang (2009) proposed a conceptual model in managing supply disruption via a time-based risk management. The time-based risk management concept focuses on time instead of cost, probabilities or impact. They consider three elements of time namely *detect* the disruption event ($D1$), *design* a response ($D2$) and *deploy* the response ($D3$) with the objectives to reduce the overall *response* time ($R1$) and thus *recovery* time ($R2$) and total impact. To cut short the response time based on the formula of $R1 = D1 + D2 + D3$ is the focus of this model which can reduce the recovery time, $R2$ after the event has occurred.

For the conclusion of this section, we discussed studies that have been conducted to design and measure the supply disruption processes. Designing the measurement processes is one of the important steps in the process of measuring the supply disruption mitigation strategies. Normally, the measurement is used as a benchmark or tool to quantify the performance of the mitigation strategies. In the next section, we present some literatures that related to the supply disruption mitigation strategies.

2.5.3 Modelling Supply Disruption Mitigation Strategies

This section discusses related inventory-supply disruptions literature which focused on the implementation of strategies in mitigating the supply disruptions. Most strategies that being presented in this section are based on the operational supply chain mitigation strategies that have been discussed in section 2.3.1.

Tomlin (2006) studied ways to mitigate supply disruptions from various schemes of disruption mitigation strategies which are acceptance, inventory and sourcing mitigation, contingent rerouting and combination of inventory mitigation and contingent rerouting. He modeled a firm's inventory system that operates under an infinite-horizon plan and periodic-review replenishment system with complete backorders of unmet demand under a simple supply chain structure of a two suppliers-single product setting. The firm has two types of suppliers namely supplier R which is is reliable but expensive with a capacity flexibility and supplier U which is unreliable but cheaper. The state of supplier R is either up or down in a period with an *all-or-nothing* source). The uncertain state of supplier U is modelled as two-state discrete Markov chain and the expected disruption length is assumed to follow a geometric probability distribution. The objective of this study is to find policy that optimal under the crisis events which focuses on the ordering policy and the base-stock level when supplier U is up. Findings of this study are the following. Tomlin believed that the reliable supplier's percentage uptime and the nature of the disruptions (rare but long versus frequent but short) are important factors in finding the optimal strategy. Based on the given percentage uptime, he discovered that sourcing mitigation is a better strategy as opposed to the inventory strategy when the disruptions become longer and less frequent. In addition, the mitigation strategy is in favour over the contingent rerouting in the event of rare disruptions. From the result of the mixed mitigation strategies of carrying extra stock in the inventory and get source partially from the reliable supplier, he showed that this strategy is optimal if the unreliable supplier has a finite capacity. If the reliable supplier has a high capacity flexibility, the contingent rerouting is the best strategy.

Wang et al. (2010) studied the supply reliability problems by investigating two types of mitigation strategies in the effort to increase supply reliability. This dual sourcing strategy is the main focus of this study. They examined random capacity and random yields models under a single-product newsvendor with unreliable suppliers in a single selling session. The aim of these models are to propose a specific strategy that in favour to the firm's particular disruption conditions and to deploy both strategies simultaneously when needed. To model the supply reliability problems, a two-stage stochastic program is used to determine the order quantity as a decision variables by looking at the success or failure of the improvement efforts at the first stage and at the second stage, the firm objective is to maximise its expected profit by determining the distribution functions for the capacity loss with an information about the status of suppliers. In the random capacity model, a decrease in the supplier cost reduces the effectiveness of dual sourcing but this dual sourcing is in favour with an increase in supply reliability heterogeneity. For random yield problem, supply reliability improvement is in favour with an increase in the reliability heterogeneity but decreases with an increase of cost heterogeneity.

Saghafian and Van Oyen (2012) analysed two types of mitigation strategies which are contracting with secondary flexible supplier and monitoring the primary supplier to monitor the risk of supply disruptions with an objective to minimise the total inventory cost. They modeled a firm that operates with a periodic inventory review where unmet demand is lost in a single-period and infinite-horizon settings. The unit cost of item from flexible supplier is higher than the unit cost from unreliable supplier. The disruption risk is assumed to be a level of threat (dynamic process) and has been modelled as a discrete-time Markov chain and Partially Observable Markov Decision Process (POMDP) is used to develop this model. From this model, they investigate the effectiveness of investing in the secondary supplier, having a recourse option and obtaining information (i.e. full or partial) on the risk of disruptions at the primary supplier. They discussed about the possibility of overspending on the secondary flexible supplier in the event of disruption. From the results, comparing a single flexible supplier and two dedicated suppliers strategies, contract with the single flexible supplier is better but the optimal capacity investment is higher. They have also discovered that, ordering to the flexible supplier is more attractive if the firm does not have information on the risk of disruption at the primary suppliers.

Jakśić and Rusjan (2009) proposed inventory control with Advance Capacity Information (ACI). They focus on the future of uncertain supply capacity information to improve the management of inventory and reduce inventory cost-related. Heuristic procedure is used to build practical and reasonable inventory policies with an objective to find the optimal policy that minimises the relevant inventory costs. In a single-period newsvendor model setting, the models are analysed based on the assumptions of unmet demand is backlogged, the fixed cost is zero and demand and supply processes are stochastic non-stationary with known distributions in each time period. The important parameter in this model is the length of the ACI horizon which represents how far in advance the available supply capacity information in revealed.

Chen et al. (2010) studied the strategies of dual sourcing and inventory management. They focus on the problem in the presence of disruptions information which can either be asymmetry or imperfection in finding an optimal sourcing strategy. They modeled the firm's inventory system that operates with two suppliers; one is unreliable but offers lower product price and the other one is reliable but offers higher price. A Bayesian model with Dirichlet prior distributions is used to model a dynamic update of supply risks knowledge to achieve mathematical tractability in Bayesian updating in a single and multiple periods analyses. From the mathematical proof, they show that single sourcing from the reliable supplier is possible in the multiple period model as opposed to the single period model which just offer a strategy to source from a single unreliable supplier or dual sourcing from the both (un)reliable suppliers. They analysed these models under three different situations namely perfect information, imperfect information and Bayesian learning. The finding results are as follows. From the total cost versus the number of periods, as the period numbers increase, the Bayesian update performed better as opposed to the imperfect initial information. From the result of the percentage errors of the three information situations against the unit order cost of the reliable supplier, the Bayesian model is in favour over the imperfect situations with an increase in this order cost. Finally, the outcome of the percentage error versus the shortage cost and holding cost analysis showed that the imperfect information and Bayesian learning are not preferable when the holding and shortage costs are relatively close. In contrast, the imperfect information is in favour than Bayesian information with an increase in the difference of holding and shortage costs. From these three information situations, Bayesian information approach appears to be a most effective in cost reduction and robust to imperfect initial information prior to the disruptions.

Hu et al. (2013) studied the effectiveness of two mitigation strategies in managing supply disruptions. They compared two strategies namely the restoration enhancement (RE) and supply diversification (SD). In the event of supply disruptions, in the RE model, they investigate the capability of the supplier to invest in capacity restoration with the incentive mechanisms from the firm and consider two conditions of incentive; incentive prior to and after the disruption. In contrast, the effectiveness of SD strategy is studied under the condition when the firm diversify the order to an expensive but reliable suppliers. They modeled the risk of disruption as a dichotomous parameter (i.e., all or nothing) with an assumption that there is no disruption with probability β. Other assumptions are the unit market price without disruption, p_1 is lower that price with disruption, p_2 (i.e. $p_1 \leq p_2$). In RE study, they discover that the firms and suppliers prefer the incentive prior to disruptions as opposed to incentive after disruptions. When comparing the RE and SD strategies, the RE is in favour over the SD strategy when the unreliable suppliers' status to operate is more predictable with a high restoration outcome.

For the conclusion of this section, we presented some literature that related to our research problem which will be the main references in developing our research model framework. There are three main issues that have been identified from these literatures which are a proper way to design the disruption parameters and performance (output) parameters, how to restructure the inventory policies in the event of the supply disruptions and a phenomenon of supplier dependency that may exist in those models. Based on these issues, we intend to develop mathematical models that cater these two elements when speaking about the supply disruptions which will be discussed in detail in the next section.

2.5.4 Conclusion

In this section, we presented some literature that related to inventory modelling studies that subject to supply disruptions. In presenting the inventory model in those literatures, at first, our discussion were focused on identifying ways of supply uncertainties being classified. Then, we reviewed how the disruptions processes have been designed so that it can be measured. Finally, we presented some related literatures and identified a number of issues that will be the references in developing our research framework. In what to follows, we will discuss and present the gaps within these literatures which related in developing the models.

2.6 Literature Summary and Gaps

Since the 9/11 attack, the research on the management of inventory when dealing with supply disruption have increased dramatically. Managing inventory when facing supply disruption in global supply chain network is becoming more important due to the rampant economics pressure and earth climate change that have caused the increases in operating costs and unpredictable natural disasters. The operational processes in the inventory system, such as the ordering process poses additional challenges due to the complexity of the supply chain network and too many characteristics of the suppliers that need to be considered when developing and proposing the optimal ordering policies. The disruption mitigation strategies in supply chain is widely being investigated, however comparatively little research has investigated the effectiveness those strategies that reviewed in this chapter. In this section, we summarised the reviewed literatures and the literature gaps are presented.

2.6.1 Literature Summary

In this chapter, we reviewed relevant literatures from two different perspective of managerial of supply disruptions and the mathematical studies that have investigated the issue on supply disruptions. A brief summary on these two perspective literatures are the following.

Supply Disruption Mitigation Strategies in Managing Inventory

The SCRM is used as a procedure or strategy in managing supply disruptions. The advantage of integrating all nodes in supply chain network when dealing with the disruptions makes the SCRM preferable to the BCP, thus most of companies and practitioners in the business market nowadays opt to the SCRM. With the SCRM, the mitigation strategies plan becomes more structured and well planned. In addition, this plan has assisted firms in their effort to reduce the disruption consequences and minimise all related costs. Several ways and approaches of mitigation strategies are recommended in this plan such as inventory management, supply diversification and flexibility, information visibility and agility. These proposed strategies are important for the firms that have off-shored their sources due to the complexness of the supply chain structures, the far distance of the sources and higher costs incurred in the supply processes. From all these strategies, we realised that the information on the disruption plays an important roles as one of requirement in re-designing the existing policies when facing the supply disruption. Since disruptions are typically unexpected adverse events, advance, accurate and quality information are important and should be a priority to be considered in mitigation strategies plan.

Supply Disruptions-Inventory Models

Before the concept of supply chain has been introduced in the business market, most of the studies in modelling the inventory systems that subject to the supply disruption focused more on the inventory problems at the operational level, such as the impact of supply disruption on the optimal replenishment policy in finding the performance of the inventory systems. For instance, the lead-time is used as a subject of problem in the event of disruption and how the changes in the delivery time can effects the performance of the fill-rate (DeCroix, 2013; Mohebbi and Hao, 2008a, 2006; Mohebbi, 2003). However, the dimension of research focus is becoming more complex and sophisticated after the supply chain concept was introduced in the business market. Therefore, there is a growing need to improve the current inventory policies. Other than the structure of the inventory system, the researchers and practitioners have to consider other factors that are vulnerable to most probably at all nodes in the supply chain network such as the source point of the disruptions in the chain and the differences in the characteristics of disruptions at each node. In addition, the problems that have being investigated not only cover the issues at the operational level, but also usually cover the problems at the strategic level. Those studies at the strategic level are focus on the development of organisational disruption mitigation policies. For instance, Saghafian and Van Oyen (2012) examine a surplus monetary investment in mitigating the supply disruption in the firm's supply chain.

Most of the mitigation policies that have been developed with mathematical models have addressed the supply disruption problems in the chain of processes, from the in-house production of the firm to the bigger picture of the firm's operation in the supply chain network. In addition, under any settings of inventory model with disruptions, the form of the optimal inventory policy most probably is unknown and a close-form method is usually in favour for the mathematical solution (Snyder et al., 2010). Various disruption mitigation strategies have been proposed in re-designing the inventory policies in the event of supply disruptions such as supply diversification, capacity flexibility and information visibility. For examples, there are studies that explore the advantages to have reliable information in the event of supply disruptions (Atasoy et al., 2012). Those strategies were integrated with the structure of the inventory system in the decision making process as the performance measurements to find the optimal inventory policies under the disruptive supply environment (**?**).

One of the popular mitigation strategies in the SCRM is a supply diversification strategy. This strategy enable the firm to have more than one supplier when placing the orders. The objectives of this strategy are to avoid a reliance on one supplier only and to have a back-up source in the event of supply disruptions. From a mathematical inventory model perspective, the researchers often prefer to choose a dual-suppliers setting in developing the research models. The outcome of the models analyses tend to compare the performance of single or dual suppliers in mitigating supply disruptions. The performance of the suppliers are usually measured by different settings of the suppliers' profile. The policies that have been proposed from the models are either single- or dual-sourcing policies.

From the literatures, we can see that the development of models with supply diversification strategy in supply disruptions-inventory studies required the researchers to identify potential suppliers from a set of candidates, depending on the nature of the firm's business and the topology of supply chain network. One of the criteria in selecting the potential suppliers is based on the profile of the suppliers (Zhou et al., 2011; Zeng et al., 2005). Several factors considered when checking the suppliers' profile which are various related costs (i.e., ordering cost and transportation cost), timing of products/services delivery, location of the suppliers, law and regulation at the location of suppliers' plant and the reliability of the suppliers when handling crisis situation (i.e. mitigation plans) (Burke et al., 2009). Based on the aforementioned selected potential suppliers profile group, we would say this group sometimes will consist of various type of suppliers or we can call it as *non-identical suppliers*.

The firms who off-shored their supplies usually have non-identical suppliers. The studies under the topic of global supply chain usually identified non-identical suppliers as suppliers who are usually some of them can offer low costs items/products but has long lead-times whereas the others provide quick response at higher costs (Su and Liu, 2015). Generally, the lower cost is often the main decision criterion for global sourcing (Su and Liu, 2015; Handfield et al., 2007) which the suppliers' location are far from the firms. But, the firms have also considered to have back-up suppliers or third-parties supplies near to its plants as an alternate supply when the cheaper supply flow is interrupted. From an operational perspective, managing global supplies with non-identical multiple-suppliers poses additional operational challenges, comparing to the firm who operates with identical multiple-suppliers (Su and Liu, 2015; Christopher et al., 2011; Tang and Tomlin, 2008; Christopher and Peck, 2004). The non-identical suppliers are usually located in more than one country and they are simultaneous members of a number of networks (Christopher et al., 2011). Since these non-identical suppliers are located in multiple geographical regions (Christopher and Peck, 2004), there are risks of disruptions, bankruptcies, breakdowns, macroeconomic and political changes (Manuj and Mentzer, 2008).

Studies that consider non-identical suppliers have widely being investigated under the environment of global supply chain. From the global supply chain studies perspective, we found that there are a number of limitations and gaps with the research in the supply disruptions-inventory modelling studies which will be discussed in detail in the next section.

2.6.2 Research Gaps

This thesis addresses the issue of the effectiveness of dual-sourcing strategy in mitigating supply disruptions by assuming that the firms get the supplies from two non-identical suppliers in global supply chain network. Throughout this thesis, we identified a number of studies that have implemented the supply diversification strategy under the research on the mathematical modelling of the supply disruption-inventory models and the global supply chain model. From these two areas of literature survey presented in the previous section, we discovered a number of limitation and gaps from conceptual and mathematical modelling perspectives.

From a conceptual perspective, we identified a little research have been done on the concept of disruption recovery in a mathematical approach. To-date, mathematical studies on disruptions discovery are seen more popular in mitigation strategies planning (i.e., warning system) as opposed to the disruption recovery. Unfortunately, in reality, those strategies can be very costly to the firms because there must be an allocation of money invested for an on-going disruption risks monitoring systems. The systems may reduce the disruption impacts, but the possibility of disruptions to occur are unknown with low probability and at some point the systems may deliver wrong signals. Because of this reason, we realised that there is a need for a more reactive strategies on the recovery process, which to be implemented immediately after disruptions in the process of recovery and to minimise the system costs associated with recovery.

We also identified one limitation in papers that studied the inventory policies subject to supply disruptions with the setting of dual or multiple suppliers. The models presented are based on the assumption that, if one supplier faces any disruption, then there are other suppliers who can step in and make up the difference at short notice. Thus, most of the studies related to supply disruption mitigation strategies assume that the supply processes from one supplier to another suppliers are independent. The assumption that the disruption event of one supplier temporarily or permanently going out of business is totally independent from other suppliers, however, this does not hold in all cases. In reality, the event that causes the disruption is likely to affect a number of different suppliers at the same time. These suppliers might have a link with their customers, suppliers, geographical location and trade rules and regulations. It is therefore not reasonable in some cases to assume that a production disruption will occur for one supplier and that the disruption will not have an effect on other suppliers. This phenomenon of suppliers dependency due to disruptions are widely being investigated in the finance research stream, but little research have been done in the area of OR/MS research.

For a mathematical modelling perspective, we focus on the re-engineering of the inventory policies' performance parameters (i.e. order-up-to level) in the supply disruption-inventory model with the global supply chain setting. In most non-identical dual-sourcing models, normally, two types of performance parameter have been considered, which are *regular* and *expedited* order-up-to parameters. According to Scheller-Wolf et al. (2003), if the inventory level is below the expedited target level, an expedited order (i.e usually from the expensive but fast supplier) is placed to bring the inventory position to this level, then a regular order (i.e usually from the cheaper but slow supplier) is placed to being the final inventory position up to the regular target level. However, in contrast, we just consider one order-up-to parameter to model the inventory policies which is the maximum inventory level. This thesis differs from a more complicated non-identical, dual-sourcing of inventory policies models, since the objective is not to study the performance of the inventory policies under the setting of dual-non-identical suppliers. Rather we investigate the effectiveness of dual-sourcing strategy in mitigating supply disruptions, under a non-identical suppliers setting. The reason to have only one order-up-to level parameter in this thesis is to simplify firm's ordering decision formula structure to avoid any complexity in finding the optimal ordering policy. We believe that one parameter is sufficient to capture managerial insights of dual sourcing strategies as one of supply disruption mitigation plan.

The results obtained in this thesis could be used to assist the firm in making disruption mitigation strategies by using the inventory and supplier management framework, regarding on additional costs associated with the risks of supply disruptions.

2.7 Research Model Framework

In this section, the map of research model framework which built based on the gaps and limitations in the previous section are briefly presented. For a better understanding, figure 2.4 illustrates the framework of research model in this thesis.

We identified three potential research areas in addressing those research gaps and limitations to study the effectiveness of dual-sourcing strategy in finding optimal inventory policies that subject to supply disruption. These three potential research areas are disruption discovery, disruption recovery and supply dependency phenomenon. Under the first of

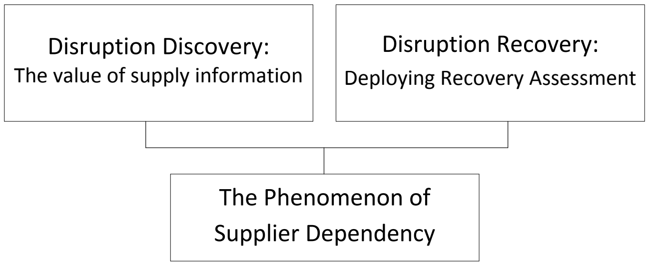


Figure 2.4. A research model framework

research area, which is disruption discovery, we will first examine the value of supply disruption information in supply disruptions-inventory models. A measurement parameter of disruption process is designed to represent the condition of the disruption information. We propose two models to investigate the performance of optimal dual-sourcing policies in various conditions of disruption informations. Then, another three models are introduced in the second research area which is disruption recovery. We intend to explore a quantitative recoverability assessment and we will examine the performance of the optimal dual-source policies based on various recoverability conditions. Finally, the phenomenon of supplier dependency is investigated based on the analyses from the previous models of disruption discovery and recovery. Several models are introduce and examine under various condition of disruption discovery and recovery. The development of all models in this thesis will be discuss in detail in the next chapter in section 3.5.

2.8 Conclusion

In this chapter, we presented related literatures on supply disruptions and inventory models that subject to disruptive supply events. From the literatures, there has been little reported in the area of understanding the effectiveness of supply diversification strategy. Therefore, we launch a study on investigating the effectiveness of supply diversification strategy specifically on a dual-sourcing strategy in managing the firm's inventory in the event of disruptions at the suppliers side. We focus on re-modelling the inventory policies based on the structure of the suppliers, the re-engineering of inventory performance parameter and supply disruption processes parameter. A modelling methodology to understand the effectiveness of the dual-sourcing strategy which can affect a firm's performance would be of considerable benefit to offer guidelines for the firms to improve the supply chain risk management capabilities.

3. Methodology

3.1 Introduction

Managing inventory that is sourced from two non-identical suppliers is a big challenge for a firm due to differences in the suppliers' profiles and the increase of disruption risks in the supply chain. In designing strategies and procedures for the firm's supply process, management needs a simple yet practical system for planning inventory purchases which covers all relevant criteria and parameters. The aim of this thesis is to develop and analyse simple tools to help management of a firm with this problem. As an initial stage of this research, we now set out a complete review of the methodology used in our analysis.

This chapter has been constructed in five sections as follows. In section 3.2, assumptions, notations and equations are introduced under a general model structure and in section 3.3, the framework of modelling the firm's ordering process under risk of events that disrupt supply with a discrete Markov decision process (DMDP) method is presented. Then, the formulation of the firm's ordering policy is discussed in section 3.4 and the description of each model that has been designed for analysis from the context of several problems discussed in Chapter 2 are presented in section 3.5. Finally, in section 3.6, we present the procedure for conducting experiments to examine the firm's optimal ordering policy.

3.2 General Modelling Assumptions and Notations

We consider a simple two-echelon supply chain with one firm and two suppliers in a single product/component setting. Each supplier is capable of supplying an infinite amount of the raw/semi materials. The firm operates under a make-to-order setting and uses the raw/ semi material to produce a semi/final product to meet demand from customers for the semi/final product. For a better understanding, we illustrate the supply chain structure in figure 3.1.

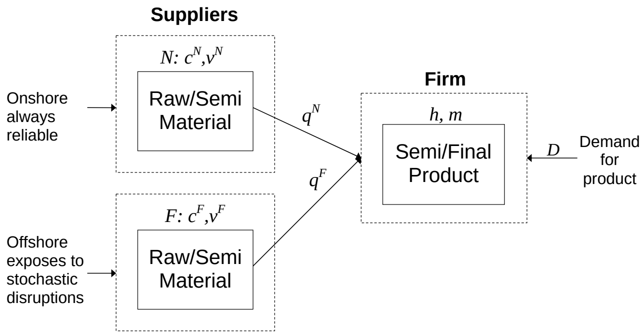


Figure 3.1. The simple supply chain structure (see pages ii-iv for a definition of all notation)

We assume one supplier, referred to as the onshore supplier, is located close to the firm while the other, referred to as the offshore supplier, is remote. The onshore supplier is denoted by N and the offshore supplier is denoted by F. A set of supplier is denoted by sp for $sp = \{F, N\}$. We assume that supplier N is perfectly reliable but, in contrast, the availability of supplier F is uncertain as this supplier is susceptible to supply chain disruption. We define the availability of supplier F as an ability of supplier F to deliver the order within a given time. The availability of supplier F can be either at the up state denoted by u or the down state denoted by w. In state u, supplier F is able to deliver complete orders on time, while in state w, nothing can be supplied. The additional notation in figure 3.1 refers to cost parameters which are introduced in section 3.2.1.

We use a discrete time model in which ordering decisions are taken at the beginning of each period after the inventory level of the firm and the state of the offshore supplier have been observed. Let Q denote the number of periods in the firm's planning horizon. We consider both finite ($Q < \infty$) and infinite ($Q = \infty$) planning horizons for the firm.

Figure 3.2 describes the sequence of events during period t, that is the interval between the point with t periods to go in the planning horizon and the point with $t - 1$ periods to go. From figure 3.2, at the beginning of period t, the firm observes the inventory level, i, and the state of supplier F, a, to decide whether inventory replenishment is needed or not. The inventory level cannot exceed the maximum storage capacity, denoted by I, at any time. The changes in i are based on the quantity of demand, D_t and the quantity of raw/semi materials supplied by supplier sp, q_t^{sp} for $sp = \{N, F\}$. We assume customers do not accept backorders, hence i is non-negative ($i \geq 0$). After observing i and a, the firm decides on the quantity to order from supplier sp, q_t^{sp} for $sp = \{N, F\}$. If supplier F is in state u, then, the firm has a dual-source option (i.e., can place the orders with supplier N and supplier F). In contrast, if supplier F is in state w, then the firm only has a single-source option (i.e., can place an order with supplier N only). Note that, in this simple model, supply from supplier F is an all-or-nothing process. Every order that the firm places with a supplier incurs two types of ordering costs: namely a fixed ordering cost (later known as the fixed cost) and a variable ordering cost proportional to the size of the order with constant of proportionality known as the unit cost. Let the fixed cost be denoted by c^{sp} and the unit cost be denoted by v^{sp} for $sp \in \{N, F\}$. The lead time of an order (i.e., the time between the firm placing an order and the order being delivered) is denoted by L^{sp} for $sp \in \{N, F\}$. We assume the leadtime of each supplier is known and constant. We further assume that the leadtime of the offshore supplier is longer than that of the onshore supplier (i.e. $L^F > L^N$). This could be due to the greater distance involved, the number of countries through which the order must pass or the different modes of transport required. Therefore, the firm will receive q_t^N items instantly when the order is placed at the beginning of the period and q_t^F items immediately before the end of the period.

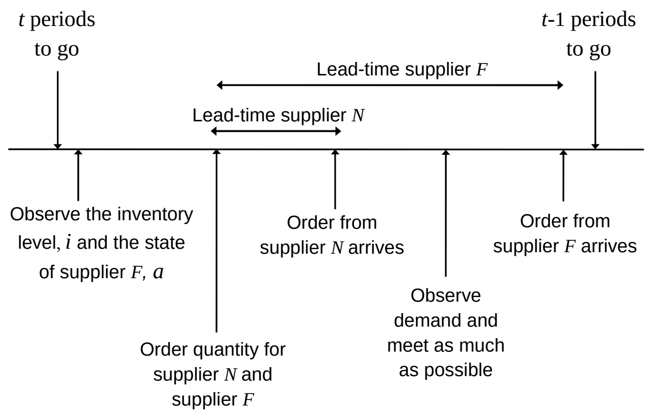


Figure 3.2. The firm's ordering process

If demand during the period, D_t does not exceed $i + q_t^N$ (i.e., $D_t \leq i + q_t^N$), then the firm can meet all demand in the period, but if demand during the period is greater than $i + q_t^N$ (i.e., $D_t > i + q_t^N$), then the unfilled demand is considered as lost sales since there are no backorders. For every unit of lost sales in a period, there is a penalty cost, denoted by m. There is also a holding cost, denoted by h, per item of inventory per time period. We will discuss all costs related to the firm's ordering process in detail in section 3.2.1.

We consider two models of customer demand. Firstly we assume demand is deterministic and occurs at a constant rate. Under this model, the demand in each period, D_t, is known and constant. To ensure the storage capacity is never exceeded with this model of demand, $q_t^N \leq I - i$ and $q_t^N + q_t^F \leq I - i - D_t$. This simple model of demand will yield ordering policies that are easy to understand. This will help with the validation of the implementation of the solution algorithms and might provide useful managerial insights. In real business market conditions, demand is rarely constant. Therefore, it is also interesting to analyse the firm's ordering decision process under uncertain demand. Secondly, we assume demand in each period is a stationary discrete random variable with known probability distribution. The probability that the demand in period t is d_t is denoted by $P(D_t = d_t)$.

3.2.1 Cost Function

The costs related to the firm's ordering decision from N and F are as follows.

Ordering costs

Under the first cost factor, the firm incurs the ordering costs for every order that it places with a supplier. We denote the ordering cost by $ORDER$. For $sp = \{N, F\}$, there is a one-off fixed cost, c^{sp}, if the firm places an order with supplier s and a variable cost, v^{sp}, that increases with the size of the order, q_t^{sp}. The equation for *ORDER* is given by:

$$ORDER = \sum_{sp \in \{N,F\}} \delta(q_t^{sp})c^{sp} + q_t^{sp}v^{sp}$$

where,

$$\delta(x) = \begin{cases} x & \text{if } x > 1 \\ 0 & \text{if } x = 0 \end{cases}$$

Note that, supplier N has lower fixed cost per order and higher unit cost per item than F (i.e., $c^N < c^F$, $v^N > v^F$). For example, this is consistent with the situation where supplier F benefits from lower labour or material costs, but is located in a different country far from the firm, so may incur higher transportation costs.

Holding cost

Under the second cost factor, the firm incurs a holding cost for inventory held during period t. This is assumed to be in proportion to the average inventory level during the period which is approximated by the simple average of the inventory level at the beginning of the period and the inventory level at the end of the period before the offshore order is delivered. According to Silver et al. (1998), the holding cost reflects the cost of having money tied up in inventory for a period of time. Let h denote the holding cost per item of inventory per time unit. We denote holding cost by *HOLD* and the equation for *HOLD* is given by:

$$HOLD = h\Big(\frac{1}{2}(i + \max(i + q_t^N - D_t, 0))\Big)$$

In this expression, i represents the inventory level at the beginning of the period and $(\max(i + q_t^N - D_t, 0)$ represents the inventory level at the end of the period before any offshore order is delivered.

Penalty cost

Finally, under the third cost factor, the firm is charged a penalty cost, m, for each unit of demand that cannot be satisfied in a period (i.e., each unit of lost sales). We denote the penalty cost by $PNLTY$ and formulate $PNLTY$ as:

$$PNLTY = m\Big(\max(D_t - i - q_t^N, 0)\Big)$$

When non-negative, the term $D_t - i - q_t^N$ inside the maximisation represents the amount of unmet demand.

3.2.2 Conclusion

In this section, we have presented the firm's ordering process and the firm's related inventory costs under the firm's simple supply chain structure. This structure will be used throughout this thesis. Under this structure, we intend to model the firm's ordering decisions under disruptive supply events at the offshore supplier and we choose the discrete-time Markov decision process (DMDP) as a method to derive the decisions that are optimal for the firm with respect to orders from the onshore and offshore suppliers. Therefore, in the next section, we present the framework for formulating the firm's ordering process with the DMDP method.

3.3 Research Analysis Instrument

In this section, we introduce a discrete-time Markov decision process (DMDP) as the research instrument that will be used to model the firm's decision making process. We will describe a DMDP in general terms, discuss the advantages of using the DMDP and provide some examples of how researchers and practitioners have used DMDP in the decision making process. This section provides the technical underpinning for the subsequent section which formulates the firm's decision problem as a DMDP.

3.3.1 Introduction to Discrete-Time Markov Decision Processes

A Markov decision process (MDP) is a mathematical optimisation technique for decision-making in uncertain situations which is capable of modelling probabilistic sequential decision process problems. In practice, the MDP is frequently used in inventory control, maintenance, manufacturing and telecommunication areas (Tijms, 2003, pg.234). The objective of the MDP model is to provide the decision maker with a proper (or optimal) *policy* and the optimal policy is supposed to be the policy that has the best performance with respect to the decision maker's criterion. Basically, the MDP is an extension of a Markov process. The basic concepts of the Markov process are those of a *state* and of a *state transition* (Tijms, 2003, pg.81). However, there are two additional factors for the MDP which are those of *actions* and those of *rewards/costs*. The Markov process can be thought of as a stochastic process used to predict the future state of the process based on the current state of the process and the assumption that the transition of state follows a certain probability distribution. The additional factors in the MDP, allow the decision maker to influence the future state of the process and the rewards/costs involved in the process through the decisions that are made.

One of the main features of model analysis using the MDP approach is the Markovian property. Due to the Markovian property, the sequential decision making process at each point in time is *memoryless*. The next state of the process depends on the current state and action only, and not states (or actions) in the past. Therefore, using the MDP technique, for a complex multi-stage decision problem in the presence of uncertainty, can provide the decision maker with efficient solutions and a compact representation of the decision problem. The decisions that are made with the MDP technique focus more on the expected future performance while considering the current state of the process and the consequences (rewards/costs) of the actions that can be taken.

In general, the discrete Markov decision process (DMDP) is a method that can be used to make an optimal decision where the decision is partly based on a random process and partly controlled by the decision maker. In this thesis, we are interested in finding the firm's optimal ordering policy under the probabilistic supply disruption affecting the offshore supplier. The *discrete* element, which refers to a decision making process that only occurs at a *fixed* time points, and the *Markov* element, which refers to the *memorylessness* property in the DMDP structure, make the DMDP a suitable method to use in this thesis. To relate these two elements to the reason why we chose the DMDP to model the firm's ordering decision under disruptive supply events; note that the firm's planning horizon is discrete and the information on which decisions are based is just the observed state of disruptive events and the observed inventory level.

The key components of the DMDP are a set of decision epochs, a set of states, a set of admissible actions, a set of one-step rewards/costs and a set of transition probabilities. Figure 3.3 shows a symbolic presentation of a general two-state Markov decision process for ordering and the description of the process is as follows. At the beginning of each of a given number of discrete time periods (e.g., period 1), the process is observed to be in some state (e.g., inventory level and state of offshore supplier) which is a member of the set of states or the state space. Such a point is called a decision epoch, where the firm will make a decision (or an action). At each decision epoch, an action must be selected from the set of admissible actions. In the firm's ordering process, the action refers to the quantity of material to be ordered from supplier N and supplier F. As a result of the chosen action, the firm faces two economic consequences. First, the current state evolves probabilistically to another state by the beginning of the next period according to a certain probability distribution, and secondly, the firm receives a one-step reward (or incurs a one-step cost). In the firm's ordering process, the one-step cost refers to cost factors that relate to the ordering process as a result of the action taken in the current state. We will describe the mathematical formulation of the firm's ordering process with these DMDP components in detail in section 3.4.1.

Based on a random sequence of states and admissible actions, the firm can prescribe a policy which is a mapping between what has happened in the past and what has to be done in the current state. Effectively, the policy provides the firm with a prescription for choosing the action in any possible future state (Puterman, 2009). Under the mapping process, there is a cost incurred by the firm and the cost is referred to as the total inventory cost related to the firm's ordering process. According to Puterman (2009) after multiple-steps of decision making processes in a system, the sequence of costs has to be viewed as a *random* sequence since the decision maker does not know the cost prior to policy selection and implementation. Therefore, the total cost after multiple-steps of the process is known as an expected total cost.

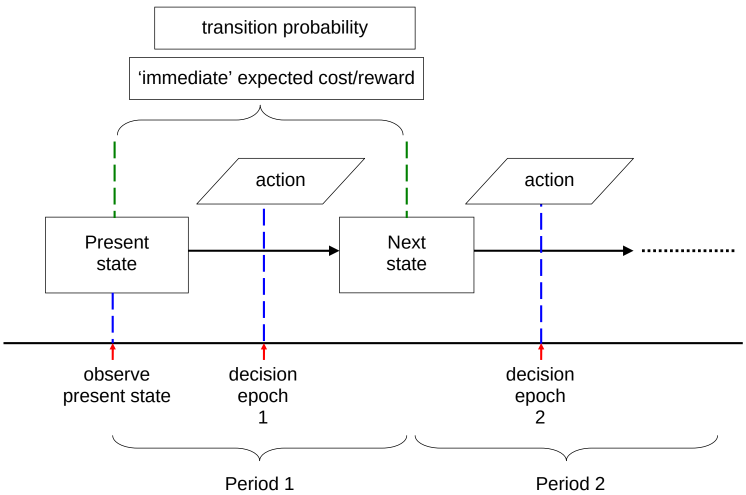


Figure 3.3. Symbolic presentation of a sequential Markov decision process

Supposedly, in one decision process, one policy in a set of policies that achieves the best performance measurement should be selected. In this thesis, the chosen optimal policy, with the best performance should achieve the smallest expected total cost value. However, there is one remaining problem in the DMDP model, how can we find the optimal policy among a set of policies for one decision process? To answer this question, the decision maker needs to evaluate the performance of each policy to measure the goodness each policy's performance. The policy evaluation refers to a process to compute the expected cost from optimality equation. In what follows, in section 3.3.1a, we explain about the optimal policy in general and in section 3.3.1b, we briefly describe several computational algorithms that can be used to find the optimal policy.

The optimal policy

An optimal policy is associated with the *Principle of Optimality* introduced by Bellman (1957). He writes:

> “ An optimal policy has the property that whatever the initial state and initial decision (action) are, the remaining decisions (actions) must constitute an optimal policy with regard to the state resulting from the first decision”

This principal means that the decision maker and the optimal policy need not be influenced by past states and actions when considering a particular decision epoch. However, there is a technical restriction with this principle; it is only valid when the number of states and the number of actions are *finite* (Tijms, 2003, pg.237).

We can obtain the optimal policy through policy evaluation and the evaluation process refers to an activity that maximises (or minimises) some combinations of expected rewards received (or expected costs incurred) over time. The combinations can be either the future rewards (or future costs) over a trajectory or the rewards (or costs) for multiple trajectories through expectation-based measures. The policy evaluation process can be conducted over a period of either a finite-horizon or an infinite-horizon. The process with finite-horizon is applicable to the decision making process that must end and, in contrast, the process with infinite-horizon is applicable to a long-run decision making process that does not stop unless there is a parameter in the model to reflect the end of the decision process.

Finding the optimal policy is associated with the computation of an optimal value from an optimality equation function. The optimality equation is a formula used to compute an optimal value that minimses the expected total cost (or maximises the expected total reward). The optimal value that represents the best cost (or the best reward) over the remainder of the planning horizon can be associated with the state resulting from the decision taken in each state at a decision epoch. In other words, the optimal decision at a decision epoch can be determined by combining the one-step cost (or reward) and the optimal cost (or reward) over the remainder of the planning horizon through the optimality equation (Porteus, 2002).

Under the infinite-horizon setting, we assume the decision maker seeks to minimise the long-run average cost per time unit. The long-run average cost per time unit for a given optimal policy can be computed using a dynamic programming computational approach. Dynamic programming (or backward induction) is used to optimise the optimality equation function that recursively evaluates the expected total costs (Puterman, 2009). A key feature of the dynamic programming approach is a backwards process that uses induction simplify the computation of an optimal policy.

Computational algorithms for DMDP

Tijms (2003) reports that there are three different computation algorithms that can be used to compute the optimal policy value for an infinite horizon model, namely *policy iteration* (PI), *value iteration* (VI) and *linear programming* (LP). However, the most widely used algorithms to compute long-run average cost per time unit optimal policy are the PI and the VI (Tijms, 2003). The first algorithm, PI works on the policy space and generates reward (or cost) for the optimal policy, whereas the second algorithm, VI approximates the maximum average reward (or minimum average cost) through a sequence of value functions (Tijms, 2003). A version of VI can also be used for finite horizon problems.

Generally, PI takes a policy and computes its value. Then, it iteratively improves the policy until it cannot be improved any further. While in VI, approximations to the optimal value function are calculated iteratively until convergence and then an optimal policy is deduced. The advantages of these two algorithms are, on the one hand, PI has a capability to converge in finite-time and, on the other hand, VI can deal with a large state space in the DMDP problem analysis (Tijms, 2003). Therefore, decision makers always have the options to choose the algorithm that is most suitable for their analyses.

In our analysis, we consider the VI algorithm as a computational approach to find an optimal policy. The advantages of the VI algorithm that led to this choice are that the algorithm is applicable to both finite and infinite horizons, the most suitable when dealing with a large state space (Tijms, 2003) and the easiest implement in any computer programming language. Moreover, if there is a periodicity issue in the computation of the optimality equation function, it can easily be solved with a simple data transformation method using a *perturbation* technique (Tijms, 2003). In general, the VI algorithm is a recursive computation that uses an iterative method which successively applies the optimality equation to calculate the value function for each period (finite horizon) or until the fixed point is reached (infinite horizon). Specifically for an infinite horizon the steps of VI are: (1) calculate the expected maximum reward (or the expected minimum cost) for each state from the optimality equation function, (2) the value functions provide lower and upper bounds on the maximum average reward (minimum average cost) and, (3) under a certain aperiodicity condition these bounds converge to the maximum average reward (or the minimum average cost) (Tijms, 2003).

3.3.2 Conclusion

In this section, we have discussed the structure of the DMDP for a better understanding of the main components of the DMDP that have been used in formulating the firm's ordering decision process. We also stated that the VI algorithm is the chosen computational algorithm that will be used in this thesis to solve the optimality equation function. Next, we formulate the firm's ordering process with the DMDP components before using this formulation to find the optimal ordering policy.

3.4 General Ordering Policy Formulation

In this section, the firm's ordering decision making process is formulated with the DMDP. We refer to Puterman (2009), Tijms (2003) and Porteus (2002) as the main references in describing the firm's ordering process via the DMDP method. The description about the firm's ordering process with the DMDP components in is presented in section 3.4.1, followed by the explanation about the policies in section 3.4.2. Then, the ordering decision is formulated with the optimality equation function in section 3.4.3 and finally the optimal ordering policy is defined in section 3.4.4.

3.4.1 The DMDP Components

This section describes the components of the DMDP model, which covers the notation, assumptions and equations in the DMDP components. The state spaces, action spaces and the transition probabilities however are explained in general and will be described in detail according to the problem description in later chapters. The DMDP components used throughout of this thesis are the following:

Decision epoch

Decision epochs arise at the beginning of each time period when the firm makes an ordering decision. The set of decision epochs, denoted by T, is given by:

$$T = \{1, 2, ...Q\} \text{ for } 1 \leq Q \leq \infty.$$

In the DMDP, the planning horizon will represent the firm's production planning horizon. Under the finite- horizon setting, we assume that t starts from 1 until Q where Q can be any finite number and under the infinite-horizon DMDP model, the value of t increases from 1 until it approaches ∞.

States

State is denoted by y which is a member of state space Y. For each state $y \in Y$, at each decision epoch, the firm will take decisions (or actions). In our model, Y is finite.

Actions

At each decision epoch, the firm is provided with all necessary information to make a choice of actions from the set of admissible actions. We denote the action as b and the set of admissible actions as $B(y)$. In our model, $B(y)$ are finite and deterministic.

Transition probabilities

As a result of choosing an action from the set of admissible actions, $B(y)$, at decision epoch t, the firm incurs a *cost* and the state, y evolves probabilistically to another state, by the next decision epoch, which we call a *transition of the state*. The new state is determined by a probability distribution. We refer to the probabilities of this distribution as *transition probabilities*. As a result of chosen action b, there is a probability of making a transition from state y to state z in one period and we denote this probability by $p_{y,z}(b)$. In some cases, it can be convenient to express transition probabilities as a matrix, called the transition matrix. In our model, the state generally has two components, inventory level and state of offshore supplier. We consider the evolution of both of these components separately.

One-step costs

At each decision epoch, for each chosen action b, the firm incurs a one-step cost which is related to the firm's inventory cost. The one-step cost represents the inventory cost during period t when action b has been chosen when the process was in state y. We assume that the firm's inventory level at the beginning of a period, denoted by i, is a component of the state y. The one-step cost is denoted by $C_t^y(b)$ and this cost under a constant demand setting is given by:

$$
\begin{aligned}
C_t^y(b) &= ORDER + HOLD + PNLTY \\
&= \sum_{sp\in\{N,F\}} (\delta(q_t^{sp})c^{sp} + q_t^{sp}v^{sp}) + \ h\Big(\frac{1}{2}(i + \max(i + q_t^N - D_t, 0))\Big) \\
&\qquad + \ m\Big(\max(D_t - i - q_t^N, 0)\Big)
\end{aligned}
$$

and the one-step cost in the ordering decision under a stochastic demand setting is given by:

$$
\begin{aligned}
C_t^y(b) &= ORDER + \Big(E_d\Big(HOLD + PNLTY\Big)\Big) \\
&= \sum_{sp\in\{N,F\}} (\delta(q_t^{sp})c^{sp} + q_t^{sp}v^{sp}) + \sum_{d_t=0}^{K} P(D_t = d_t)\Big\{h\Big(\frac{1}{2}(i + \max(i + q_t^N - d_t, 0))\Big) \\
&\qquad + \ m\Big(\max(d_t - i - q_t^N, 0)\Big)\Big\}
\end{aligned}
$$

where K denotes the support of the probability distribution of demand.

In both one-step cost equation functions, the summation of the one-step cost (as a result of action b), $C_t^y(b)$ consists of the ordering cost, $ORDER$, the holding cost, $HOLD$ and the penalty cost, $PNLTY$. Note that, an explanation of these costs has been reported in section 3.2.1. For the one-step cost in the model with stochastic demand, the values of $HOLD$ and $PNLTY$ depend on the demand probability distribution, $P(D_t = d_t)$ as explained in section 3.2.1.

3.4.2 Policies

A policy prescribes the action that should be taken in every possible future state at every possible future decision epoch and the prescription is associated with a *decision rule*. The output of the DMDP model provides the firm with a prescription for the choice of action on the quantities to be ordered from the suppliers, q_t^{sp} where $sp = \{N, F\}$ at each decision epoch, for any possible future inventory level and state of the offshore supplier.

Before we define the policies in detail, we will briefly discuss the decision rule definition. The decision rule is a function defined on the state space Y that specifies an admissible action from $B(y)$ for each state y. It encapsulates the actions taken at a specified decision epoch (Porteus, 2002, pg.48). The decision rule is denoted by Δ_t. Under decision rule, Δ_t, the chosen action for each state $y \in Y$ at decision epoch t is given as $\Delta_t^y \in B(y)$. A list of admissible decision rules for each decision epoch then prescribes a *policy* for the DMDP.

In this thesis, the ordering policy is denoted by π where the policy, π lists the chosen admissible decision rules Δ_t at every decision epoch, t. Therefore, let

$$\pi = (\Delta_1, \Delta_2, ..., \Delta_Q).$$

The objective of the DMDP model analysis is to seek a policy that minimises the expected ordering cost. For infinite horizon models we seek a stationary policy that chooses the same decision rule Δ^* at each decision epoch. In this case, let

$$\pi = (\Delta^*, \Delta^*, \ldots, \Delta^*).$$

In the next section, we will discuss the objective function of the DMDP model which is related to the computation of the optimality equation function.

3.4.3 Optimality Equation Function

Define $V_t^y(\pi)$ to be the expected cost of policy π when there are t periods to go and the process is in state y. From the preceding discussion it follows that:

$$V_t^y(\pi) = C_t^y(\Delta_t^y) + \sum_{z \in Y} \Big(p_{y,z}(\Delta_t^y) V_{t-1}^z(\pi) \Big)$$

From the above equation, as a result of the chosen π, technically, $C_y^t(\Delta_t^y)$ denotes the 'one-step' cost at decision epoch t in state y and $\sum_{z \in Y} (p_{y,z}(\Delta_t^y) V_{t-1}^z(\pi))$ denotes the total expected future cost given the transition structure of the Markov process. For a finite horizon we seek a policy π to minimise $V_Q^y(\pi)$ for initial state y, while for an infinite horizon we seek to minimise $\lim_{Q\to\infty}(V_Q^y(\pi)/Q)$.

Using a dynamic programming approach based on the principle of optimality, this optimisation can be performed in stages. Define V_t^y to be the minimum expected cost with t periods to go when the process is in state y. By the principle of optimality:

$$V_t^y = \min_{b\in B(y)} \left\{ C_t^y(b) + \sum_{z\in Y} p_{yz}(b) V_{t-1}^z \right\}$$

This equation is known as the optimality equation for the problem.

In our DMDP, we seek to minimise the expected total inventory cost (later known as expected cost) of all supplies received when the firm takes the decision according to a given ordering policy. This cost involves two types of costs; (1) the one-step inventory cost, $C_t^y(b)$ and (2) the expected minimum cost resulting from the next state at the next decision epoch of the ordering decision making process, V_{t-1}^z. The optimality equation is formulated according to the customer demand model under the general structure. Under the model with constant demand, the optimality equation is given by:

$$\begin{aligned} V_t^y &= \min_{b\in B(y)} \left\{ C_t^y(b) + \sum_{z\in Y} (p_{y,z}(b) V_{t-1}^z) \right\} \\ &= \min_{b\in B(y)} \left\{ ORDER + PNLTY + HOLD + \sum_{z\in Y} \left(p_{y,z}(b) V_{t-1}^z \right) \right\} \end{aligned}$$

where

V_t^y = expected minimum cost when in state y with t periods to go.

Under the stochastic demand model, the optimality equation is given by:

$$\begin{aligned} V_t^y = \min_{b\in B(y)} \Bigg\{ & ORDER + \sum_{d_t=0}^{K} P(D_t = d_t) \big(PNLTY + HOLD \big) \\ & + \sum_{z\in Y} p_{y,z}(b) V_{t-1}^z \Bigg\} \end{aligned}$$

3.4.4 Optimal Policy

The aim is to find an optimal policy among all the policies that have been provided in the decision making process. An optimal policy, denoted by π^* prescribes an action in every state $y \in Y$ and at every decision epoch t. For a finite horizon problem, when the optimality equation is solved for state y at decision epoch t, the action chosen is an element of an optimal policy. Let Δ_t^* be the decision rule consisting of the actions which minimise the optimality equation for each state at decision epoch t. The policy consisting of these decision rules is optimal for the finite horizon model.

For an infinite horizon model, where the aim is to minimise the long-run average cost per period, the policy that chooses decision rule Δ_t^* at every decision epoch approaches a stationary optimal policy as $t \to \infty$. The process referred to here as solving the optimal equation involves calculating the expected minimum cost, V_t^y, and is the basic step of the value iteration (VI) algorithm discussed in the previous section. In section 3.6.1, we shall explain how to derive optimal policies using the VI algorithm.

3.4.5 Conclusion

In this section, a complete review of the DMDP as the instrument to derive the firm's ordering policies is presented and the DMDP has been formulated according to the given inventory system settings. In the next section, we shall explain a number of models that have been developed to provide insight on the impact of offshore supplier disruption.

3.5 Design of Model of Analysis

In this section, we provide an overview of the analysis conducted in this thesis. A number of models are developed and compared to provide insight on the impact of offshore supplier disruption on the firm's ordering decisions. For a better understanding, the development of models in this thesis is illustrated in figure 3.4 and an explanation of each model is provided in the following.

3.5.1 Routine Sourcing Model

We start the analysis with a preliminary analysis which we call a *routine ordering* model or Model 1. Model 1 is the most basic model in the sense that there is no disruption to the offshore supplier. The objective of this model is to find an optimal ordering policy for a firm who sources from two non-identical suppliers (i.e., different ordering cost and lead-time). We use a discrete-time Markov decision process (DMPD) method with the objective of minimising the expected cost to determine the optimal routine ordering policy. In this model, we assume that there is no disruption at supplier F and both suppliers, N and F, always can be relied on to deliver the order on time. We expect that the firm will order more from supplier F and only order from supplier N to meet an immediate demand when the inventory level is unexpectedly low. The result from this model is used as the benchmark to assess the next model, which we call a *crisis sourcing* model.

3.5.2 Crisis Sourcing Model

The crisis sourcing model, or Model 2, has been developed with an assumption that supplier F is unreliable and there is a risk that this supplier F is unable to deliver an order on time. The firm observes the state of supplier F at the beginning of each period. We model the state of supplier F with a Markov chain process where the state of supplier F can either be up or down. When up, supplier F will deliver any order placed in that period in full and on time. When down, supplier F is unable to deliver any items in the period. We also use the DMDP method to determine the optimal ordering policy under a number of scenarios for the probability of supplier F failing (i.e., moving from the up to down state between periods) and the probability of supplier F recovering (i.e., moving from the down to up states between periods). This model will help us determine the relationship between the characteristics of disruption at supplier F and the optimal ordering policy. In this model, the uncertainty of the state of supplier F plays an important role for the firm when determining how much to order and from which suppliers to order. From this model, we expect that, if the transition probability from up to down is high and the transition probability from down to up state is low, the firm would prefer to order more from supplier F when it is up and will be required to order from supplier N too. We also expect the expected inventory cost to be higher in Model 2 than in Model 1. The result from this model is used as the benchmark to assess the following models, which cover the first part of the research analysis in this thesis by exploring the value of supply disruption information towards the firm's ordering policies.

3.5.3 Value of Supply Disruption Information

For this part of our analysis, we introduce two models which we call Model 3 and Model 4. We developed these two models based on the supply disruption information focusing on information about the length of disruption. In Model 3, we assume that *the firm knows in advance how long each disruption is going to last* and in Model 4, *the firm knows the probability distribution of the length of the disruption, but does not know how long the disruption is going to last.* With these two models, we aim to provide the firm with insight on the value of a strong strategic alliance with the offshore supplier characterised by more information about the lengths of disruptions. The optimal ordering policies under these two models are also determined using the DMDP method. These model are basically extensions of Model 2 with multiple Down states corresponding in some way to different lengths of disruption. We expect the firm to be able to use the additional information about supply disruption to make more efficient ordering decisions. Therefore, we expect there to be less ordering from supplier N and fewer lost sales. Both models require as parameters the probability of a disruption occurring to supplier F (i.e., the probability of supplier F moving from the up state to one of the down states) and the probability distribution of the length of a disruption.

3.5.4 Modelling the Supply Disruption Recovery

In the second part of our main research analysis, we aim to provide the firm with a strategic decision making framework when considering the process of disruption recovery. The idea is that the process of recovery by the offshore supplier following disruption consists of a sequence of distinct phases. We introduce another three models, referred to as Model 5, Model 6 and Model 7 which differ in the assumptions made about the length of each phase of the recovery process. As in the first part of the research analysis, the optimal ordering policies under these three models are also determined using the DMDP method. In all three models, we assume that *the firm knows the number of recovery phases*. In model 5, similar to Model 2 the length of each phase is modelled by a constant hazard rate, (i.e., the probability that the phase will end in this period). Model 6 extends the idea of Model 3 so that the length of a phase of the recovery is known as soon as the preceding phase is complete. Model 7 is a combination of the ideas underpinning Model 4 and Model 5 structures. In Model 7, we assume that *the firm does not know the length of any phase of the disruption in advance, but it knows the probability distributions of the lengths of each phase of the recovery process*. Model 5 requires as parameters the probability of F failing and the hazard rates (or probabilities of moving to the next phase of recovery) for each phase of the recovery process. Models 6 and 7 require as parameters the probability of supplier F failing and the probability distributions of the lengths of each phase of the recovery process.

3.5.5 The Order Pressure Model

In the final part of our research analysis, we aim to investigate an 'order pressure' scenario at supplier N when there is disruption to supply from supplier F. We assume that *when there is disrupted supply from supplier F, this disruption will create order pressure at supplier* N. For example, this could occur because many firms are affected by the disruption and all these firms are now trying to source supply from other suppliers. Based on this assumption, we consider two variants of models 3, 4, 6 and 7 above, namely Variant f and Variant p. In Variant f, we assume that there is no order pressure in the chain of supply and supplier N can deliver every order in full and on time. While in Variant p, we assume that there is order pressure in the chain of supply and, when supplier F is down, supplier N only delivers a proportion of the order with lead time L^N and delivers the remainder with lead time L^F. As with all other models, the optimal ordering policies under these two variants are also determined using the DMDP method.

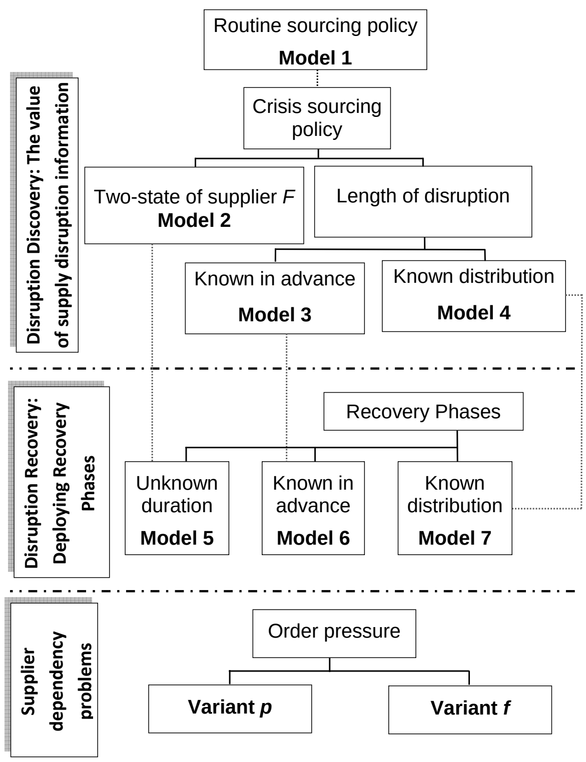


Figure 3.4. Summary of models for analysis

3.6 Procedure of Experimental Analysis

We now explain how we shall use the methodology to conduct the experiments in this thesis. Generally, the process to derive the optimal policies is based on the computation of the expected minimum cost under the optimality equation function using the Value Iteration (VI) algorithm. In addition, in some circumstances, based on finding optimal policies under the infinite horizon DMDP model, we are also interested to measure the performance with respect to *fill rate* and *average inventory level* via a simulation technique.

We start this section with a brief explanation of how we design the experiments according to the nature of the firm's demand and the firm's planning horizon in section 3.6.1. Then, we present details of the VI algorithms in section 3.6.2 followed by the description of the perturbation technique in section 3.6.3 for use in the event of a problem with periodicity. After that, all the steps related to the fill rate and average inventory level computations are explained in section 3.6.4 and finally, the numerical values used in the experiments are presented in section 3.6.5.

3.6.1 Design of Numerical Experiments

This research uses a scenario based approach in conducting the experiment. Four different settings of the firm's inventory system based on the parameters of the firm's customer demand and the firm's production planning horizon are used in examining the optimal ordering policy. The demand can be either constant or stochastic and the planning horizon can be either finite or infinite. The analysis will consider models with the four settings as follows:

a. The optimal ordering policy with finite-horizon and constant demand.
b. The optimal ordering policy with finite-horizon and stochastic demand.
c. The optimal ordering policy with infinite-horizon and constant demand.
d. The optimal ordering policy with infinite-horizon and stochastic demand.

For a better understanding, we illustrate the analyses of the four model settings in table 3.1. In table 3.1, we give a short name for each model and the short name of the models are based on the combination of the firm's planning horizon categories (i.e., Fin for Finite and Inf for Infinite) and customer demand models (i.e., $Cons$ for Constant and Sto for Stochastic). For example, the case of $FinCons$, is used to represent finite planning horizon category and constant demand type. In later chapters, these short names will be used to present the results of each model.

Table 3.1. Four different model settings

		Demand	
		Constant	Stochastic
Planning Horizon	Finite	$FinCons$	$FinSto$
	Infinite	$InfCons$	$InfSto$

In the analysis of every model, we will investigate how conditions relating to the disruptive supply event affect the firm's optimal ordering policy by examining the output of the optimal ordering decisions and the minimum expected cost. In some cases, we are also interested to check the performance of the optimal policy by examining the values of fill rate and average inventory level. These will be derived from simulation as explained in detail in section 3.6.4.

In what follows, we present the details of the VI algorithms used to find the firm's optimal ordering policies.

3.6.2 Value Iteration (VI) Algorithm

Technically, the VI algorithm to find an optimal policy consists of five steps. The five steps of the VI algorithm, are presented in the next section. Under the finite-horizon DMDP setting, the process to derive the optimal policy involves all five steps except Step 2, but in contrast, the analysis under the infinite-horizon DMDP setting will consider all steps. Step 2 refers to the computation of a bound between the minimum average reward and the value iteration calculation at decision epoch t. We follow Tijms (2003, pg.261) in explaining the steps of the VI algorithms. Note that, in this thesis, the steps of the VI algorithm have been implemented in Java programming language.

The steps of the VI algorithm

The steps of the VI algorithm as follows.

Step 0 : (*Initialisation*). Under both finite and infinite-horizon planning settings, set $t = 1$ as an initial iteration number and suppose this iteration number represents the decision epoch of the DMDP model. Let the expected cost when $t = 0$ be equal to 0 for all states, $V_0^y = 0$, for each $y \in Y$. Another additional parameter under the infinite planning horizon setting is a tolerance value to detect convergence after t-iterations. This parameter is denoted by ε where $\varepsilon > 0$.

Step 1 : (*Value-iteration step*). For each state $y \in Y$, compute the optimality equation function:

$$V_t^y = \min_{b \in B(y)} \left\{ C^y(b) + \sum_{z \in Y} \left(p_{y,z}(b) V_{t-1}^z \right) \right\} \qquad \text{for each } y \in Y.$$

Let Δ_t be any optimal decision rule such the action $b = \Delta_t^y$ minimises the right-hand side of the optimality equation for V_t^y for each state y. At iteration t, we find V_t^y for some (finite) t in the finite-horizon DMDP model.

Step 2 : (*Bounds of minimum average cost*). Under the infinite-horizon DMDP setting, compute the bounds:

$$J_t = \max_{y \in Y} (V_t^y - V_{t-1}^y)$$

$$H_t = \min_{y \in Y} (V_t^y - V_{t-1}^y)$$

where

H_t = maximum difference in the minimum expected cost for any state between iterations t and $t-1$ and,

J_t = minimum difference in the minimum expected cost for any state between iterations t and $t-1$.

Step 3 : (*Stopping test*). Under the finite-horizon DMDP setting, if $t = Q$, stop with optimal policy given by $\pi = (\Delta_1, \Delta_2, \ldots, \Delta_Q)$. Under the infinite-horizon DMDP setting, if $0 \leq J_t - H_t \leq \varepsilon$, stop with optimal stationary policy $\pi = (\Delta_t, \Delta_t, \ldots)$.

Step 4 : (*Continuation*). Set $t = t + 1$ and return to step 1.

When the algorithm is stopped either after $t = R$ under the finite-horizon DMDP setting or after t iterations under the infinite-horizon DMDP setting, we now have the values of q^{sp} for $sp = \{N, F\}$ for all states and all decision epochs, V_t^y under the finite-horizon DMDP setting and minimum average cost $g = (J_t + H_t)/2$ under the infinite-horizon DMDP setting.

3.6.3 Perturbation Technique

Under the constant demand infinite-horizon DMDP setting, the model is likely to be periodic. In a Markov chain, a state is periodic if the number of periods between successive visits to the state is always a multiple of a fixed integer greater than 1. In the event of periodicity the convergence of upper bound and lower bound to the minimum average cost is not guaranteed. Following Tijms (2003), the periodicity issue can be solved by transforming the data through a *perturbation* of the one-step transition probabilities. The perturbed transition probabilities are given by:

$$\overline{p_{ij}} = \tau p_{ij} \qquad \text{for } \; j \neq i$$

$$\overline{p_{ii}} = \tau p_{ii} + 1 - \tau$$

for some constant τ with $0 < \tau < 1$.

The perturbed problem is aperiodic because, under every stationary policy, there is a non-zero probability of remaining in each state between one period and the next. It is also possible to show that the minimum average cost per period for the perturbed model is the same as in the original DMPD model (Tijms, 2003). The Markov decision model can be perturbed by following the steps below. Note that, this perturbed DMDP model has also been implemented in the Java programming language.

Step 1 : Choose the value of τ where τ is some constant with $0 < \tau < 1$.

Step 2 : Define the perturbed one-step transition probabilities, $\overline{p}_{y,z}(b)$ by:

$$\overline{p}_{y,z}(b) = \begin{cases} \tau p_{yz}(b) & y \neq z, \; b \in B(y) \text{ and } y, z \in Y \\ \tau p_{yz}(b) + 1 - \tau & y = z, \; b \in B(y) \text{ and } y, z \in Y. \end{cases}$$

Note that, the state space Y, action spaces $B(y)$ for $y \in Y$ and immediate costs $C_t^y(b)$ for $b \in B(y)$ and $y \in Y$ remain unchanged in the perturbed DMDP model. Based on this perturbed DMDP model, we now can derive an optimal ordering policy via the VI algorithm computation.

The steps of the VI algorithm are as in section 3.6.2, except for the computation of the optimality equation function in Step 2. The modified computation is given by

$$V_t^y = (1-\tau)V_{t-1}^y + \min_{b \in B(y)} \left\{ C_t^y(b) + \tau \sum_z \left(p_{y,z}(b) V_{t-1}^z \right) \right\}$$

With this simple modification, we will compute the minimum average cost and derive an optimal ordering policy.

3.6.4 Estimation of Fill Rate and Average Inventory Level

Under the risk of disruption to the offshore supplier and stochastic demand, the firm may face a situation where the inventory level is not sufficient to satisfy all of the demand during a period. We call such an event a *shortage* event. Under our assumptions, shortage events result in lost sales and the inconvenience to customers is modelled using a penalty cost. In such cases, it is common practice to also consider the performance of the optimal policy in terms of a *fill rate* and an *average inventory level*. In this thesis, a simulation method is used to estimate the values of fill rate and the average inventory level.

The fill rate is a Type-II service level which measures the quantity-oriented performance that describes the proportion of total demand being satisfied within a reference period without backorders or lost sales (Silver et al., 1998). It also has been recognised as the true service measure as it measures exactly how much demand was met (Silver et al., 1998). The higher the value of the fill rate, the better the performance of the ordering policy and vice versa. In simulation, the fill rate refers to the proportion of total demand satisfied per period. Each time period, t, we randomly generate an observation from a discrete probability distribution to represent the demand during that period, d_t, and calculate the value of shortage in that period, denoted by sh_t. Following the assumptions and notation of the preceding sections, the shortage value is given by

$$sh_t = \max\left(d_t - (i_t + q_t^N), 0\right)$$

where sh_t is the amount of demand that cannot be satisfied by the total inventory available during the period. The total inventory available during the period is the initial inventory level plus the order quantity from the onshore supplier, N. If $d_t - (i_t + q_t^N) > 0$, then there is a shortage event and the value represents the units of demand that cannot be satisfied in the period. The fill rate for the period is $1 - \frac{sh_t}{d_t}$. In the simulation experiment, the expected shortage value per period is denoted by $E(sh_t)$ and the total expected demand value per period is denoted by $E(d_t)$.

The simulation experiment also allows us to estimate the average inventory level which is another interesting performance measure. The average inventory level refers to the average start of period inventory level during the simulation run. We then use replications of the simulation experiment to compute a confidence interval for point estimators of the average fill rate and the average inventory level.

Simulation Experiment

In what follows, we present the steps used to conduct the simulation experiment. Note that, in this thesis, the simulation experiment has also been implemented in the Java programming language.

Step 0 : (*Initialisation*). Set the values for the state of F, a_1, and the inventory level, i_1, for the start of the simulation experiment. Set $t = 1$.

Step 1 : (*Look up the optimal ordering decisions*). Based on the current state (i_t, and a_t), look up the quantities to be ordered from supplier N and supplier F (i.e., q_t^N,q_t^F) from the optimal ordering policy.

Step 2 : (*Generate random demand*). Generate the random demand value, d_t, from the discrete probability distribution of the demand per period.

Step 3 : (*Calculate the shortage value*). Based on the current inventory, i_t, and the amount of demand, d_t, compute the shortage value, sh_t, from:

$$sh_t = \max\left(d_t - (i_t + q_t^N), 0\right)$$

Step 4 : (*Update the inventory level*). The inventory level at the start of the next period is given by

$$i_{t+1} = \max\left((i_t + q_t^N) - d_t, 0\right) + q_t^F.$$

The value is calculated as the residual inventory after demand has been satisfied plus the quantity of order from supplier F.

Step 5 : (*Update state of the offshore supplier, F*). The probability distribution of the state of supplier F in the next period is determined by the transition probabilities, $p_{a_t,z}$ from the Markov model of the state of supplier F. The state of supplier F in the next period is generated at random from this discrete probability distribution.

Step 6 : (*Continuation*). Set $t = t + 1$. If t is greater than the total length of the simulation run, stop the simulation. Otherwise repeat from step 2.

Note that random observations from discrete probability distributions can be generated from a $Uniform(0, 1)$ pseudo-random number using the following procedure (Ross, 2007). Let $P(o)$ for $o = 0, 1, \ldots, O$ denote the probability distribution and let r denote a $Uniform(0, 1)$ pseudo-random number created with the Java function *Math.random*(). For an example, please refer to Sedgewick and Wayne (2007).

Step 1 : (*Initialisation*). Set $o = 0$ and $p = 0$.

Step 2 : Calculate the probability of the random variable is at most o by adding $P(o)$ to the current value of p: Set $p = p + P(o)$.

Step 3 : If $r < p$, stop and return o. Otherwise set $o = o + 1$ and repeat from Step 2.

When the simulation algorithm is stopped, we have a list of the inventory level, i_t, the values of the demand, d_t and the shortage, sh_t, in each period of the simulation. To reduce the effect of the initial state on the results of the simulation, it is common practice to include a warm-up period during which the values observed are not included in the estimation of the measures of interest (Brooks and Robinson, 2001). Assume that the simulation runs for a total of Q periods and the first M of these are used as a warm up ($M < Q$). We can estimate the average demand per period with the formula:

$$E(d_t) = \frac{\sum_{t=M+1}^{Q} d_t}{M - Q}$$

and the average shortage per period using the formula:

$$E(sh_t) = \frac{\sum_{t=M+1}^{Q} sh_t}{M - Q}$$

With these two average values, we then estimate the average fill rate. The average fill rate is denoted by P_2 and is given by:

$$P_2 = 1 - \frac{E(sh_t)}{E(d_t)}$$

The average fill rate, P_2, can be interpreted as the probability that the firm is able to satisfy a unit of demand arising during a period. It is estimated as 1 minus the ratio of $E(sh_t)$ and $E(d_t)$. The average inventory level is denoted by I_A and can be calculated using the formula:

$$I_A = \frac{\sum_{t=M+1}^{Q} i_t}{Q - M}.$$

This simulation experiment is run several times, each run known as a replication, to construct a confidence interval for the point estimator of average fill rate and average inventory level.

Constructing a confidence interval

The values of fill rate and average inventory level will be presented by the point estimators under the confidence interval values with 95% confidence that the true point estimator across the number of replications falls within the confidence interval based on the obtained sample. The size of sample in the simulation experiment is referred to as the number of replications and each replication refers to a different run of the simulation experiment using different streams of random numbers (Brooks and Robinson, 2001). Note that, we will not discuss in detail the theory of confidence intervals in this thesis, but we refer to Law (2014) for the construction of the confidence interval in simulation model analysis.

For the confidence interval, we use a mean (or an average) as the point estimator. Therefore, refer to Brooks and Robinson (2001), the confidence interval can be calculated using the following formula:

$$\text{Confidence interval} = \text{Mean} \pm \frac{\text{Standard deviation,} t_{n-1,\frac{\alpha}{2}}}{\sqrt{n}}$$

where

$$\begin{aligned} n &= \text{number of replications} \\ t_{n-1,\frac{\alpha}{2}} &= \text{value from students } t\text{-distribution with } n-1 \text{ degrees of freedom and} \\ &\quad \text{significance level } \frac{\alpha}{2}. \end{aligned}$$

The standard deviation can be calculated by:

$$\text{Standard deviation} = \sqrt{\frac{\sum_{i=1}^{n}(\text{Results}_i - \text{Mean})^2}{n-1}}$$

In this thesis, we will present the fill rate and the average inventory level values according to the above confidence interval formula structure with $t_{n-1,\frac{\alpha}{2}} = 1$. For example, if the mean value is equal to 0.97 and the standard error value, $\frac{\text{Standard deviation}}{\sqrt{n}}$ is equal to 0.0003, then we will present the confidence interval as 0.97 ± 0.0003. Confidence intervals for arbitrary significance levels can easily be determined from this information.

3.6.5 Choice of Parameters in Numerical Experiments

In what follows, we explain how parameter values for the models developed have been chosen for use in the numerical experiments in subsequent chapters.

Time Parameters

In the finite-horizon models, the length of the planning horizon is considered to be a month and decisions are taken each day. Thus, the number of periods is $T = 30$. In the infinite-horizon models, the time period is still considered to be a day, but the length of planning horizon is arbitrary.

Recall that the replenishment time of supplier F is longer than supplier N to represent the different locations of these suppliers relative to the firm. Thus, set the lead-times of supplier N and supplier F at each decision epoch in each model, $L^N = 0$ and $L^F = 1$ ($L^N < L^F$).

State Space Parameters

The state of the process, y, for each model in the DMDP modelling framework comprises two parameters: namely inventory level, i, and the state of supplier F, a, for $a \in \{0, 1, \ldots, A\}$ for $0 \leq A \leq \infty$. In Model 1, state of supplier F is always up, so $A = 0$ and $y = (i, 0)$. In all other models, supplier F may be up or down, so $A > 0$ and $y = (i, a)$. The state of supplier F varies with each model depending on the problems addressed by the model. For instance, in Models 2, 3 and 4 are $y(i, A_j)$, the state represents the risk of supply disruption and in Models 5, 6 and 7, the state represents the quantitative assessment of the phased recovery process. In the DMDP model analysis, we model changes in the inventory level and changes in either the states of supplier F separately. A detailed description of the state space, Y, for each model will be discussed in subsequent chapters.

Cost Parameters

We discusses a special case of the holding cost, h, as follows.

We define the holding cost, h, as:

$$h = \text{holding cost per item per time unit}$$

and suppose h represents the cost to hold the items in the firm's inventory. This cost is related to the unit ordering cost of supplier N and supplier F (i.e., v^N and v^F) as it depends on the cost of capital that is tied up in the inventory. Refer to Silver et al. (1998), let

$$x = \text{holding cost per unit of capital tied up in inventory}$$
$$\text{(i.e., used to buy current inventory) per year.}$$

Therefore, for $sp \in \{N, F\}$, the holding cost is given by

$$x * v^{sp} \text{ per item per year for items supplied by supplier } sp.$$

But, items from supplier N and supplier F in the firm's inventory are identical. So, we do not know which is which. Therefore, we need to identify a single holding cost. In order not to underestimate the total cost of holding inventory with a cheaper value of the ordering cost in the firm's business operation, we chose the variable ordering cost of the onshore supplier, v^N (recall that, $v^N > v^F$). Therefore, h is given by:

$$h = \frac{(x * v^N)}{\text{number of time unit in a year}}$$

Often in periodic review models, the time unit is set equal to the length of one time period. Therefore, we set the number of time periods in a year equal to 52 weeks.

We may set x values to vary between 0.25 and 0.55 (Alfares, 2007). In this thesis, the value of x is chosen to be equal to 35% ($x = 0.35$) and suppose that this value is sufficent enough to present the real holding cost in the industrial practice.

Demand Parameters

The parameter of demand is not the main interest to be studied in this thesis, thus the models are tested with only one value and one type of probability distribution in the deterministic and stochastic demand models respectively. In the deterministic demand models, demand in each period is assumed to be equal to 5, $D_t = 5$. In the stochastic demand models, the probability of demand in each period, $P(D = d_t)$, is assumed to follow a truncated Poisson distribution, $P(d_t) \sim Pois(\lambda, K)$, where λ denotes the demand rate and K represents the value at which the distribution is truncated and therefore the maximum value of demand to be considered. The value of λ is categorised into three cases: namely small, medium and large. The small demand case represents a really slow moving product, thus set $\lambda = \{1, 2 \text{ or } 3\}$. The medium case represents the demand condition that corresponds to the deterministic demand case, thus set $\lambda = 5$. The large demand case represents a fast moving product, thus set $\lambda = \{10, 20 \text{ or } 30\}$. Note that, the various values of λ are chosen for the large demand case so that we can obtain the probability distribution that is almost symmetric, i.e., approximately normal. We truncate the distribution at $K = 50$, so that in all cases the sum of the demand probabilities is very close to 1.

Value Iteration Parameters

In the perturbed DMDP model with constant demand, refer to the one-step transition probabilities (see section 3.6.3 on one-step transition probabilities), there is a parameter of τ. The value of τ is set to $\tau = 0.5$. In the infinite-horizon model, there is a parameter of ε in the optimality equation which refers to the tolerance value for the convergence of the value iteration process. The value of ε is set to $\varepsilon = 0.001$. The values of τ and ε used in this thesis are as suggested by Tijms (2003).

Base set of parameters

To conduct the experiment, assumptions have been made on the values used for some parameters of the DMDP model. These values will be the base value settings when we conduct the experiments for each model. We need a base set of parameters value that gives a reasonable looking ordering policy as a base policy before we can compare the results of the models and examine the sensitivity of the models to changes in the values of the parameters. For a better understanding, the base values used are summarised in table 3.2.

For a base case we assume that, the maximum capacity of inventory, I is equal to 70 and demand in both deterministic and stochastic demand models are equal to 5, thus $D = 5$ and $\lambda = 5$.

When dealing with cost parameters, a common approach is to fix one cost at one unit and take everything else relative to that. In our analysis, we take all costs relative to the unit cost of F and fix $v^F = 1$. Recall that, supplier F is cheaper than supplier N, thus, the ordering costs, v^{sp} for $s \in \{N, F\}$, are assumed to be equal to $v^F = 1$ and $v^N = 2$. This means that the offshore supplier sells the items at half the cost per unit of the onshore supplier which is good enough to represent a very substantial discount in supply. The fixed costs, c^{sp} for $sp \in \{N, F\}$, are assumed to be equal to $c^N = 5$ and $c^F = 10$. In this way the base cost settings ensure the suppliers' cost ratios are 2:1 in both fixed and ordering costs.

Penalty cost, m, is set to be equal to 8 with the idea that the firm is penalised with a high cost when there is a lost sale in a period. If the goods are sold at a mark up of 100%, the profit would only be 1-2 per item. So, a lost sale results in loss profit of 1-2 plus goodwill cost for future lost sales. Hence, $m = 8$ is a high enough value for the base case. The parameter of x in the holding cost function is assumed to be equal to $x = 0.35$ and this value is sufficient enough to represent the real holding cost in industrial practice. We assume 52 periods in a year so that a time period represent a period of one week in the base case, thus $t = 13$.

Table 3.2. The values of the base set of parameters

Parameters	I	D	λ	m	h	c^N	c^F	v^N	v^F
Values	70	5	5	8	$\frac{0.35*v^N}{13}$	5	10	2	1

3.6.6 Conclusion

In this section, we have reported a step-by-step procedure to derive the optimal policy using the VI computational approach and discussed the parameter values used in the numerical experiments.

3.7 Conclusion

In this chapter, we have provided a complete review of the methodology used to conduct our research analysis. We started with an explanation of the development of a general simple supply chain model that consists of a firm and two non-identical suppliers, including the notation, assumptions and cost equations in the firm's inventory model. Then, we introduced the discrete Markov decision process (DMDP) method as the instrument to model the supply disruption problem in the firm's inventory model. The explanation of the DMDP method covers the description of the model and a procedure for finding optimal policies. We also briefly described the structure of the models used in our research analysis. Finally, we explained the basis for the numerical experiments that will be used in our model analysis. In the next chapter, we will present our preliminary results based on the analysis of Model 1 and Model 2.

4. The Ordering Policies

4.1 Introduction

In this chapter, we present a simple analysis to study the firm's ordering policy under a normal supply chain operation without considering the risk of disruption at the offshore supplier. We consider this simple analysis as a preliminary study which we call a *routine ordering* model (later on known as Model 1). Supply disruption risk is then introduced in a *crisis ordering* model (later known as Model 2) where the reliability of the offshore supplier now is subject to disruption. Note that, the onshore supplier is always perfectly reliable in Model 2. In Model 2, we examine the impact of disruption to order deliveries on the firm's ordering policy as a result of unexpected adverse events experienced by the offshore supplier. We investigate, by numerical experiments, how the disruption affects the firm's ordering decision and the minimum expected inventory cost of the optimal ordering policy.

Most quantitative studies that examine the inventory models with non-identical suppliers traditionally focus on determining dynamic inventory policies. These inventory policies are typically characterised by one or two target inventory positions with an aim to minimise the inventory cost. For example, Veeraraghavan and Scheller-Wolf (2008) and Scheller-Wolf et al. (2003) study a dual-index policy with two target levels that performs close to optimality. Basically, the dual-index policy represents a policy that tracks inventory over regular and expedited lead-times based on two target levels of inventory positions. In every period, if the expedited inventory position is below expedited order-up-to target level, it is brought back to this level by placing an expedited order. After the expediting order is made, regular orders are placed, restoring the regular inventory position to its regular target level. In our models, the order quantities are constrained by the maximum storage capacity of the firm but are otherwise arbitrary functions of the inventory level and, in the case of finite horizons, the number of periods remaining in the planning horizon. We want to investigate the effectiveness of a dual-sourcing strategy in mitigating supply disruptions, in a setting with non-identical suppliers. Ordering strategies with a simple structure (such as a base-stock policy or dual-index policy) have the practical advantage of being easier to implement, but imposing such a strategy would complicate the DMDP model. We believe that our approach is sufficient to capture managerial insights of dual sourcing strategies as one aspect of the supply disruption mitigation plan.

This chapter has been constructed in three sections as follows. The routine ordering model and crisis ordering model are explained in section 4.2 and section 4.3 respectively. Then, the conclusion for this chapter is presented in section 4.4.

4.2 The Routine Ordering Model

In Model 1, we assume that the onshore and offshore suppliers are perfectly reliable and the ordering decisions made are solely based on cost and lead-time considerations. The objective of Model 1 is to minimise the expected cost of satisfying customer demand by ordering from the onshore and offshore suppliers. We expect that the firm will order more from the offshore supplier due to the lower cost and that orders from the onshore supplier will be considered only when inventory is needed quickly due to shorter lead time. Model 1 has been designed with simple assumptions to avoid complexity appearing in the process of formulating the ordering process. In Model 1, we are interested to find the firm's ordering policy that minimises expected total inventory cost after taking into account the operating profile of the onshore and offshore suppliers. We model the firm's ordering policy with the DMDP modelling framework and full details are presented in the next section. The optimal ordering policy of this model will be a benchmark for Model 2.

The structure of this section is as follows. We describe Model 1 and its assumption in section 4.2.1 and section 4.2.2, followed by formulation of the ordering decision problem via the DMDP in section 4.2.3. Then, the results and findings are reported in section 4.2.4 and section 4.2.5. Finally, the conclusion for Model 1 is presented in section 4.2.6.

4.2.1 Model Description

The firm has chosen to implement a dual-sourcing strategy on an on-going basis in its supply management. Recall the firm's dual-sourcing structure in section 3.2, both the onshore supplier, supplier N, and the offshore supplier, supplier F, are assumed to be perfectly reliable. During the ordering process, the firm needs to split the orders between suppliers supplier N and supplier F based on different fixed and unit ordering costs and lead-times. Therefore, the firm's ordering decisions have to consider two conditions on the costs and the lead-time as follows:

a. Supplier supplier N has lower fixed cost per order, c^N and higher unit cost per item, v^N than supplier supplier F (i.e., $c^N < c^F$, $v^N > v^F$).

b. The lead-time for supplier supplier N is shorter than the lead-time for supplier supplier F (i.e., $0 = L^N < L^F = 1$).

Based on these two conditions, we can say that the firm sources from two non-identical suppliers: supplier supplier N is an expensive but fast supplier and supplier supplier F is a cheap but slow supplier.

4.2.2 Model Assumptions

The assumptions of Model 1 are as follows:

a. Both supplier F and supplier N are always available.

b. The firm's inventory planning horizon is discrete.

c. Demand is either deterministic or stochastic. The stochastic demand distribution follows the truncated Poisson distribution, $P(d_t) \sim Pois(\lambda, K)$.

d. Customers do not accept backorders, thus the firm encounters lost sales. The firm is charged with a penalty cost for each unit of demand that cannot be satisfied in a period, $PNLTY$.

e. The firm incurs a holding cost for inventory held during period t, $HOLD$.

4.2.3 Model Formulation

In this section, we explain the formulation of the problem analysed in Model 1 as a DMDP model and present the optimality equation. Note that, the explanation will cover both types of demand setting in the firm's inventory system: constant and stochastic demands.

Components of the DMDP for Model 1

The components of DMDP for Model 1 are as follows:

Decision epoch

A decision is taken at the beginning of each period. Let t denote the decision epoch when there are t periods remaining in the planning horizon, $t = 1, 2, \ldots, T$.

States

Suppliers supplier N and supplier F are perfectly reliable, thus at each decision epoch, the firm only reviews the inventory level, i. The state of Model 1 is denoted by i and the state space Y is given by:

$$Y = \{0, 1, ...I\}.$$

The inventory level can assume any value between 0 and maximum of inventory level, I.

Actions

Based on the current state i, the firm decides on the quantities to be ordered from suppliers supplier N and supplier F. The action is denoted by $b = (q^N, q^F)$ and the set of admissible actions, $B(i)$, is given by:

$$B(i) = \left\{(q^N, q^F) : \quad q^N, q^F \geq 0 \quad \& \quad q^N + q^F \leq I - i\right\} \quad \text{for} \quad 0 \leq i \leq I.$$

By ensuring that at each decision epoch, the sum of the quantities ordered from suppliers supplier F and supplier N, $q_t^N + q_t^F$, must be less than or equal to $I - i$, we ensure the capacity constraint cannot be violated even in the case of zero demand.

Transition probabilities

The inventory stored at the beginning of a period and the order from supplier supplier supplier N are used to meet as much demand during the period as possible. Any excess demand results in lost sales. The quantity ordered from supplier F is delivered at the end of the period. Therefore if action $b = (q^N, q^F)$ is chosen in state i and the demand during the period t is D_t, the state at the next decision epoch will be $\max(0, i + q^N - D_t) + q^F$.

Under the constant demand setting, there is no uncertainty in the problem and transitions are deterministic. In this case:

$$p_{i,j}(b) = \begin{cases} 1 & \text{if} \;\; j = \max(i + q^N - D_t, 0) + q^F \\ 0 & \text{otherwise} \end{cases}$$

Under the stochastic demand setting, the only uncertainty in the problem is the customer demand. In this case:

$$p_{i,j}(b) = \begin{cases} \sum_{d=i+q^N}^{\infty} P(D_t = d) & \text{if } j = q^F \\ P(D_t = i + q^N + q^F - j) & \text{if } q^F < j \le i + q^N + q^F \\ 0 & \text{otherwise} \end{cases}$$

One-step costs:

The one-step cost function, as a result of choosing action b in state i, consists of the ordering cost, $ORDER$, the holding cost, $HOLD$ and the penalty cost, $PNLTY$. In the case of stochastic demand, the values of $HOLD$ and $PNLTY$ depend on the random demand during the period. The one-step costs for Model 1 with the constant and stochastic demand settings are based on the same assumptions as the one-step costs for the models in section 3.4.1e. See this section for a detailed explanation. This one-step cost is denoted by $C_t^i(b)$ when $b = (q^N, q^F)$ is chosen in state i at decision epoch t. If the demand during the period t is D_t, the state at the next decision epoch will be $\max(0, i + q^N - D_t) + q^F$. Under the constant demand setting, there is no uncertainty in the problem and the one-step cost is given by:

$$\begin{aligned} C_t^i(b) &= ORDER + HOLD + PNLTY \\ &= \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \ h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\ &\quad + \ m\Big(\max(D_t - i - q^N, 0)\Big) \end{aligned}$$

Under stochastic demand, the one-step cost is given by:

$$\begin{aligned} C_t^i(b) &= ORDER + \Big(E_{D_t}\Big(HOLD + PNLTY\Big)\Big) \\ &= \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t)\Big\{h\Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) \\ &\quad + \ m\Big(\max(d_t - i - q^N, 0)\Big)\Big\} \end{aligned}$$

Optimality equation

We introduce a value function, $V_t(i)$, for every decision epoch, t, at every state of inventory level, i, to represent the minimum cost over the final t periods of the planning horizon when the inventory level is i at decision epoch t. The optimality equation for Model 1 with constant demand is given by:

$$V_t(i) = \min_{b \in B(i)} \Big\{ \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + m\Big(\max(D_t - i - q^N, 0)\Big) \\ + h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) + V^i_{t-1}\Big(\max(i + q^N - D_t, 0) + q^F\Big)\Big\}$$

and the optimality equation function for Model 1 with stochastic demand is given by:

$$V_t(i) = \min_{b \in B(i)} \Big\{ \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d=0}^{\infty} P(D_t = d_t)\Big\{m\Big(\max(d_t - i - q^N, 0)\Big) \\ + h\Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) + V^i_{t-1}\Big(\max(i + q^N - d_t, 0) + q^F\Big)\Big\}\Big\}$$

From these optimality equations, we seek optimal order quantities from supplier N and supplier F at each decision epoch.

4.2.4 The Optimal Ordering Policies

In this section, we report the ordering policies according to the planning horizon of the DMDP model. We start the report with the optimal ordering policies under the finite horizon planning setting in section 4.2.4a which covers the finite-horizon Model 1 with constant demand (later known as M1*FinCons*) and stochastic demand (later known as M1*FinSto*). Then, we report the optimal ordering policies under the infinite horizon-time setting in section 4.2.4b covering the infinite-horizon Model 1 with constant demand (later known as M1*InfCons*) and stochastic demand (later known as M1*InfSto*). Note that, the numerical experiments use the base values stated in section 3.6.1, which are as follows:

Table 4.1. The values of the base set of parameters

Parameters	I	D_t	λ	m	h	c^N	c^F	v^N	v^F
Values	70	5	5	8	$\frac{0.35 * v^N}{13}$	5	10	2	1

The optimal ordering policies under the finite-horizon Model 1 setting

The ordering policies of Model 1 under the finite horizon setting with constant and stochastic demands are as follows.

The ordering policy of M1FinCons

Figure 4.1 shows that if the inventory level, i falls below $D_t = 5$ in any period, then the firm will almost always place the orders with both supplier N and supplier F. The only exceptions to this are when the time remaining in the planning horizon is very short ($t = 1$ or 2). However, from figure 4.2, if i is between $D_t = 5$ and $2D_t - 1 = 9$, the firm anticipates that i will fall below D_t between the current and the next decision epoch (period between decision epochs t and $t-1$), and places an order with supplier F only. If i is adequate to cover demand in current and next periods ($i \geq 2D_t$), the firm will never place any order to supplier N or supplier F.

The ordering policy of M1FinSto

Figures 4.3 and 4.4 show that when i is less than or equal to 11, the firm will always place an order with supplier F unless there is very little time remaining in the planning horizon ($t \leq 3$). However, the firm only orders from supplier N if the the inventory level is very low, i is less than 7 ($i < 7$). From figure 4.5, if i is high enough to cover demand in the short term, the firm never places any order with supplier F or supplier N.

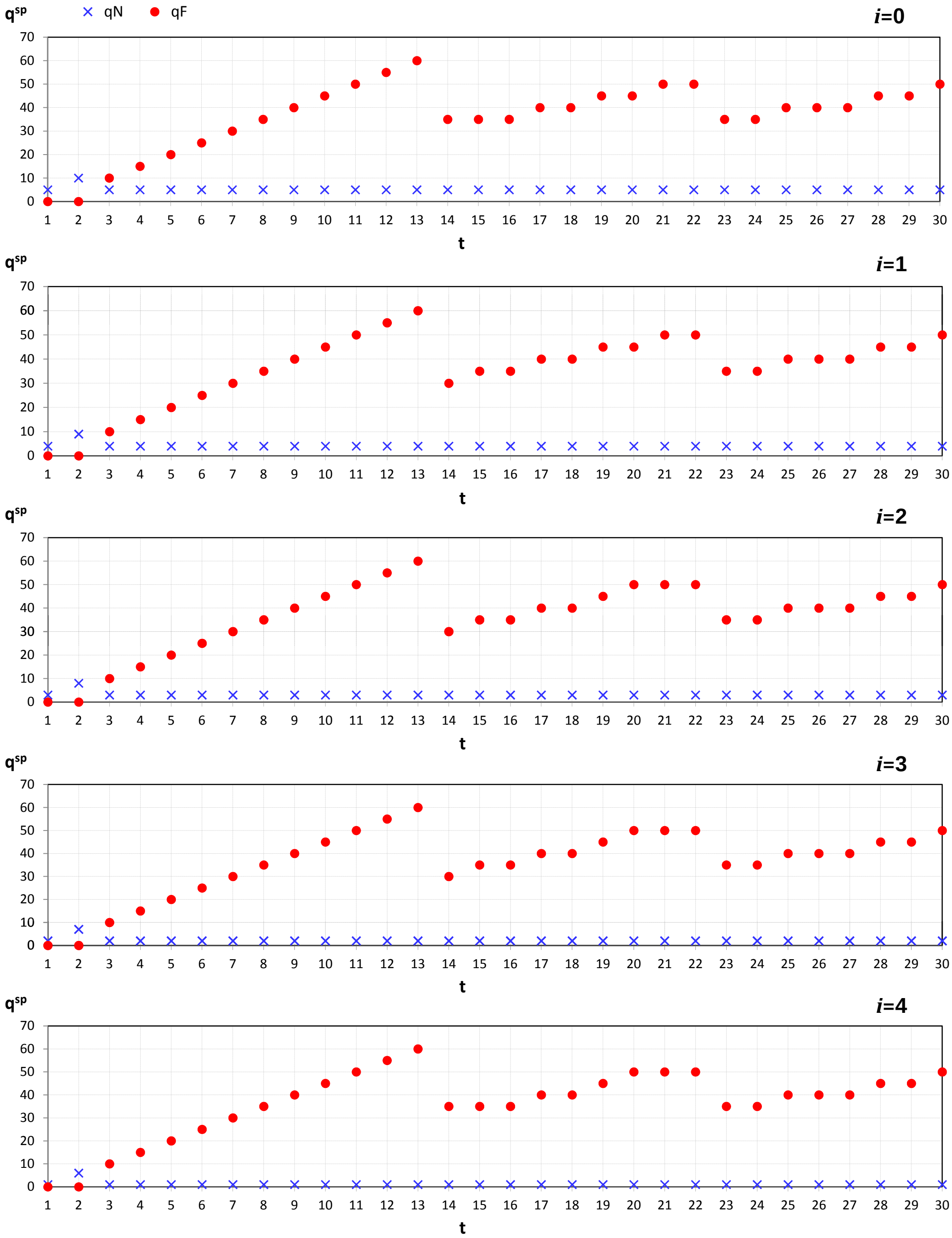


Figure 4.1. M1*FinCons*: The ordering decision from supplier N and supplier F over period, t for $0 \le i \le 4$

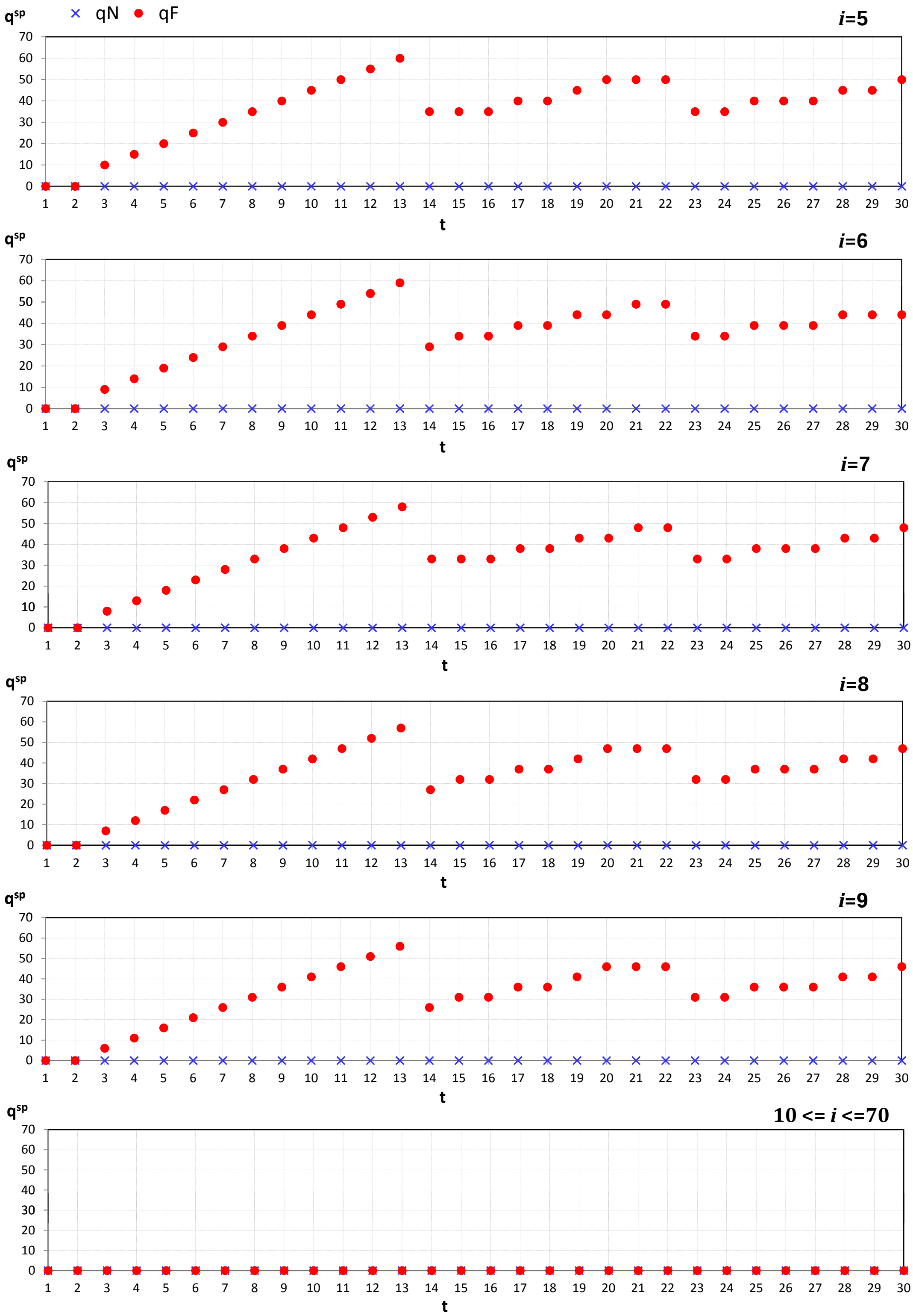


Figure 4.2. M1*FinCons*: The ordering decision from supplier N and supplier F over period, *t* for $5 \le i \le 70$

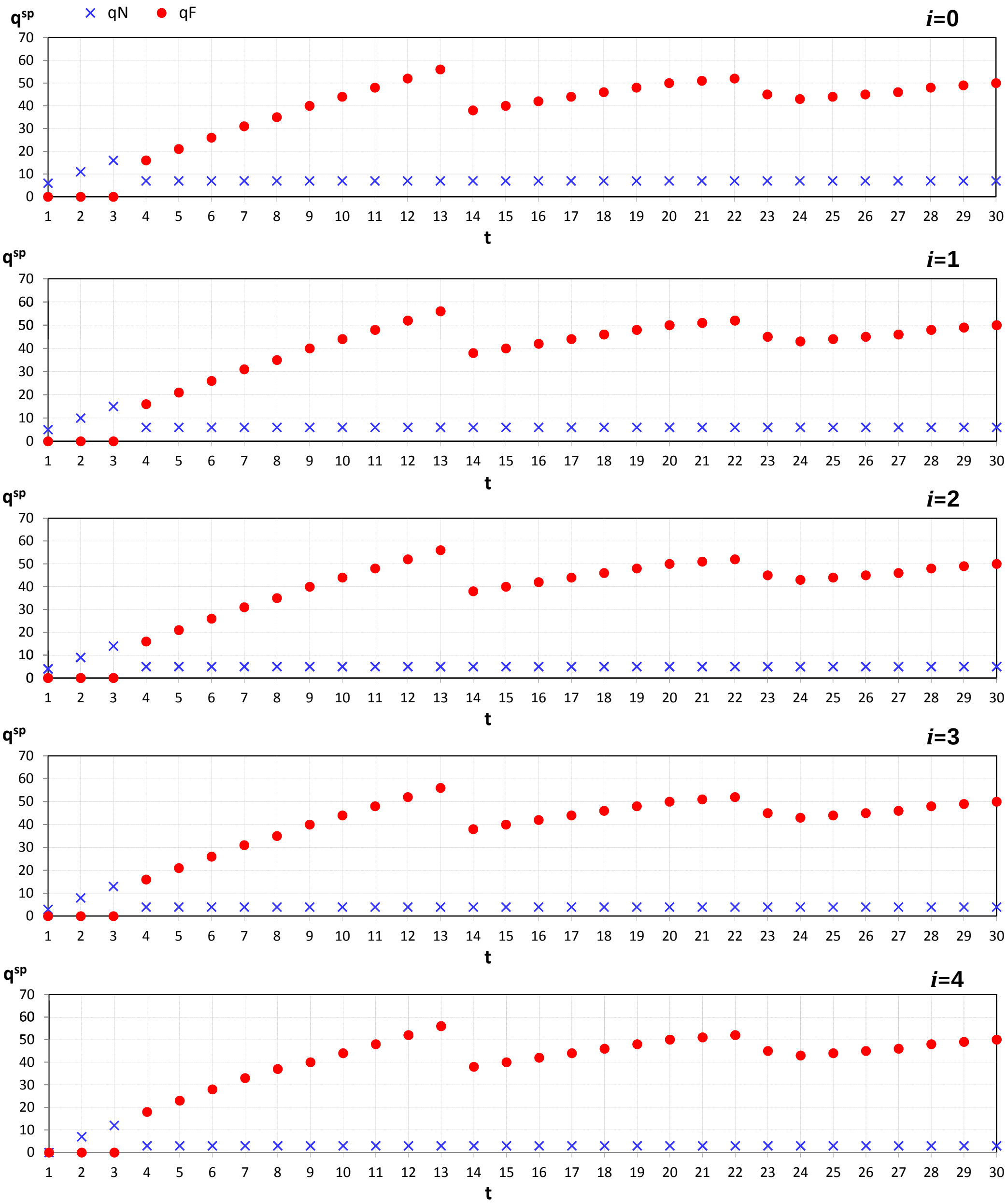


Figure 4.3. M1 *FinSto*: The ordering decision from supplier N and supplier F over period, *t* for $0 \leq i \leq 4$

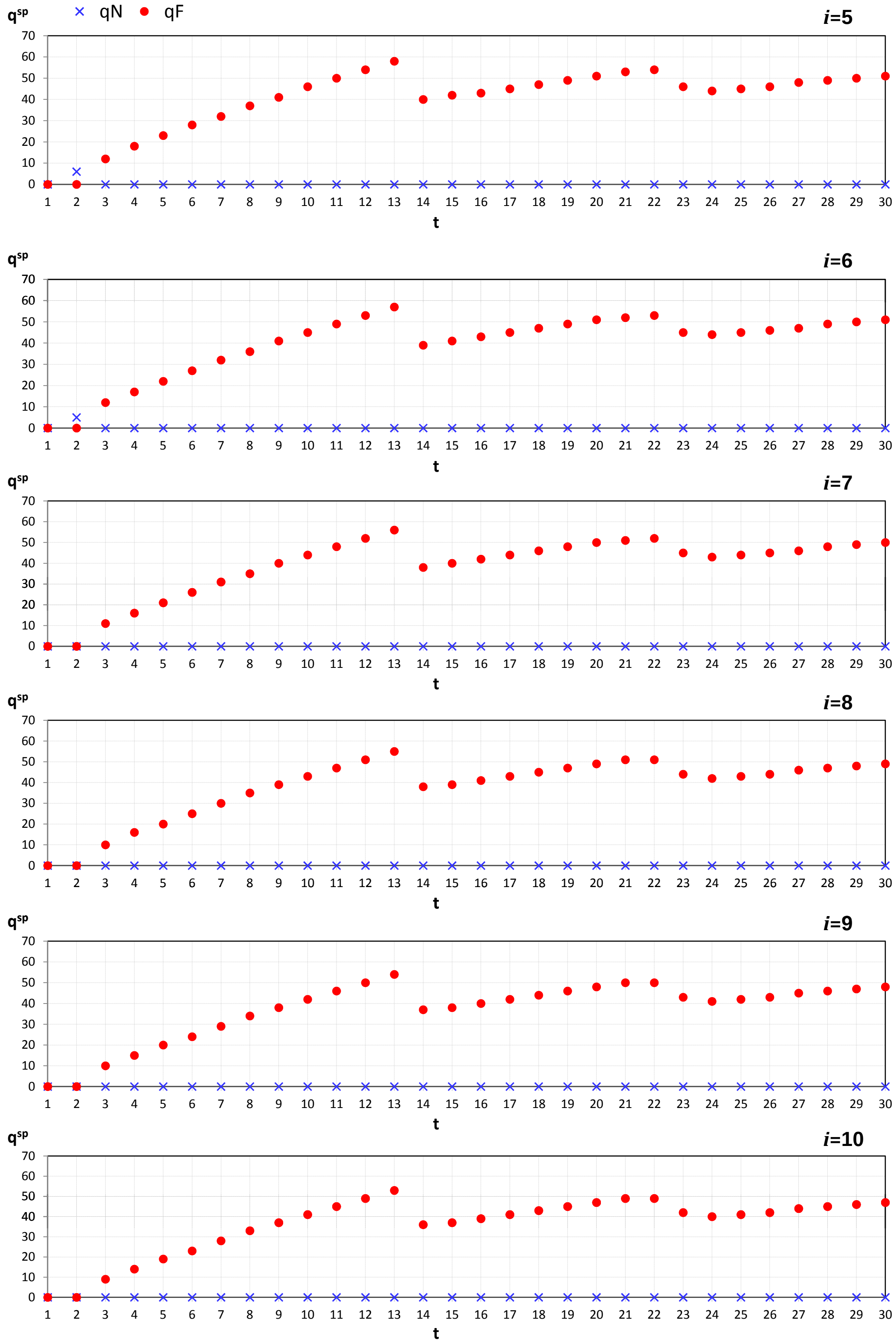


Figure 4.4. M1 *FinSto*: The ordering decision from supplier N and supplier F over period, t for $5 \leq i \leq 10$

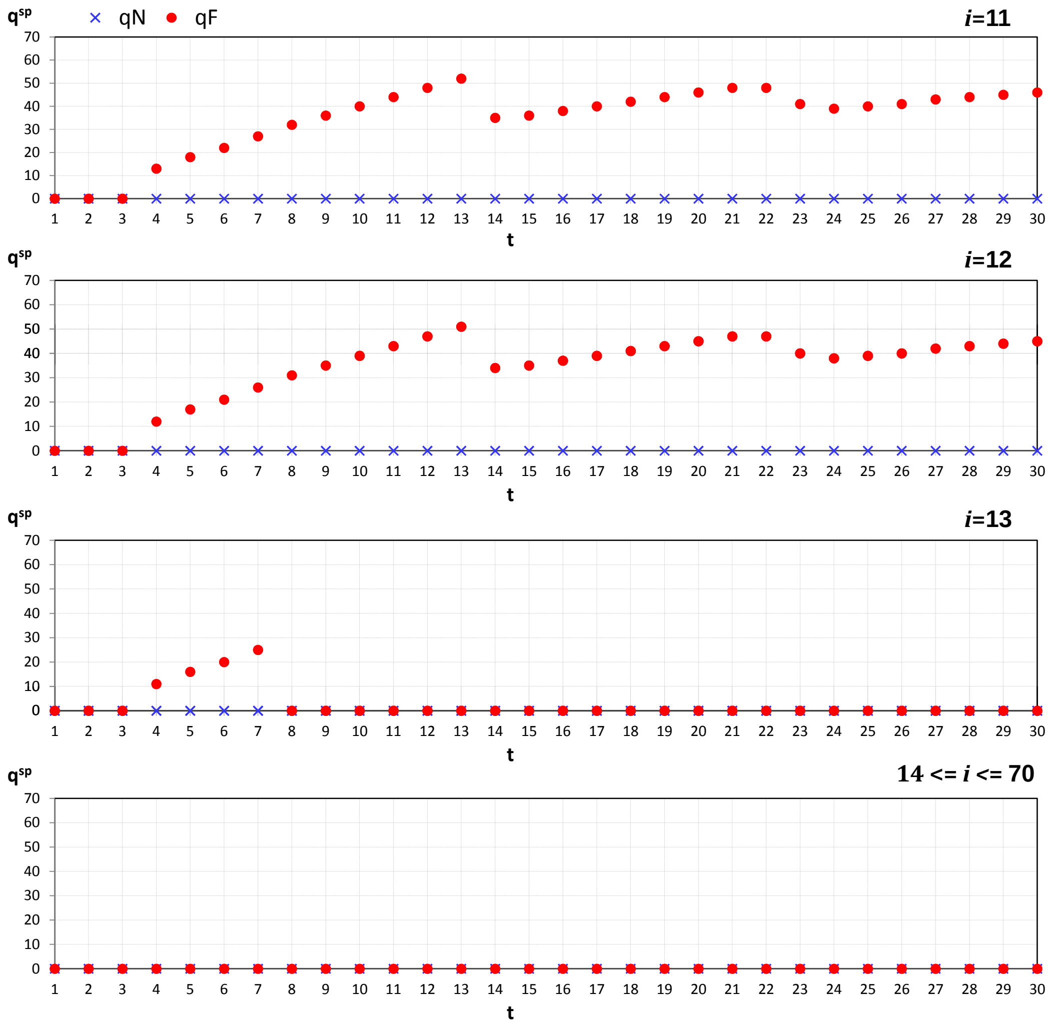


Figure 4.5. M1*FinSto*: The ordering decision from supplier N and supplier F over period, t for $11 \le i \le 70$

Discussion

Under the finite-horizon model with the constant demand setting, if the inventory level is below the quantity of demand, the firm will always place an order with the offshore supplier and only order from the onshore supplier when the inventory level is close to zero. However, if the inventory level is above the quantity of demand but the firm is expecting the inventory level to fall below the demand by the next decision epoch, the firm will place an order with the offshore supplier. If the inventory level is enough to satisfy the demand over the next two periods, the firm will never order anything from the onshore and offshore suppliers. Compare this optimal policy with the policy under the stochastic demand setting. The firm will only order from the onshore supplier if the inventory level is close to zero unless the decision epoch is close to the end of the planning horizon. Also the firm will always order from the offshore supplier if the inventory level is not sufficient to satisfy the expected demand in the current period and the next. Qualitatively the ordering policies in the two cases are similar. However, the relationship between the ordering policy and demand is easier to see in the constant demand case. We conclude that the constant demand model can provide useful insight about the underlying process. The expected cost in every period under the stochastic demand setting is always higher than the same cost in every period under the constant demand setting. This is to be expected as, although the expected demand per period is the same in both models, the uncertainty in the stochastic case makes it harder to satisfy demand.

The ordering policies under the infinite-horizon Model 1 setting

The ordering policies of Model 1 under the infinite horizon planning setting with constant and stochastic demands are as follows:

The ordering policy of M1InfCons

Figure 4.6 shows that when the inventory level, i, falls below the quantity of demand ($i < 5$), the firm will place orders with supplier N and supplier F. The quantity ordered from supplier N is used to satisfy the demand and the quantity ordered from supplier F is used to increase the inventory level at the next decision epoch to 45 which is well below of the maximum inventory level. However, if i is at or slightly above D ($5 \leq i \leq 9$), the firm anticipates that i will fall to below D, at the end of this period and places an order with supplier F. The quantity ordered from this supplier is chosen to increase the inventory level to 45 items at the next decision epoch. If i is high enough to cover D, for at least two periods ($i \geq 10$), the firm will never place any order to supplier N or supplier F. Note that the long run average cost per period, $g = 7.32$ and the fill rate is 100%.

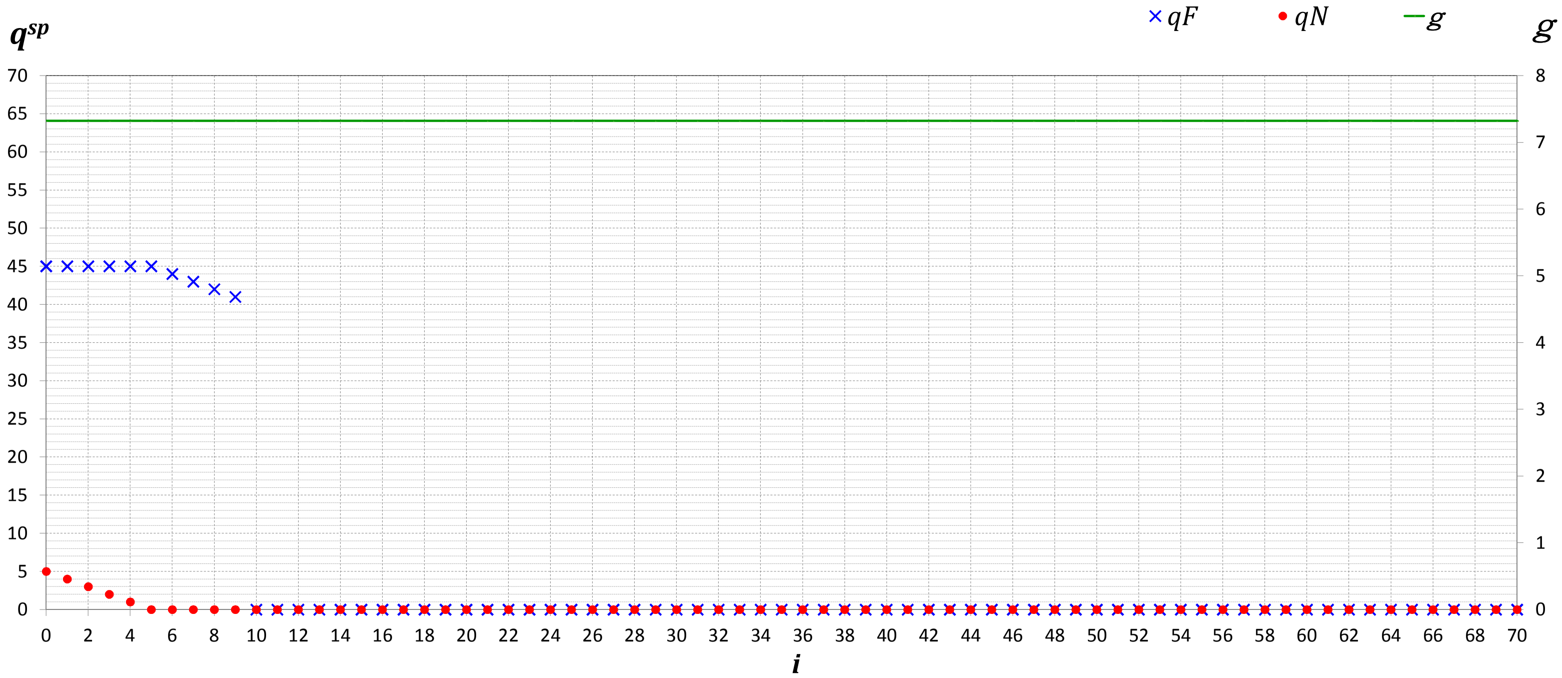


Figure 4.6. The ordering policy of Model 1 with constant demand

The ordering policy of M1InfSto

Figure 4.7 shows that when the inventory level, i, is close to zero ($i < 5$), the firm will place orders with supplier N and supplier F. The quantity ordered from supplier N is used to satisfy demand during the period. The offshore order quantity is 47 while the onshore order quantity decreases as the inventory level increases, so that the inventory position is fixed at 54. Hence, the inventory level at the next decision epoch is at most 54 which is well below the maximum inventory level. If i is slightly higher ($5 \leq i \leq 12$), the firm anticipates that i might fall to a low level by the end of this period and places an order with supplier F. The quantity ordered from this supplier is chosen to increase the inventory position to 54. If i is high enough to cover a sufficient amount of demand for the next two periods, the firm will never place any order with supplier N or supplier F. Note that the long run average cost, $g = 7.69$ and fill-rate, $P_2 = 0.9978 \pm 0.0001$.

Discussion

Under the infinite-horizon setting, the properties of optimal policies of constant and stochastic demand models are more or less the same. The firm only orders from the onshore supplier when there is an immediate shortage or a high risk of a shortage in the short term and orders from the offshore supplier when there is no urgent demand to be satisfied. However, when the inventory is high enough to satisfy demand in the short term, the firm will never order anything from the onshore or offshore suppliers. As expected, the long-run average costs under stochastic demand setting is higher than under constant demand setting. Compared to the finite horizon models, the policies are easier to describe and understand due to the absence of end of planning horizon effects. This is an important advantage of the infinite horizon models.

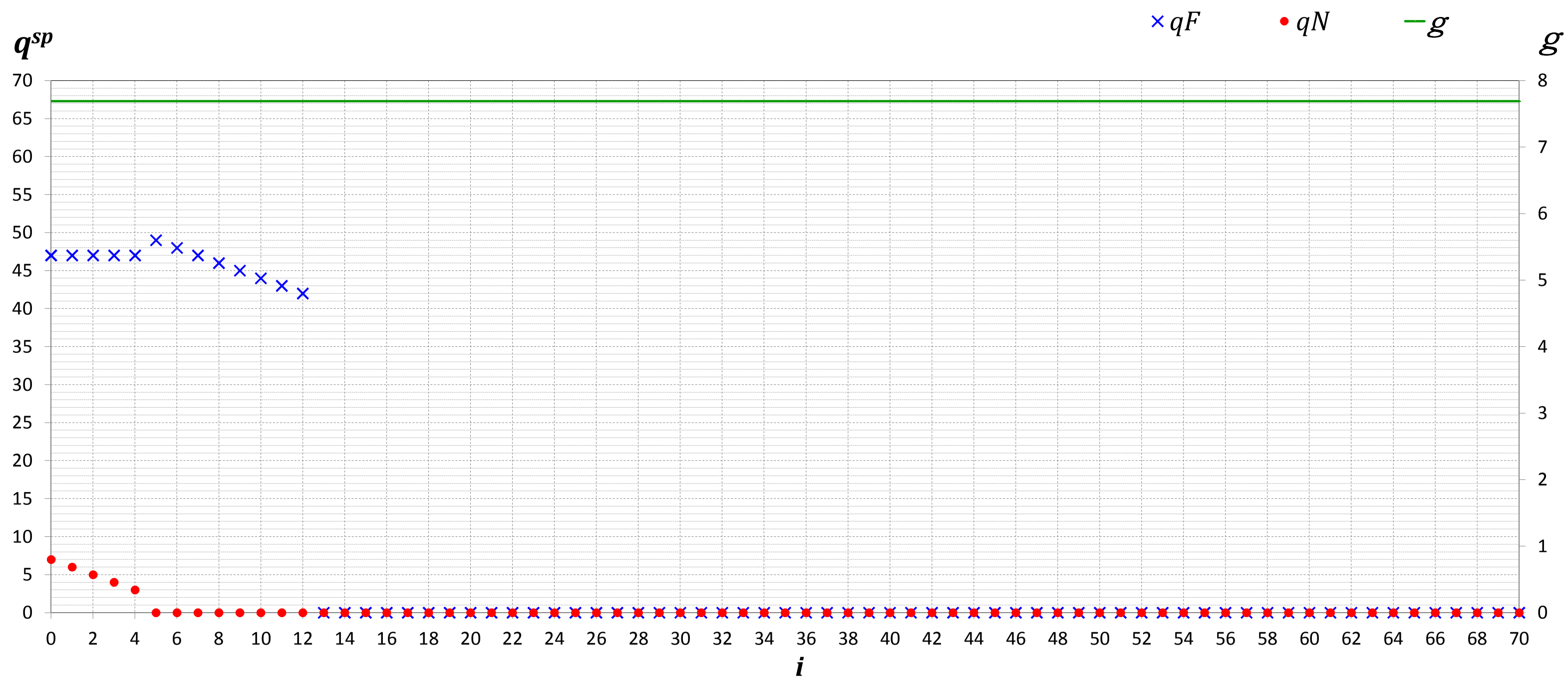


Figure 4.7. The ordering policy of Model 1 with stochastic demand

4.2.5 The Sensitivity Analyses

In this section, we report on the sensitivity of the optimal policies to the average demand, λ, for the stochastic demand models and various costs. We first look into the impact of varying the cost parameters on the ordering policies, explicitly the holding cost, h, the penalty cost, m and the fixed ordering cost of supplier N, c^N. The optimal policies under the various costs are compared with the optimal policies under the base case to examine the effect on the ordering decisions, q^{sp} for $sp = \{N, F\}$, the expected costs of the optimal policy, g, under the finite and infinite horizon DMDP models. The values used are given in table 4.2.

For the holding cost, h, we consider the range of x (where x is the holding cost as a proportion of the value of inventory) with values between 0.15 and 0.55 with an increment of 0.10 units. Thus, in addition to the base case with $x = 0.35$, we consider 4 cases denoted by $h1$, $h2$, $h3$ and $h4$ with $x = 0.15$, $x = 0.25$, $x = 0.45$ and $x = 0.55$, respectively. For the penalty cost, m, we consider the range of values between 2 and 14 with an increment of 2. Thus, in addition to the base case with $m = 8$, we consider 6 cases denoted by $m1$, $m2$, $m3$, $m4$, $m5$ and $m6$ with $m = 2$, $m = 4$, $m = 6$, $m = 10$, $m = 12$ and $m = 14$, respectively. For the fixed ordering cost of the onshore supplier, c_N, we consider the range of values between 1 and 9 with an increment of 1. Thus, in addition to the base case with $c_N = 5$, we consider 8 cases denoted by $c1$, $c2$, $c3$, $c4$, $c5$, $c6$, $c7$ and $c8$ with $c_N = 1$, $c_N = 2$, $c_N = 3$, $c_N = 4$, $c_N = 6$ and $c_N = 7$, $c_N = 8$ and $c_N = 9$, respectively.

We also look at the sensitivity of the value of λ in the stochastic demand models. For the stochastic demand models, we consider various λ values to represent categories of product which are slow and fast moving products. Small values of λ are used to represent the slow moving product, thus $\lambda = \{1, 2, 3\}$ and greater values for the fast moving product, thus $\lambda = \{10, 20, 30\}$. Cases $d1$ to $d3$ represent the values of λ for a slow moving product, with $\lambda = 1, 2$ and 3 for cases $d1$, $d2$ and $d3$ respectively. Cases $d4$ to $d6$ represent the values of λ for a fast moving product, with $\lambda = 10$, 20 and 30 for cases $d4$, $d5$ and $d6$ respectively. The sensitivity test on various λ is summarised in table 4.3. To conduct this experiment, we change one parameter at a time and keep the other parameters unchanged.

Table 4.2. Various sets of costs for sensitivity test

Case	h	m	c_N	D/λ
Base	0.053846	8	5	5
$h1$	0.023077	8	5	5
$h2$	0.038462	8	5	5
$h3$	0.069231	8	5	5
$h4$	0.084615	8	5	5
$m1$	0.053846	2	5	5
$m2$	0.053846	4	5	5
$m3$	0.053846	6	5	5
$m4$	0.053846	10	5	5
$m5$	0.053846	12	5	5
$m6$	0.053846	14	5	5
$c1$	0.053846	8	1	5
$c2$	0.053846	8	2	5
$c3$	0.053846	8	3	5
$c4$	0.053846	8	4	5
$c5$	0.053846	8	6	5
$c6$	0.053846	8	7	5
$c7$	0.053846	8	8	5
$c8$	0.053846	8	9	5

Table 4.3. Various cases of λ for sensitivity test

Case	Base	d1	d2	d3	d4	d5	d6
λ	5	1	2	3	10	20	30

The sections that follow first present results on the sensitivity to h parameter values, then results relating to the sensitivity to m parameter values, followed by the results on the sensitivity to c_N parameter values. Finally, the sensitivity to λ parameter values is presented.

The sensitivity of the holding cost, h

Under the infinite-horizon setting, from figure 4.8 and figure 4.9, for both constant and stochastic demand models, the long-run average costs, g, increase as holding cost increases and decrease as holding cost decreases. This is of course to be expected and provides a degree of verification that the java programs have been implemented correctly. The quantity ordered from the onshore supplier remains the same for all h cases in both constant and stochastic demand models and so is not shown in figure 4.8 and figure 4.9. This can be explained by noting that the firm only orders from the onshore supplier in an immediate emergency and so the items ordered will never need to be held in inventory for a significant period of time. In contrast, the quantity ordered from the offshore supplier decreases as the holding cost increases. Again this is to be expected. Under the finite-horizon setting, the impact of h on the minimum expected cost, $V_t(i)$ and ordering decision, (q^N, q^F) are similar to the infinite-horizon setting and so are not shown in figure 4.8 and figure 4.9.

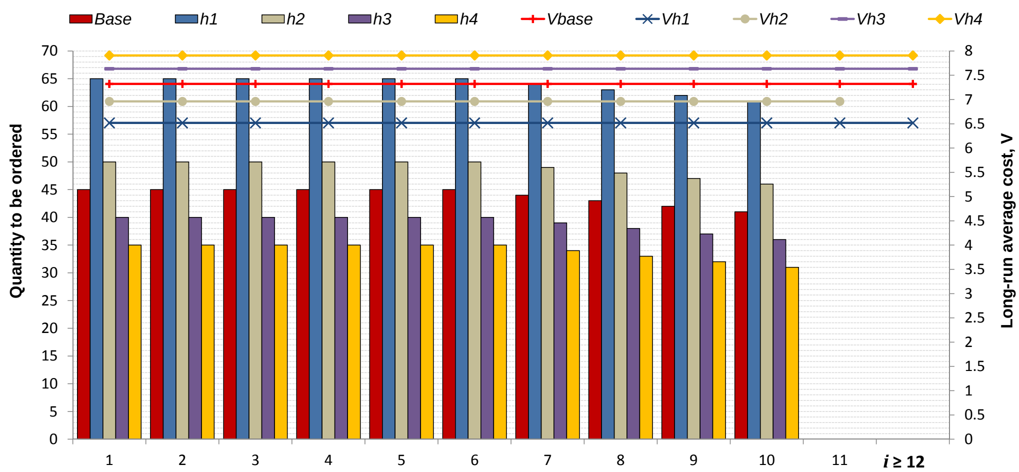


Figure 4.8. The sensitivity of q^F and g for various h under constant demand setting

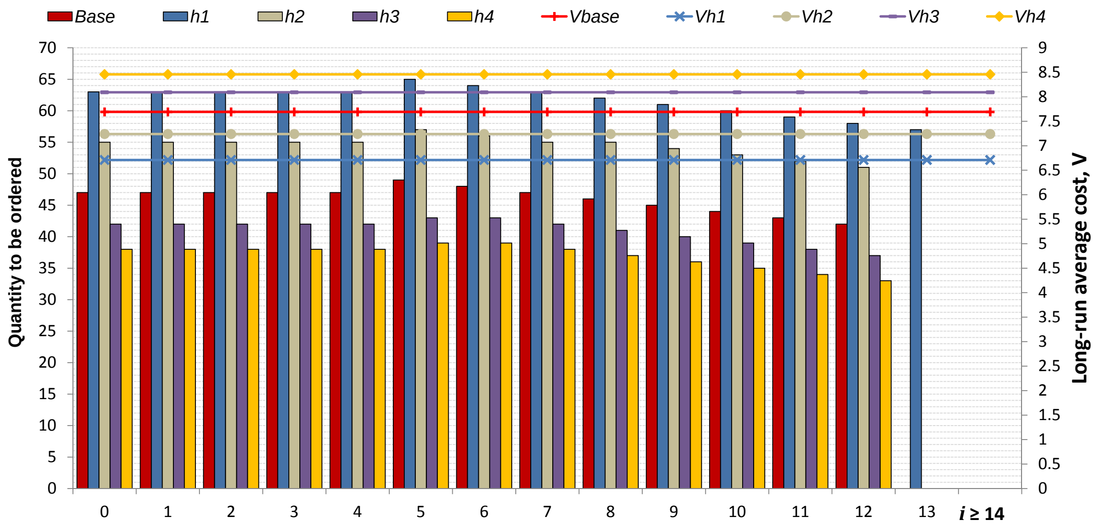


Figure 4.9. The sensitivity of q^F and g for various h under stochastic demand setting

The sensitivity of the penalty cost, m

Under the infinite-horizon setting, for constant demand model, from figure 4.10 and figure 4.11, the long-run average cost, g, and quantity ordered from supplier F, q^F, remain the same for all m cases. At first this seems an unexpected result, however in the base case with constant demand, the firm's order policy ensures that all demand is met. While we might expect that order will change as m gets smaller, it seems that the values of m are not small enough to affect the ordering policy. For stochastic demand model, from figure 4.10 and figure 4.12, for example if $i = 0, 3, 5$ or 10, g and $q^N + q^F$ increase with increases of m and decrease with decreases of m.

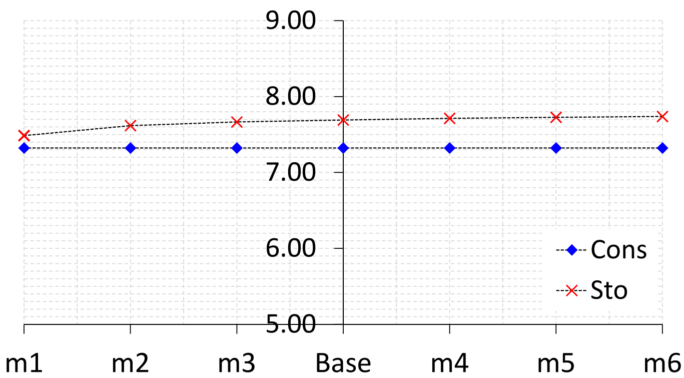


Figure 4.10. The sensitivity of g for various m under constant and stochastic demand settings.

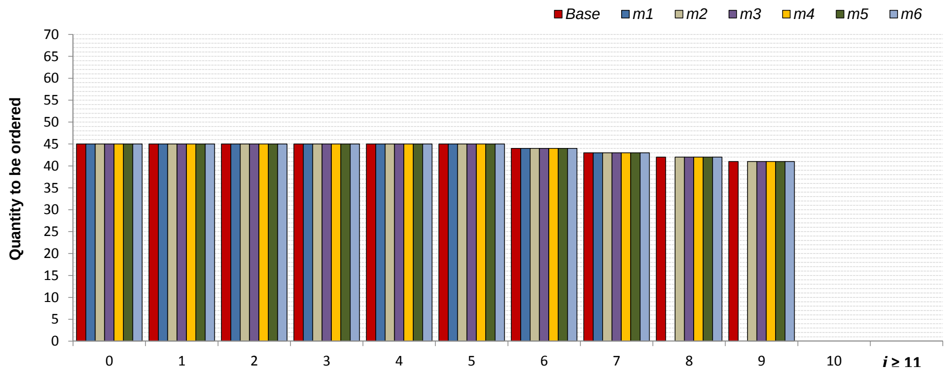
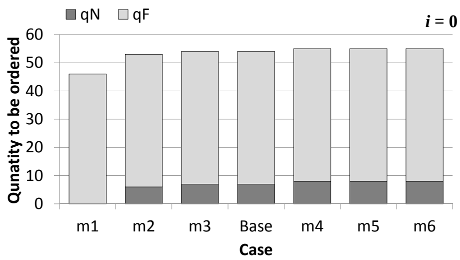
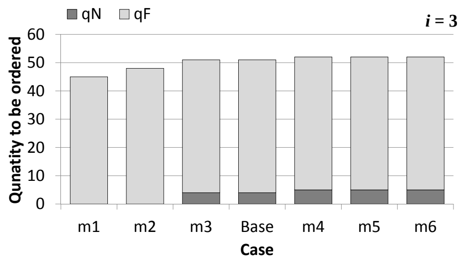


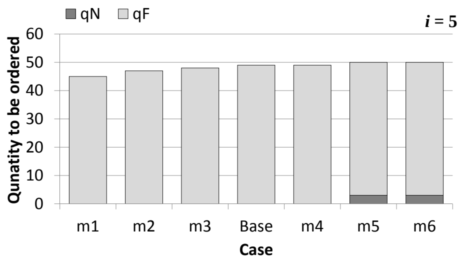
Figure 4.11. The sensitivity of q^F for various m under constant demand setting.



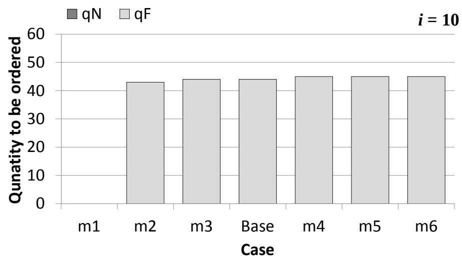
(a) $q^N + q^F$ if $i = 0$



(b) $q^N + q^F$ if $i = 3$



(c) $q^N + q^F$ if $i = 5$



(d) $q^N + q^F$ if $i = 10$

Figure 4.12. The sensitivity of $q^N + q^F$ for various m under stochastic demand setting.

This is of course to be expected and provides a degree of verification that the java programs have been implemented correctly. Under the finite-horizon setting, the impact of m on g, q^N and q^F are similar to the infinite-horizon setting and so are not shown in figure 4.10, figure 4.11 and figure 4.12.

The sensitivity of the ratio of the fixed ordering costs

Figure 4.13 shows that as this fixed order cost for the onshore supplier changes, there is no difference in the long-run average cost, g, under constant demand setting, but there are slight changes under stochastic demand setting. With stochastic demand, the long-run average cost, g, slightly decreases with a decrease in the ratio of the fixed ordering cost, c^N/c^F, and slightly increases with an increase in c^N/c^F. This is of course to be expected and provides a degree of verification that the java programs have been implemented correctly. For the constant demand model, the result seems surprising as we might expect the pattern of variation of g to be similar to the stochastic demand model. However, the optimal policy for constant demand is such that, in the long run, the firm never needs to order from the onshore supplier and so the long run average cost will not depend on the fixed order cost. Under the finite-horizon setting, the impact of c^N/c^F on the minimum expected cost, $V_t(i)$ are similar to the infinite-horizon setting and so are not shown in figure 4.13.

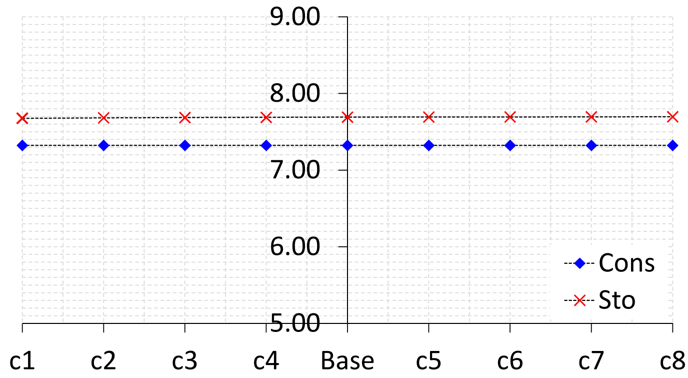


Figure 4.13. The sensitivity of g for various c^N/c^F under constant and stochastic demand settings.

The sensitivity of λ

For a slow moving product, from figures 4.14 and 4.15, quantities ordered from supplier N and supplier F decrease with decreases in λ. The results are to be expected, quantities ordered from both the onshore and offshore suppliers decrease when demand is getting smaller. For a fast moving product, from figures 4.14 and 4.15, quantity ordered from supplier N increases with increases in λ, but quantity ordered from supplier F decreases with increases in λ. This is also to be expected, the firm needs to order more from the onshore supplier to satisfy immediate demand since quantity ordered from the offshore supplier arrives later, thus quantity ordered from the onshore supplier is preferable than the offshore supplier when the average demand increases. The long run average cost, g, and fill-rate, P_2, for each case of λ is tabulated in table 4.4. As we expected, g increases with the increases in λ and the ordering policy in each λ case is able to satisfy at least 99% of demand. For a slow moving product, from figures 4.14 and 4.15, quantities ordered from supplier N and supplier F decrease with decreases in λ. The results are to be expected, quantities ordered from both the onshore and offshore suppliers decrease when demand is getting smaller. For a fast moving product, from figures 4.14 and 4.15, quantity ordered from supplier N increases with increases in λ, but quantity ordered from supplier F decreases with increases in λ. This is also to be expected, the firm needs to order more from the onshore supplier to satisfy immediate demand since quantity ordered from the offshore supplier arrives later, thus quantity ordered from the onshore supplier is preferable than the offshore supplier when the average demand increases. The long run average cost, g, and fill-rate, P_2, for each case of λ is tabulated in table 4.4. As we expected, g increases with the increases in λ and the ordering policy in each λ case is able to satisfy at least 99% of demand.

Table 4.4. The values of g and P_2 for various λ

Case	d1	d2	d3	Base	d4	d5	d6
g	2.18	3.68	5.06	7.69	13.95	27.59	43.37
P_2	0.9948 ±0.0005	0.9949 ±0.0003	0.9974 ±0.0003	0.9978 ±0.0001	0.9981 ±0.0001	0.9954 ±0.0001	0.9918 ±0.0002

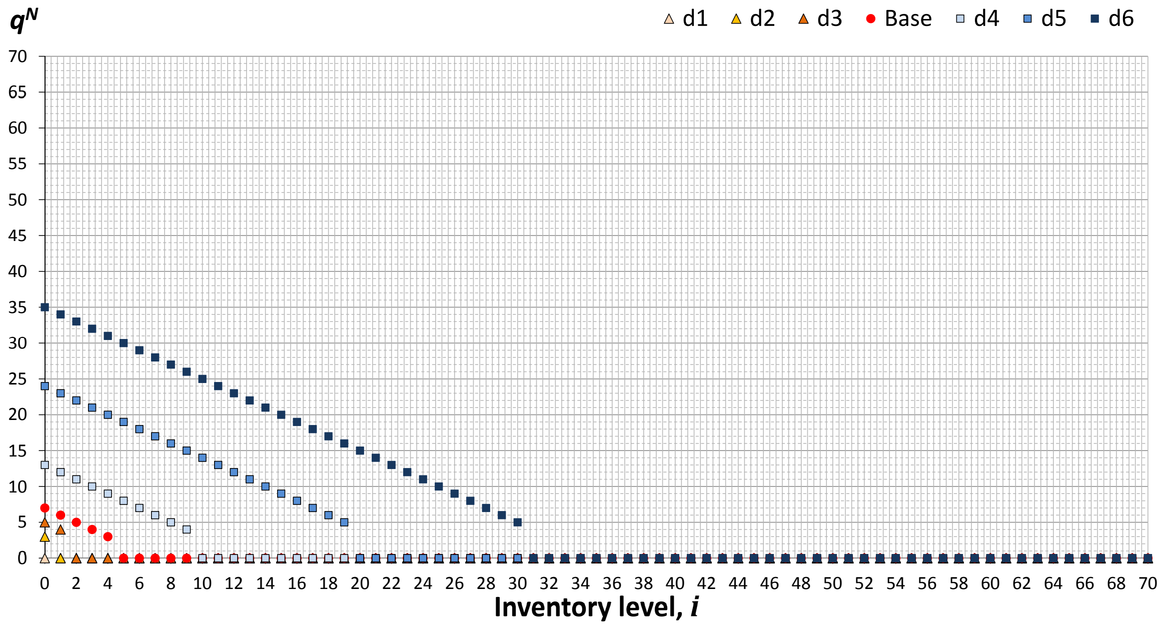


Figure 4.14. The sensitivity of q^N for various λ under stochastic demand setting.

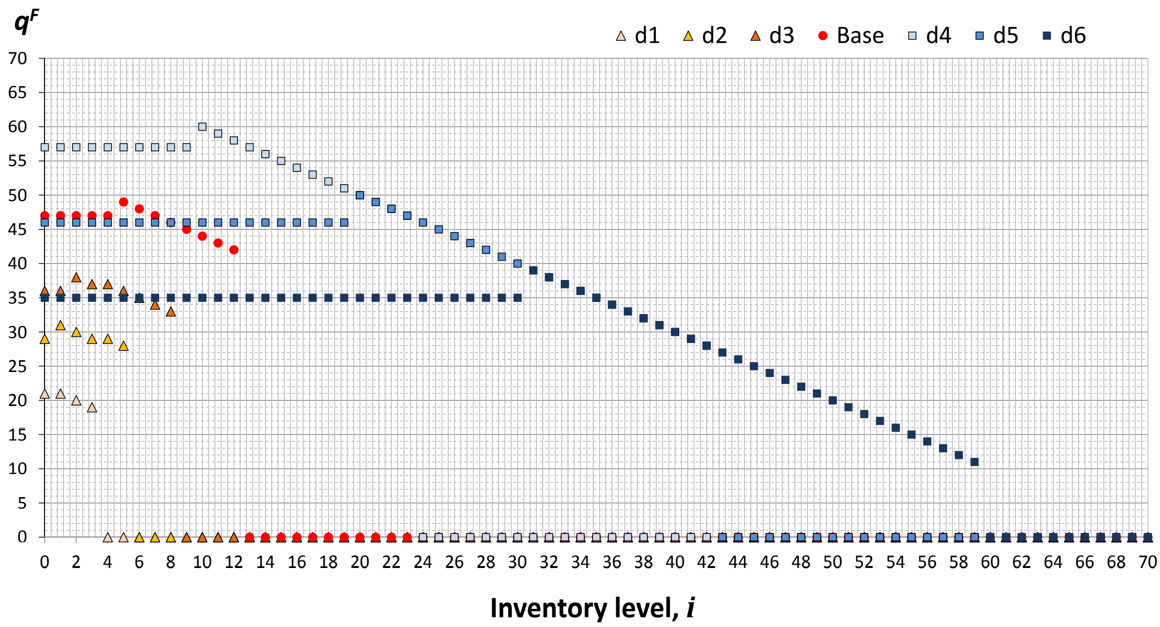


Figure 4.15. The sensitivity of q^F for various λ under stochastic demand setting.

Discussion

From the sensitivity analyses, under the constant demand setting, we can see that, only the change in the holding cost has an impact on the ordering policy. In contrast under the stochastic demand model, the holding cost, the penalty cost and the ratio of fixed costs have impacts on the ordering policy. The sensitivity between the ordering policy and various costs is easier to see in the stochastic demand case. We conclude that the stochastic demand model provides useful insight about the changes in various costs. This is to be expected as the stochastic case is more sensitive due to the existence of the probability distribution of demand. From the results, we can conclude that the offshore supplier becomes more attractive as the penalty cost decreases and the holding cost decreases. Whilst, the onshore supplier becomes more attractive as the fixed cost of the onshore supplier decreases.

4.2.6 Conclusion

Model 1 has been developed with very simple assumptions to demonstrate the optimal ordering policy different types of demand and the different planning horizon settings. Supposedly, the property of the optimal policies from Model 1 represents the ordering decision for the firm who has chosen to implement non-identical dual-sourcing strategy on an on-going basis to hedge any shortage in the inventory and with the aim to always satisfy the customer's demand. From the findings, the firm will only order from the onshore supplier if there is demand that cannot be satisfied with stock in the inventory, since the order from the offshore supplier will not arrive until later. However, if there is no shortage in the inventory, placing order with the offshore supplier is more favourable since this supplier is more attractive with lower ordering cost. The results from Model 1 have provided us with some basic understanding on how the firm who has implemented global dual-sourcing strategy can manage the inventory system under different conditions of the production horizon plan and type of demand. We would say, the offshore supplier acts as a main source and the onshore supplier as an emergency backup source in the event of shortage in the inventory.

The findings are adequate even though Model 1 has been developed for a very simple setting without considering other complex structures affecting the inventory system. As we explained at the beginning of this chapter, the inventory system of Model 1 is different from other complex models of the inventory system with non-identical dual-source option in the literature (e.g., dual-index policy). Model 1 has been designed as simple as it is with the idea to avoid any complexity in finding the optimal ordering policy. We believe that our approach is sufficient enough to capture managerial insights. Model 1 will be considered as a benchmark model for subsequent models where we will consider the element of supply disruption in modelling the ordering decision for the firm.

In conclusion, a benchmark of the firm's global optimal policy is established for further analyses on the firm's ordering process under the environment of disruptive supply events by considering supply disruption at the offshore supplier. For this reason, therefore, in the next section, we introduce the crisis ordering model or Model 2.

4.3 The Crisis Ordering Model

Model 2 presents a model involving a risk of supply disruption at the offshore supplier. To define the supply disruption process at the supplier in Model 2, we refer to the context of an *inbound supply risk*. The inbound supply risk can be defined as any events/activities that involve the disruption of inbound supply that can have significant detrimental effects on the purchasing firm (Wu et al., 2006; Zsidisin et al., 2004). Therefore, we think of disruptions at the offshore supplier as any disruptive event at the supplier that disrupts the supply process between the offshore supplier and the firm. The disruption process at the offshore supplier is represented by the status of this supplier and the status is assumed to be at either up or down. Order delivery from the offshore supplier is consider as an *all-or-nothing* process. If the supplier is in the up state, the firm will receive a complete order and if the supplier is in the down state, the firm will receive nothing.

The state of the offshore supplier is modelled as a two-state Markov chain process and we investigate numerically how the firm's ordering decision is affected by the transition matrix of the Markov chain. The values of the transition matrix represent any measurement value that can quantify the risk of supply disruption at the offshore supplier. For example, the values represent the frequency of the disruption and the length of the supply disruption. For example, if the probabilities of moving from up to down and of remaining in the down state are small, they may represent a rare disruption event of short length. However, if the probability of moving from up to down is small and the probability of remaining in the down state is large, the probabilities may represent a rare disruption event of long length. In Model 2, we seek to determine the optimal ordering policy via the DMDP. We expect that, on the one hand, if the offshore supplier is in the up state but the risk of disruption to this supplier is high, then the firm will want to order more from this supplier. On the other hand, if the offshore supplier is in the down state and the likelihood of this supplier recovering from the disruption in the short term is low, then the firm will increase the quantity to be ordered from the onshore supplier.

The structure of this section is as follows. We describe the two-state Markov chain model of the state of the offshore supplier in Model 2 in section 4.3.1 followed by the assumptions of Model 2 in section 4.3.2. Then, the formulation of the ordering decision problem under supply disruption via the DMDP is presented in section 4.3.3 and the transition probability values used when conducting the numerical experiment in section 4.3.4. The results and findings are reported in section 4.3.5, section 4.3.6 and section 4.3.7. Finally, the conclusion for Model 2 is presented in section 4.3.8.

4.3.1 Model Description

In Model 2 the firm seeks to split the order between the onshore supplier, N, and the offshore supplier, F, based on the risk of supply disruption at supplier F. During normal operations of supplier F, the firm can order from both suppliers. During a disruption at supplier F, the firm can only order from supplier N. The Markov model of the state of supplier F is as follows. The state of supplier F is assumed to be either in up state, u, or down state, w. If supplier F is in state u, then a complete order will be delivered to the firm, otherwise if in state w, then nothing will be supplied. When disruption occurs at supplier F, the state of supplier F moves from state u to w. Otherwise, supplier F remains in the same state u. When supplier F recovers from disruption, the state of supplier F moves from state w to u. Otherwise, while the recovery process continues, supplier F remains in the same state w. For a better understanding, the transitions between states u and w for supplier supplier F are illustrated in figure 4.16.

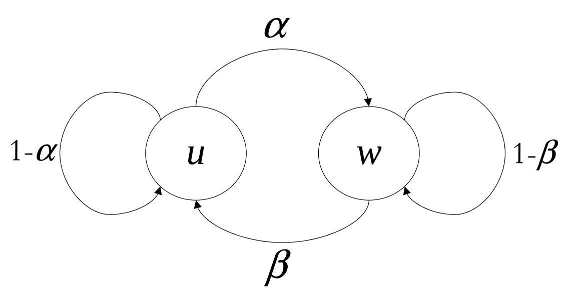


Figure 4.16. The transitions between states of supplier F

In figure 4.16, α represents the probability that there is a disruption at supplier supplier F (later known as a *supply disruption probability*) and so the state of supplier supplier F moves from state u to w. β represents the probability that supplier F recovers from the disruption (later known as a *disruption recovery probability*) and so the state of supplier F moves from state w to u. Whenever the state of supplier F is in state u or w, the state remains the same with probabilities $1-\alpha$ or $1-\beta$ respectively.

4.3.2 Model Assumptions

The assumptions of Model 2 are as follows:

a. The option of sourcing from supplier F is at risk of disruptions. The order from supplier F arrives either in full or not at all. However, the status of supplier N is perfectly reliable.

b. The rate of the transition of supplier F's state is known and fixed.

c. The firm's inventory planning horizon is discrete.

d. Demand is either deterministic or stochastic. The stochastic demand distribution follows the truncated Poisson distribution, $P(d_t) \sim Pois(\lambda, K)$.

e. Customers do not accept backorders, thus the firm encounters lost sales. The firm is charged with a penalty cost for each unit of demand that cannot be satisfied in a period, $PNLTY$.

f. The firm incurs a holding cost for inventory held during period t, $HOLD$.

4.3.3 Model Formulation

In this section, we explain the formulation of the problem analysed in Model 2 as a DMDP model and present the optimality equation.

Components of the DMDP for Model 2

The DMDP components in Model 2 are as follows:

Decision epoch

A decision is taken at the beginning of each period. Let t denote the decision epoch when there are t periods remaining in the planning horizon, $t = 1, 2, \ldots, T$.

States

At each decision epoch, the firm reviews the inventory level, i, and the state of supplier F, a. The parameters i and a comprise the state of the process y, such that $y = (i, a)$. The state space, Y, of Model 2 is given by:

$$Y = \Big\{(i, a) : \; i \in \{0, 1, \ldots, I\} \;\; \& \;\; a \in \{u, w\}\Big\}.$$

Actions:

Based on the current state, the firm then decides on the quantities to order from supplier N and supplier F. The action is denoted by $b = (q^N, q^F)$ and the set of admissible actions, $B(y)$ is given by:

$$B(i, u) = \;\; \Big\{(q^F, q^N) : \;\; q^F, q^N \geq 0 \quad \& \quad q^F + q^N \leq I - i\Big\} \quad \text{for} \quad 0 \leq i \leq I.$$

$$B(i, w) = \;\; \Big\{(0, q^N) : \quad q^N \in \{0, \ldots\ldots, I - i\}\Big\} \quad \text{for} \quad 0 \leq i \leq I.$$

Under the admissible action set $B(i, u)$, the firm can choose to order up to $I - i$ items either from supplier N only or from supplier F only or from both the suppliers. While under the admissible action set $B(i, w)$, the firm can choose to order up to $I - i$ items from supplier N only.

Transition probabilities:

We model changes in the inventory level, i, and changes in the states of supplier F, a, separately. The transition matrix describing changes in the inventory level, i, depends on the order quantities and is the same as in previous models. See section 4.2.3a for a full description. The transition matrix describing changes in the state of supplier F follows from figure 4.16 above. The transition matrix is denoted by X and formally presented below.

$$X = \begin{array}{c c} & \begin{array}{cc} u & w \end{array} \\ \begin{array}{c} u \\ w \end{array} & \begin{pmatrix} 1 - \alpha & \alpha \\ \beta & 1 - \beta \end{pmatrix} \end{array}$$

One-step costs:

As in Model 1, the one-step cost function, as a result of choosing action b in state y consists of the ordering cost, $ORDER$, the holding cost, $HOLD$ and the penalty cost, $PNLTY$. In the one-step cost function with stochastic demand, the values of $HOLD$ and $PNLTY$ depend on the random demand during the period. The one-step costs for Model 2 with the constant and stochastic demand settings are based on the same assumptions as the one-step costs for the models in previous chapters. See section 3.2.1 for a detailed explanation.

The one-step cost of choosing action b at decision epoch t when the system is in state y is denoted by $C_t^y(b)$. Under a constant demand setting, this cost is given by:

$$
\begin{aligned}
C_t^y(b) &= ORDER + HOLD + PNLTY \\
&= \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \; h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\
&\quad + \; m\Big(\max(D_t - i - q_t^N, 0)\Big)
\end{aligned}
$$

and under a stochastic demand setting it is given by:

$$
\begin{aligned}
C_t^y(b) &= ORDER + \Big(E_{D_t}\Big(HOLD + PNLTY\Big)\Big) \\
&= \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t)\Big\{h\Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) \\
&\quad + \; m\Big(\max(d_t - i - q^N, 0)\Big)\Big\}
\end{aligned}
$$

Optimality equation

Let $V_t(i, a)$ be the minimum expected cost over the remainder of the planning horizon when the inventory level is i and the state of the offshore supplier is a at decision epoch t. The optimality equation for Model 2 with constant demand is given by:

$$\begin{aligned}
V_t(i,a) = \min_{b\in B(i,a)}\Bigg\{ &\sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + m\Big(\max(D_t - i - q^N, 0)\Big) \\
&+ h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\
&+ \Big(X_{a,u} \times V_{t-1}(\max(i + q^N - D_t, 0) + q^F, u)\Big) \\
&+ \Big(X_{a,w} \times V_{t-1}(\max(i + q^N - D_t, 0) + q^F, w)\Big)\Bigg\}.
\end{aligned}$$

The optimality equation with stochastic demand is given by:

$$\begin{aligned}
V_t(i,a) = \min_{b\in B(i,u)}\Bigg\{ &\sum_{sp\in\{N,F\}} \delta(q^{sp})c^{sp} + q^{sp}v^{sp} + \sum_{d_t=0}^{\infty} P(D_t = d_t)\Bigg\{m\Big(\max(d_t - i - q^N, 0)\Big) \\
&+ h\,\Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) \\
&+ \Big(X_{a,u} \times V_{t-1}(\max(i + q^N - d_t, 0) + q^F, u)\Big) \\
&+ \Big(X_{a,w} \times V_{t-1}(\max(i + q^N - d_t, 0) + q^F, w)\Big)\Bigg\}\Bigg\},
\end{aligned}$$

Using these optimality equations, we seek to minimse $V_t(i,a)$ and find the optimal quantities to order from supplier N and supplier F, depending on the values in the transition matrix, X. We are interested to investigate numerically how the values in the transition matrix, and therefore the characteristics of the disruption, can affect the firm's ordering decisions.

Note that, the analysis of the model with stochastic demand involves two-sided uncertainty involving a supply disruption and a stochastic demand. To avoid any confusion between supply and demand uncertainties when interpreting the finding result, we assume that supply and demand uncertainties are stochastically independent.

An equilibrium distribution of the Markov chain model

Let $\pi = (\pi_u, \pi_w)$ be an equilibrium distribution of the Markov chain model of the state of the offshore supplier. If is known (Tijms, 2003), that π must satisfy:

$$\pi_u = (1-\alpha)\pi_u + \beta\pi_w \tag{4.1}$$

$$\pi_w = \alpha\pi_u + (1-\beta)\pi_w \tag{4.2}$$

$$1 = \pi_u + \pi_w \tag{4.3}$$

Since $0 < \alpha < 1$ and $0 < \beta < 1$, the Markov chain is aperiodic and ergodic. Therefore, there is a unique equilibrium distribution which describes the long run behaviour of the Markov chain. From 4.1,

$$\pi_u = \frac{\beta}{\alpha}\pi_w.$$

Substitution of this expression for π_u in 4.3 and solving for π_w gives:

$$\pi_w = \frac{\alpha}{\alpha+\beta}$$

Hence $\pi_u = \frac{\beta}{\alpha+\beta}$. The probabilities π_u and π_w can be interpreted as the long-run proportion of time that supplier F is up and down respectively.

4.3.4 Choice of Parameters Values

In this section, we present various transition probability values used for the numerical analysis. Our objective of the analysis is to analyse how the optimal policy changes with different transition probabilities. In this numerical study, we combine a few values of supply disruption probabilities, α, and disruption recovery probabilities, β, and generate 25 cases, as shown in table 4.5. For other parameters in the optimality equation, we use the base values that have been presented in section 3.6.1.

We number the cases according to supply disruption probabilities, α, and disruption recovery probabilities, β, values. For example for case 1A, number **1** is used to represent the corresponding sets of α values and letter **A** is used to represent the corresponding sets of β values. Five values for α were selected to represent different disruption frequencies ranging from very frequent (α = 0.9) to rare (α =0.1). Similarly, five values for beta were selected to represent different lengths of disruption ranging from very short (β = 0.9) to long (β = 0.1). Twenty five scenarios corresponding to all possible combinations of these values of alpha and beta are considered in the analysis. The enumerations of these numbers and letters have been sorted into ascending α and β values. The higher the numbers or letters, the bigger the α and β values are and vice versa.

In addition, we are also interested to examine the impacts of the values of the expected length of a disruption, the expected length of an interval of normal service and the proportion of time for which the offshore supplier is up to the optimal policy. The values of these four parameters are tabulated in table 4.5. The expected length of disruption is the mean of a geometric distribution with parameter β, which denoted by $1/\beta$. Similarly for the expected length of a period of normal service, which denoted by $1/\alpha$. The proportion of time for which the offshore supplier is up or down, which is denoted by π_u or π_w respectively, can be obtained from the equilibrium distribution of the Markov chain, as shown is section 4.3.3c.

From this numerical study, we illustrate the effects of the transition probabilities, case by case, on the three areas namely the firm's optimal ordering decisions, the cost of optimal policies and the performance of the optimal policy under the stochastic demand model (i.e., fill rate and average inventory). To do the experiment, we analyse Model 2 with the combination of α and β values, case by case, as in table 4.5.

In what follows, we first present results on the effects of the cases on the properties of the ordering decisions, then results relating to the effects on the properties of the costs of policies, and finally the results on the effects of the fill rate and the average inventory under the stochastic demand model analysis.

Case	α	β	$\frac{1}{\alpha}$	$\frac{1}{\beta}$	π_u	π_w
1A	0.10	0.10	10.0	10.0	0.50	0.50
1B	0.10	0.30	10.0	3.33	0.75	0.25
1C	0.10	0.50	10.0	2.00	0.83	0.17
1D	0.10	0.70	10.0	1.43	0.88	0.13
1E	0.10	0.90	10.0	1.11	0.90	0.10
2A	0.30	0.10	3.33	10.0	0.25	0.75
2B	0.30	0.30	3.33	3.33	0.50	0.50
2C	0.30	0.50	3.33	2.00	0.70	0.30
2D	0.30	0.70	3.33	1.43	0.25	0.75
2E	0.30	0.90	3.33	1.11	0.75	0.25
3A	0.50	0.10	2.00	10.00	0.17	0.83
3B	0.50	0.30	2.00	3.33	0.38	0.63
3C	0.50	0.50	2.00	2.00	0.50	0.50
3D	0.50	0.70	2.00	1.43	0.64	0.36
3E	0.70	0.90	2.00	10.00	0.17	0.83
4A	0.70	0.10	1.43	10.00	0.13	0.88
4B	0.70	0.30	1.43	3.33	0.30	0.70
4C	0.70	0.50	1.43	2.00	0.42	0.58
4D	0.70	0.70	1.43	1.43	0.50	0.50
4E	0.70	0.90	1.43	1.11	0.56	0.44
5A	0.90	0.10	1.11	10.00	0.10	0.90
5B	0.90	0.30	1.11	3.33	0.25	0.75
5C	0.90	0.50	1.11	2.00	0.36	0.64
5D	0.90	0.70	1.11	1.43	0.44	0.56
5E	0.90	0.90	1.11	1.11	0.50	0.50

Table 4.5. 25 cases based on various combination of α and β values

4.3.5 The Impacts of Transition Probabilities on the Properties of the Optimal Ordering Decisions

In this section, we explain how supply disruption probabilities, α, and disruption recovery probabilities, β, values can affect the firm's ordering decisions. We first discuss the result of the ordering decision under the finite-horizon setting in section 4.3.5a covering the finite-horizon Model 2 with constant demand (later known as M2*FinCons*) and stochastic demand (later known as M2*FinSto*). Then, we report the ordering policies under the infinite-horizon setting in section 4.3.5b covering the infinite-horizon Model 2 with constant demand (later known as M2*InfCons*) and stochastic demand (later known as M2*InfSto*).

The optimal ordering decisions with the finite-horizon model

The optimal ordering policies under the finite-horizon planning are as follows.

The ordering policy of M2FinCons

If supplier F is in state u, the firm only places an order with supplier N if there is an immediate shortage of inventory (i.e., $i < D$) at every decision epoch in all cases. In these situations, the quantity ordered from supplier N is just enough to meet the immediate shortage (i.e., $D - i$). This aspect of the policy is the same as for Model 1. However, the optimal order placed with supplier F does depend on the supply disruption and disruption recovery probabilities. This is illustrated in figures 4.17, 4.18, 4.19, 4.20 and 4.21. For example, when $i = D = 5$, from figure 4.17, in case 1 ($\alpha = 0.1$) for all β cases, the firm will always place the order with supplier F unless there is very little time remaining in the planning horizon ($t \leq 2$) because the firm has expected i will fall below D between the current and the next decision epoch and the ordered quantity decreases as β increases. In other α cases, from figures 4.18, 4.19, 4.20 and 4.21 the ordering policy is the practically identical to case 1 for each β case for decision epochs in the second half of the planning horizon ($t < 15$).

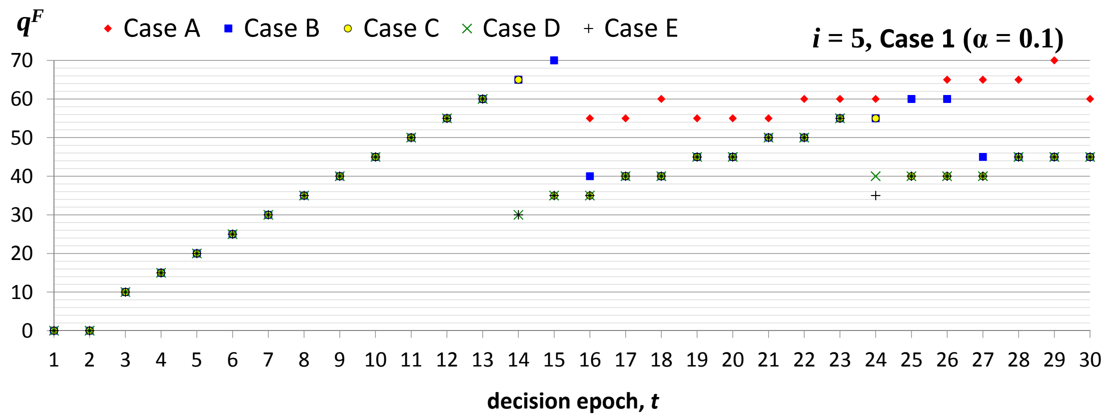


Figure 4.17. M2*FinCons*, state u: The optimal order from supplier F in Case 1

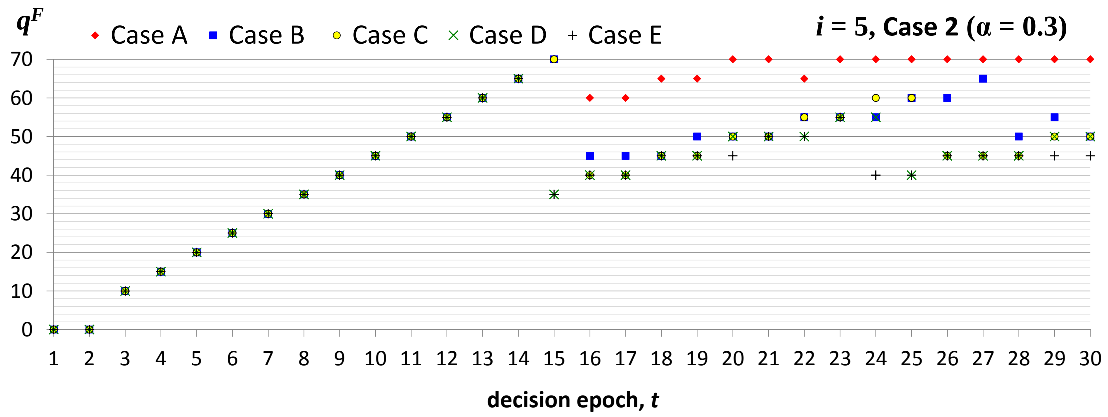


Figure 4.18. M2*FinCons*, state u: The optimal order from supplier F in Case 2

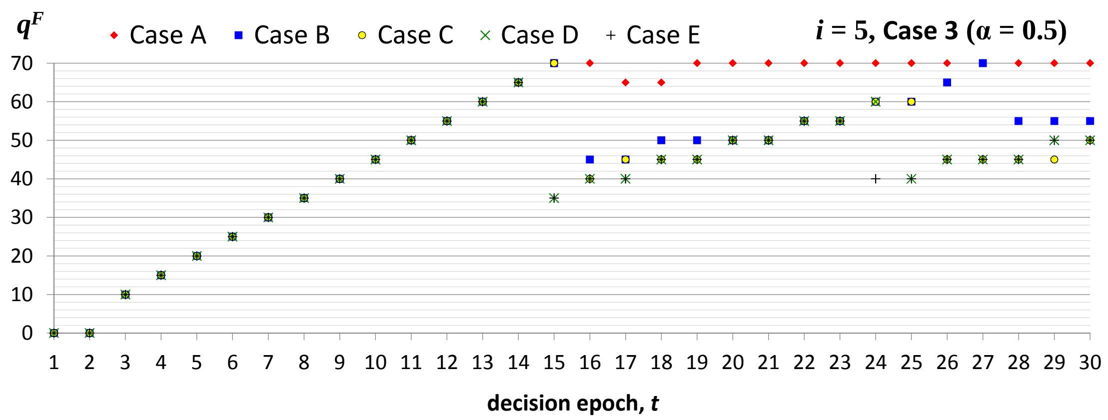


Figure 4.19. M2*FinCons*, state u: The optimal order from supplier F in Case 3

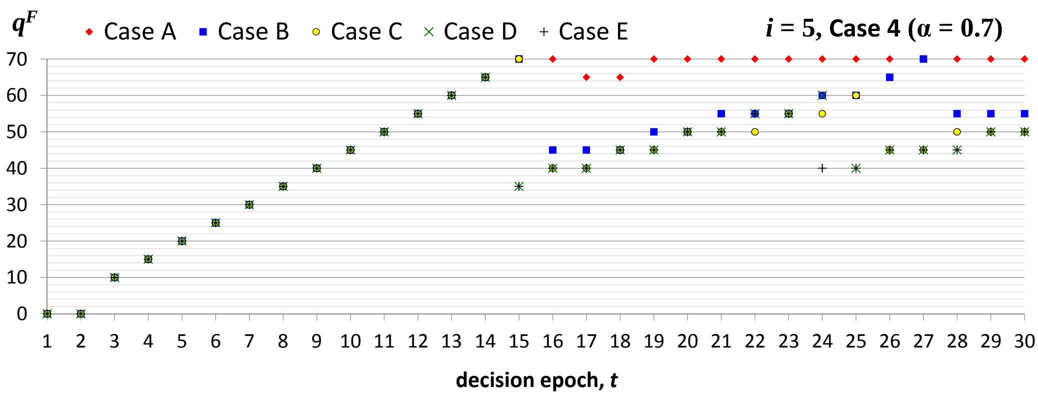


Figure 4.20. M2*FinCons*, state u: The optimal order from supplier F in Case 4

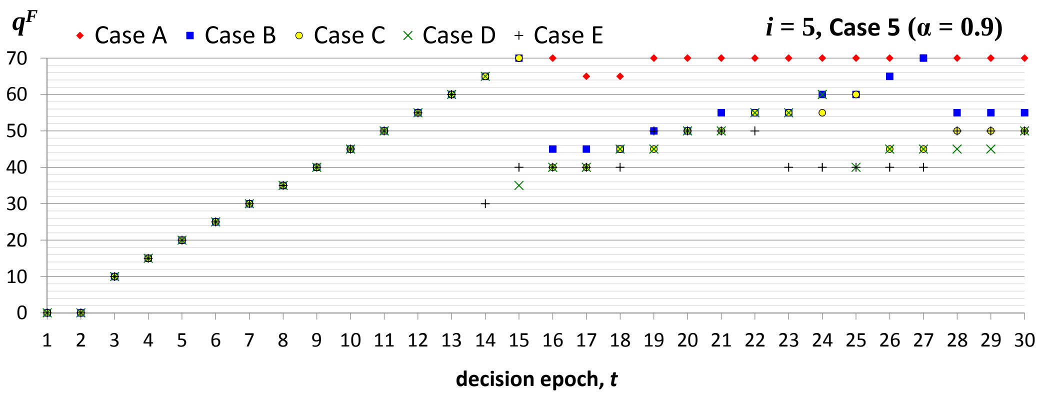


Figure 4.21. M2*FinCons*, state u: The optimal order from supplier F in Case 5

However, in the first half of the planning horizon ($15 \leq t \leq 30$), the firm will increase the quantity ordered from supplier F as α increases for each β and will decrease the quantity ordered from supplier F as β increases for each α. Assuming that for most decision epochs the order policy is generally order up to S when i is below s, the order policy for supplier F for all cases of α and β and a fixed decision epoch (i.e., $t = 10, 20$ and 30) are tabulated in tables 4.6, 4.7 and 4.8. The tables show that the order-up-to level and reorder point are non-decreasing in the probability of supply disruption. Hence, as the risk of a disruption occurring increases, the firm will keep a larger stock of items from the offshore supplier. The tables also show that the order up to level and the reorder point are non-increasing in the probability of disruption recovery. Hence, as the expected length of disruption decreases, the firm will keep a smaller stock of items from the offshore supplier.

Table 4.6. M2*FinCons*, state u: Optimal ordering policy for supplier F in $t = 10$

β α	**A** 0.1	**B** 0.3	**C** 0.5	**D** 0.7	**E** 0.9
1 0.1	(19,45)	(9,45)	(9,45)	(9,45)	(9,45)
2 0.3	(34,45)	(22,45)	(14,45)	(11,45)	(11,45)
3 0.5	(37,45)	(24,45)	(18,45)	(14,45)	(14,45)
4 0.7	(37,45)	(24,45)	(18,45)	(14,45)	(14,45)
5 0.9	(39,45)	(29,45)	(19,45)	(14,45)	(14,45)

Table 4.7. M2*FinCons*, state u: Optimal ordering policy for supplier F in $t = 20$

β α	**A** 0.1	**B** 0.3	**C** 0.5	**D** 0.7	**E** 0.9
1 0.1	(13,46)	(9,45)	(9,45)	(9,45)	(9,45)
2 0.3	(30,70)	(18,50)	(14,50)	(11,50)	(10,45)
3 0.5	(34,70)	(19,50)	(14,50)	(14,50)	(13,50)
4 0.7	(38,70)	(22,50)	(16,50)	(14,50)	(14,50)
5 0.9	(39,70)	(24,50)	(18,50)	(14,50)	(14,50)

Table 4.8. M2*FinCons*, state u: Optimal ordering policy for supplier F in $t = 30$

β α	**A** 0.1	**B** 0.3	**C** 0.5	**D** 0.7	**E** 0.9
1 0.1	(9,60)	(9,45)	(9,45)	(9,45)	(9,45)
2 0.3	(29,70)	(15,50)	(14,50)	(10,50)	(9,45)
3 0.5	(34,70)	(19,55)	(14,50)	(14,50)	(13,50)
4 0.7	(38,70)	(19,55)	(15,50)	(14,50)	(14,50)
5 0.9	(39,70)	(21,55)	(16,50)	(14,50)	(14,50)

If supplier F is in state w, the firm now only has an option to order from supplier N and the quantity ordered from this supplier N under this situation is bigger than the order quantity during normal operation (i.e., when supplier F is in state supplier N). From figures 4.22, 4.23 and 4.24, when $i = 0$, in each α case, the quantity ordered decreases when β increases. We can see that the properties of the ordering policy for supplier N under the crisis event are almost the same for each α case, thus we can conclude that the order policy under this situation depends on the disruption recovery probability, but not the supply disruption probability. In addition, the firm will order more from supplier N when the probability of disruption recovery is very low (i.e., $\beta = 0.1$) since the disruption is expected to last longer.

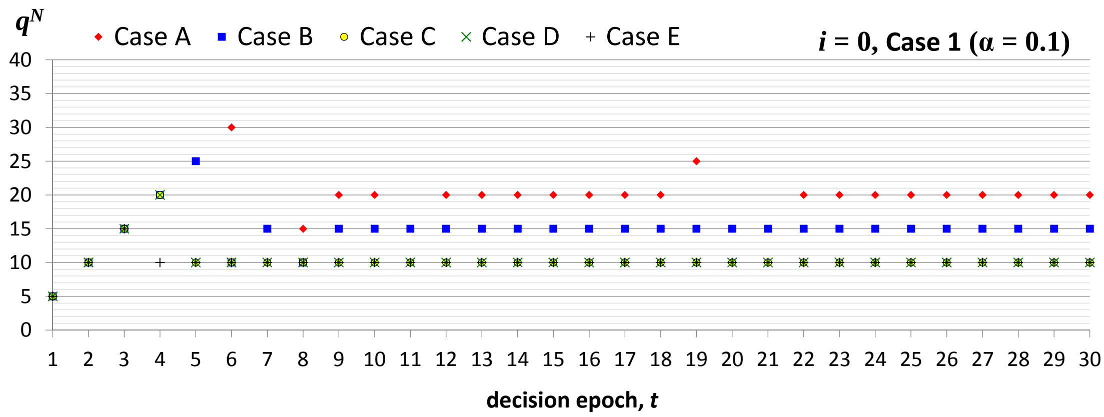


Figure 4.22. M2*FinCons*, state w: The optimal order from supplier F in Case 1

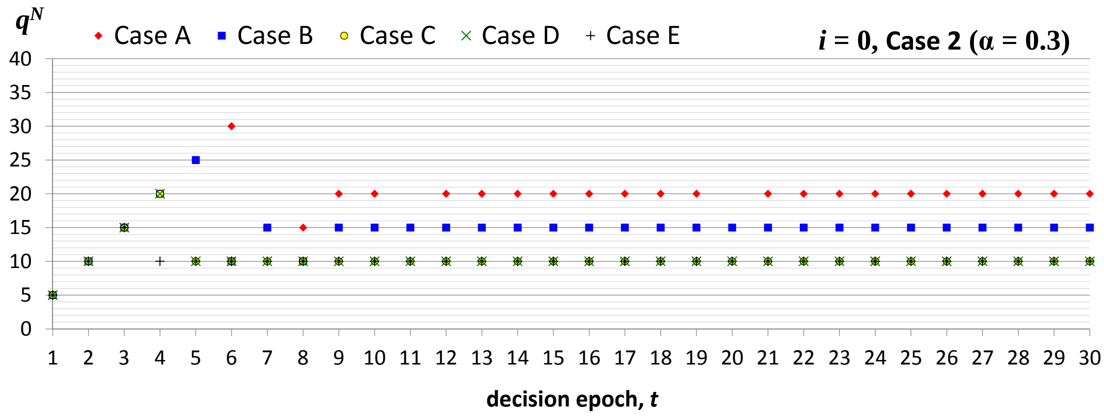


Figure 4.23. M2*FinCons*, state w: The optimal order from supplier F in Case 2

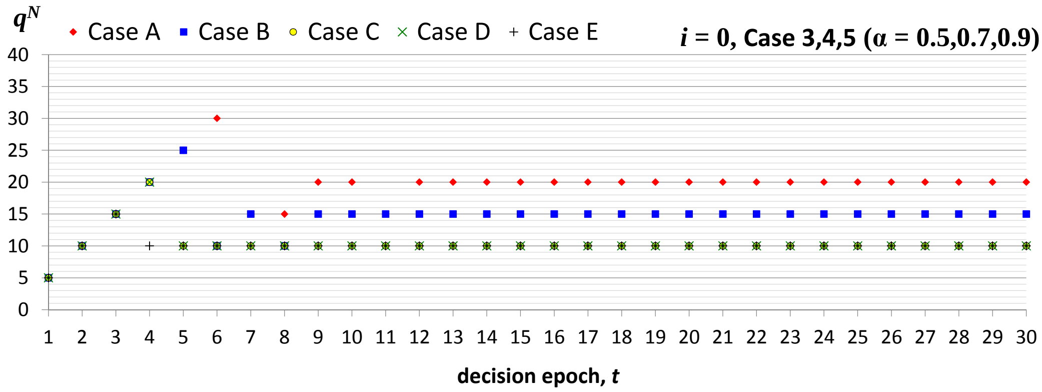


Figure 4.24. M2*FinCons*, state w: The optimal order from supplier F in Cases 3,4 and 5

The ordering policy of M2FinSto

If supplier F is in state u, the firm only places an order with supplier N when the inventory level is low (i.e., $i < 8$) in each case at every decision epoch. However, at the end of the planning horizon ($t \leq 3$), the firm will order more from this supplier, as illustrated in figure 4.25. In these situations, the quantity ordered from supplier N is just enough to meet the immediate shortage.

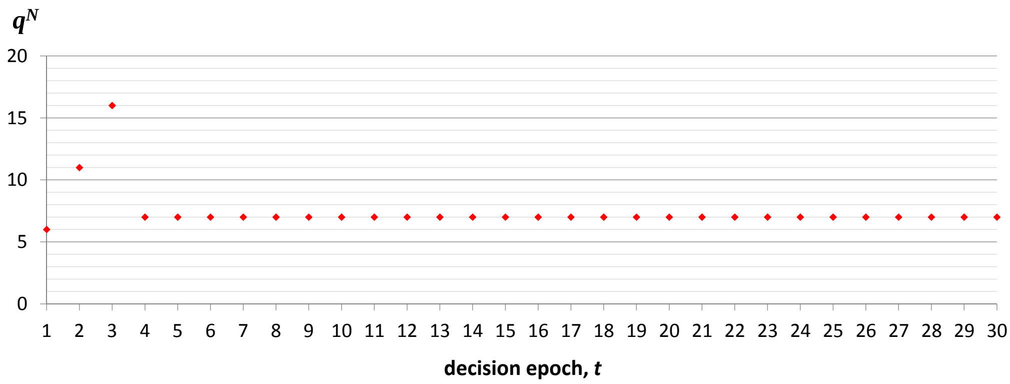


Figure 4.25. M2*FinSto*, state u: The optimal order from supplier N in each case.

However, the optimal order placed with supplier F does depend on the supply disruption and disruption recovery probabilities. This is illustrated in figures 4.26, 4.27, 4.28, 4.29 and 4.30. For example, when $i = 0$, from figure 4.26, in case 1 ($\alpha = 0.1$) for all β cases, the firm will always place the order with supplier F unless there is very little time remaining in the planning horizon ($t \leq 2$) if the firm expects i will fall too low between the current and the next decision epoch and the ordered quantity decreases when β increases. In other α cases, from figures 4.27, 4.28, 4.29 and 4.30, the incentive to order from supplier F increases due to the increasing risk of supply disruption. At decision epochs in the second half of the planning horizon ($t < 15$), the quantity ordered from the offshore supplier hardly changes with the probabilities of supply disruption and disruption recovery. This might be explained by the firm stocking up for the remainder of the planning horizon. However, in the first half of the planning horizon ($15 \leq t \leq 30$), the firm will increase the quantity ordered from the offshore supplier with an increase in supply disruption probability for each disruption recovery probability and will decrease the order quantity with an increase in disruption recovery probability for each supply disruption probability.

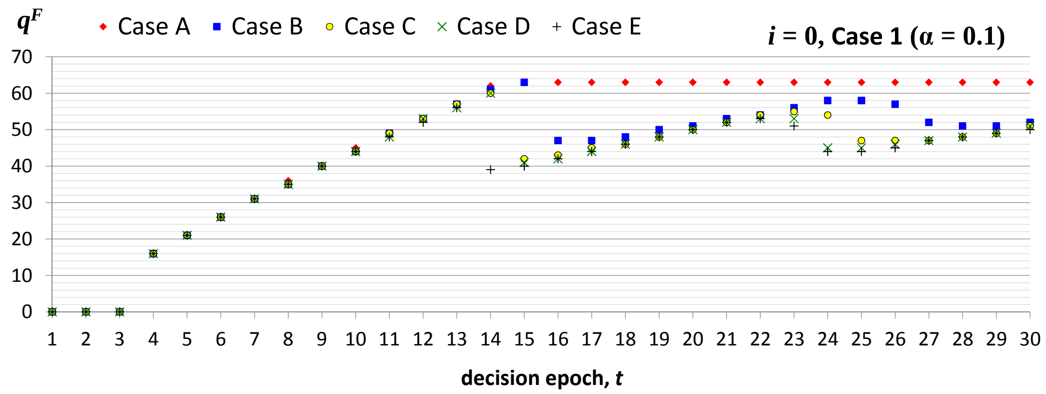


Figure 4.26. M2*FinSto*, state u: The optimal order from supplier F in Case 1

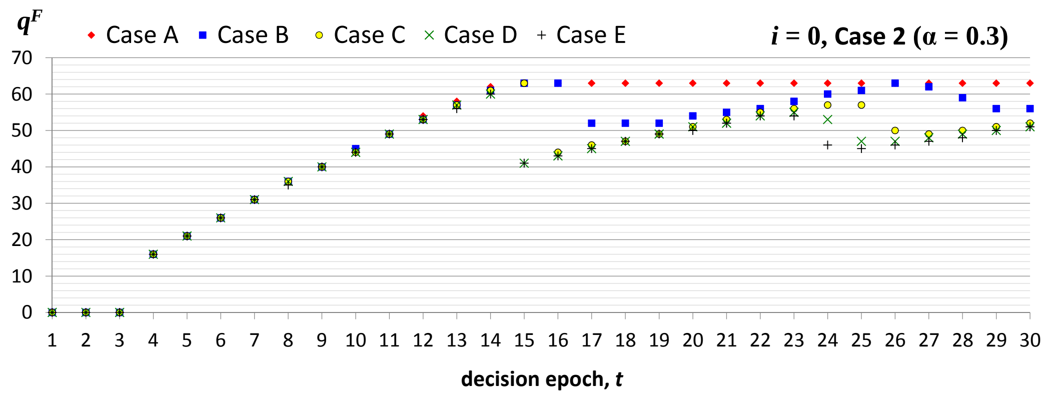


Figure 4.27. M2*FinSto*, state u: The optimal order from supplier F in Case 2

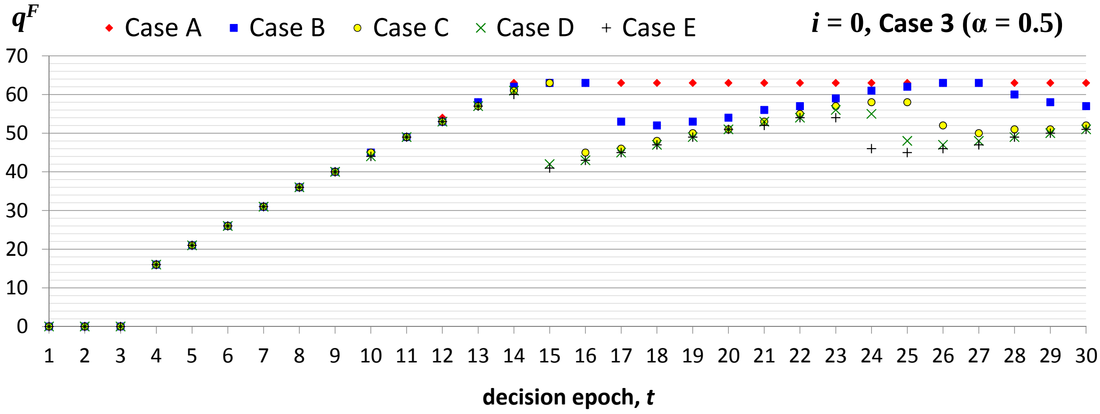


Figure 4.28. M2*FinSto*, state u: The optimal order from supplier F in Case 3

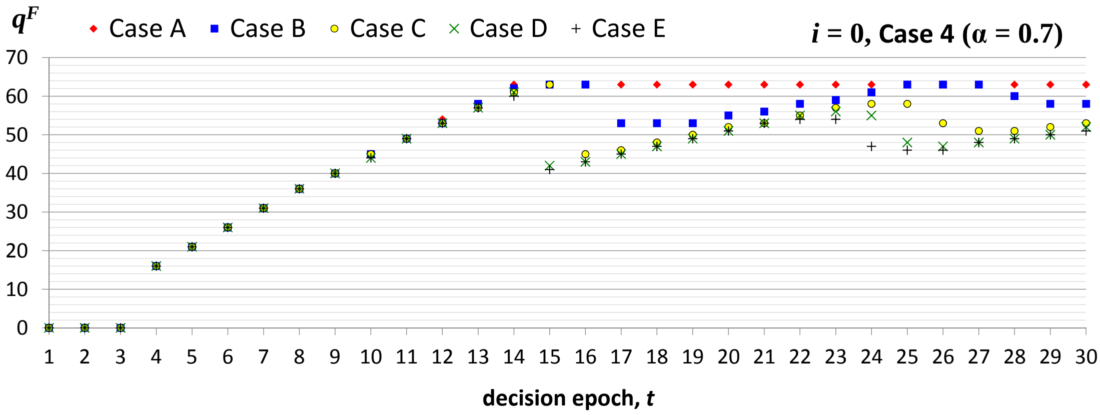


Figure 4.29. M2*FinSto*, state u: The optimal order from supplier F in Case 4

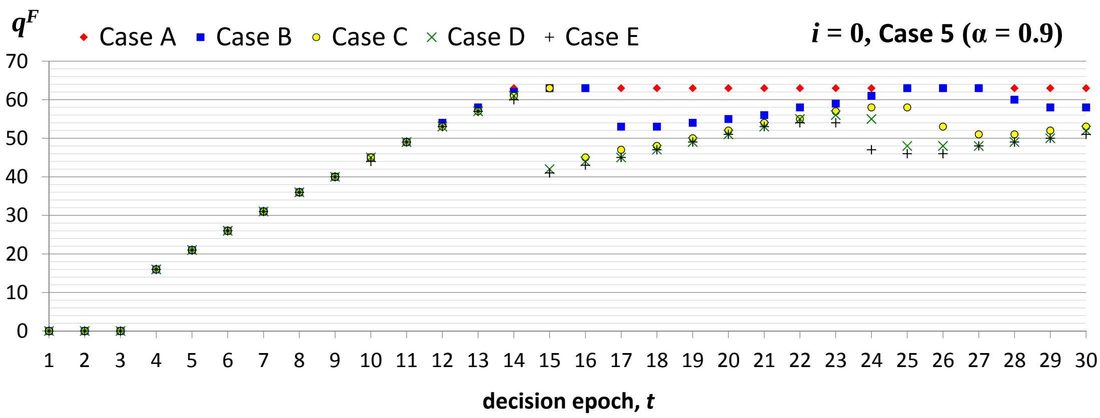


Figure 4.30. M2*FinSto*, state u: The optimal order from supplier F in Case 5

Assuming that for most decision epochs the order policy is as an (s, S) policy, the order policies for supplier F for all cases α and β and a fixed decision epoch (i.e., $t = 10;\ 20$ and 30) are tabulated in tables 4.9, 4.10 and 4.11. We observe similar trends as in the case of constant demand.

Table 4.9. M2*FinSto*, state u: Optimal ordering policy for supplier F in $t = 10$

β α		**A** 0.1	**B** 0.3	**C** 0.5	**D** 0.7	**E** 0.9
1	0.1	(18,45)	(15,44)	(14,44)	(14,44)	(14,44)
2	0.3	(32,45)	(23,45)	(18,44)	(16,44)	(15,44)
3	0.5	(34,45)	(26,45)	(20,45)	(17,44)	(16,44)
4	0.7	(36,45)	(28,45)	(21,45)	(18,44)	(17,44)
5	0.9	(36,45)	(29,45)	(22,45)	(19,45)	(17,44)

Table 4.10. M2*FinSto*, state u: Optimal ordering policy for supplier F in $t = 20$

β α		**A** 0.1	**B** 0.3	**C** 0.5	**D** 0.7	**E** 0.9
1	0.1	(16,63)	(15,51)	(14,50)	(14,50)	(14,50)
2	0.3	(33,63)	(21,54)	(17,51)	(16,51)	(15,50)
3	0.5	(38,63)	(38,63)	(19,51)	(17,51)	(16,51)
4	0.7	(40,63)	(25,55)	(20,52)	(18,51)	(17,51)
5	0.9	(41,63)	(26,55)	(21,52)	(18,51)	(17,51)

Table 4.11. M2*FinSto*, state u: Optimal ordering policy for supplier F in $t = 30$

β α		**A** 0.1	**B** 0.3	**C** 0.5	**D** 0.7	**E** 0.9
1	0.1	(16,63)	(15,52)	(14,51)	(14,51)	(14,51)
2	0.3	(33,63)	(20,56)	(17,52)	(15,51)	(15,51)
3	0.5	(38,63)	(23,57)	(19,52)	(17,51)	(16,51)
4	0.7	(40,63)	(25,58)	(20,53)	(18,52)	(16,51)
5	0.9	(41,63)	(26,55)	(21,53)	(18,52)	(17,51)

If supplier F is in state w, in most cases, the firm will place the order with supplier N when the inventory level is relatively low at all decision epochs and the quantity ordered decreases at the end of the planning horizon ($t < 6$). From figures 4.31 and 4.32 when $i = 5$, in each α case, as β increases, the incentive to order decreases due to the shorter expected length of a disruption. We can see that the properties of the ordering policy for supplier N under the stochastic demand model are similar to the constant demand model, thus we can conclude that the order policy under crisis event depends on the probability of disruption recovery, but not the probability of supply disruption. In addition, the firm will order more from supplier N when β is very low (i.e., $\beta = 0.1$) since the disruption is expected to last longer.

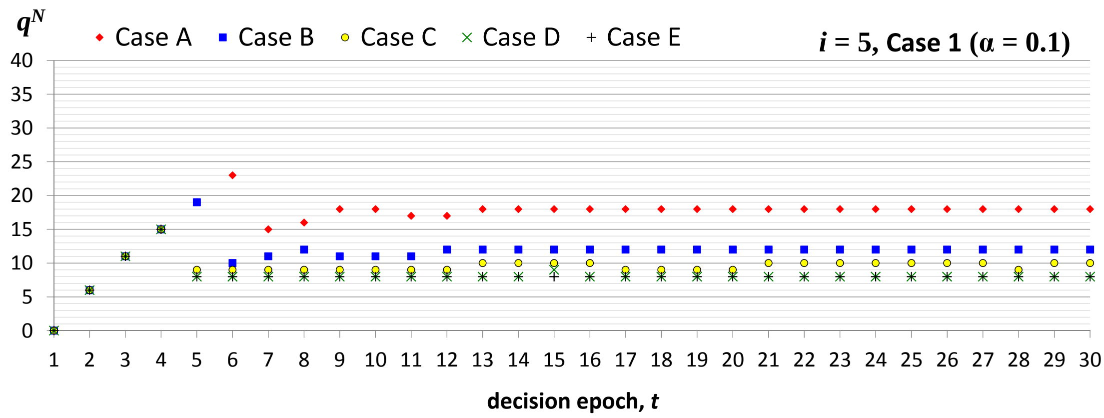


Figure 4.31. M2*FinSto*, state w: The optimal order from supplier N in Case 1

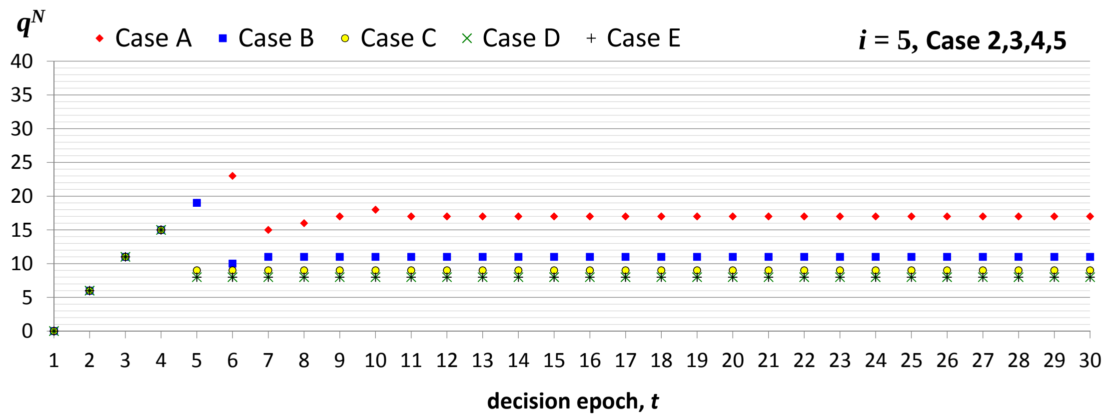


Figure 4.32. M2*FinSto*, state w: The optimal order from supplier N in Cases 2,3,4 and 5

Discussion

Under the finite-horizon plan, in both constant and stochastic demand settings, if the offshore supplier is operating as usual (or under a routine operation), the characteristics of the ordering policy is not much different from those of the optimal ordering policy of Model 1. If the inventory level falls below some threshold level, the firm will always place an order with the onshore supplier to satisfy immediate demand and also order from the offshore supplier to increase the inventory level and satisfy demand in later periods. However, if the firm expects the inventory level to fall below some threshold before the end of the period, placing an order with the offshore supplier is to meet demand in future periods more favourable due to the lower unit cost. If the inventory level is high enough to satisfy demand for at least two periods or until the end of the planning horizon, the firm will not order anything from the onshore and offshore suppliers at every period. We have observed that the supply disruption probability and the disruption recovery probability influence the optimal ordering policy. As the supply disruption probability increases, the firm maintains a larger inventory of items sourced from the offshore supplier. As the disruption recovery probability increases, the firm orders less from the offshore supplier.

If the offshore supplier is unable to deliver an order (or under a crisis operation), the firm can only place the order with the onshore supplier. As for Model 1, the firm will only order from the onshore supplier when faced with an immediate shortage. However, in contrast to Model 1, the firm will not just order enough to meet the immediate shortage. The firm will order additional items to satisfy demand during the remainder of the disruption.

Based on the experiment with reliability of the offshore supplier, the supply disruption probability has more impact on the optimal ordering from the offshore supplier than from the onshore supplier. However, the disruption recovery probability, it has more effect on the onshore supplier than the offshore supplier or it has a considerable affect on the optimal ordering from both the offshore and onshore suppliers. From our observation, and in agreement with our theory, as the chance for the offshore supplier to recover from disruption gets higher, the firm keeps less safety stock even when there is has a high possibility that the supplier will fail again in the near future.

The optimal order decisions with the infinite-horizon model

The optimal order decisions under the infinite-horizon planning setting is as follows.

The ordering policy of M2InfCons

If supplier F is in state u, the firm only orders from supplier N if the inventory level, i, is relatively low, ($i \leq 5$) in all cases, as illustrated in figure 4.33. However, in contrast, the ordering decision from supplier F does vary from case to case. From figure 4.34, in case 1 ($\alpha = 0.1$), the firm reduces the quantity ordered from supplier F with increases in β if i is less than $2D$ ($i \leq 10$). We can see from figure 4.34 that the optimal ordering policy from the offshore supplier is effectively an (s, S) policy. When $i < D$, the items in inventory and the order from supplier N are used to meet demand during the period exactly and no items are carried forward. The order from supplier F then ensures that there are S items in inventory at the beginning of the next period. When $D \leq i < s$, demand this period is satisfied from inventory and $i - D$ items of inventory are carried forward to the next period. The firm also orders $S - i + D$ items from supplier F to bring the inventory level up to S at the beginning of the next period. The optimal ordering policy for supplier F has this form in all cases.

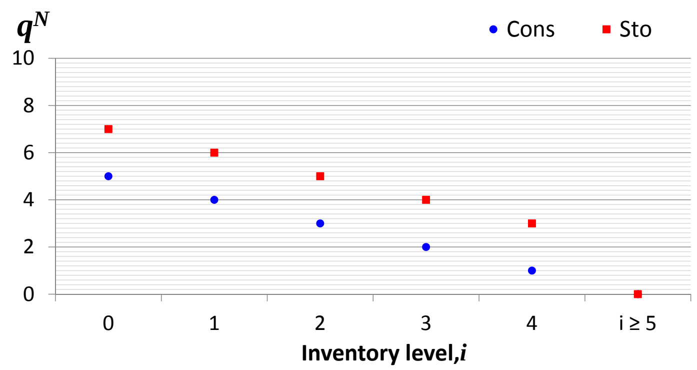


Figure 4.33. M2*InfCons*, state u: The optimal order from supplier N

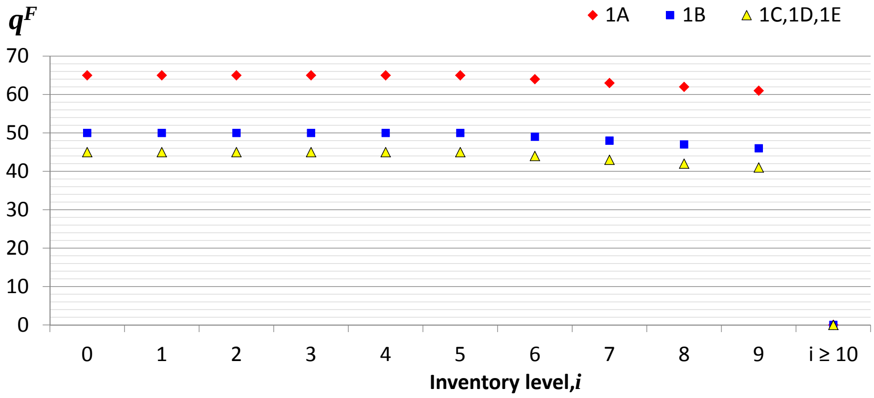


Figure 4.34. M2*InfCons*, state u: The optimal order from supplier F when $\alpha = 0.1$.

Table 4.12 shows how the parameters vary in the 25 cases considered. From table 4.12 we see that the incentive to order from supplier F increases with increases in α and decreases in β, especially in case A, ($\beta = 0.1$).

Table 4.12. M2*InfCons*, state u: Optimal ordering policy for supplier F

	β α	**A** 0.1	**B** 0.3	**C** 0.5	**D** 0.7	**E** 0.9
1	0.1	(10,65)	(10,50)	(10,45)	(10,45)	(10,45)
2	0.3	(30,70)	(15,55)	(14,50)	(12,50)	(10,45)
3	0.5	(35,70)	(20,55)	(15,50)	(15,50)	(14,45)
4	0.7	(39,70)	(20,60)	(15,50)	(15,50)	(15,50)
5	0.9	(40,70)	(22,60)	(16,50)	(15,50)	(15,50)

If we look at the relationship between the proportion of time for which supplier F is either up or down and the optimal policy, from figure 4.35, we see that order up to level, S increases with an increase in expected down time, π_w. However, from figure 4.36, reorder point, s decreases with an increase in expected up time, π_u. From figures 4.35 and 4.36, we can see that, there is a relationship between the proportion of time and the optimal ordering from supplier F. We can conclude that, the firm will increase the quantity ordered from the offshore supplier if the expected down time increases and the point of inventory level at which the firm to places an order with this supplier will be reduced if the expected up time increases. The increase in order up to level with proportion of time supplier F is down is approximately linear. The relationship between the reorder point and the proportion of time supplier F is up is non-linear.

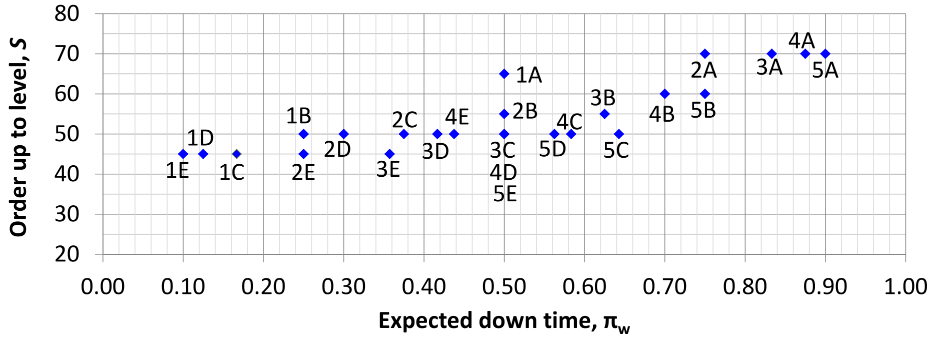


Figure 4.35. M2*InfCons*, state u: The relationship between the expected down time and order up to level.

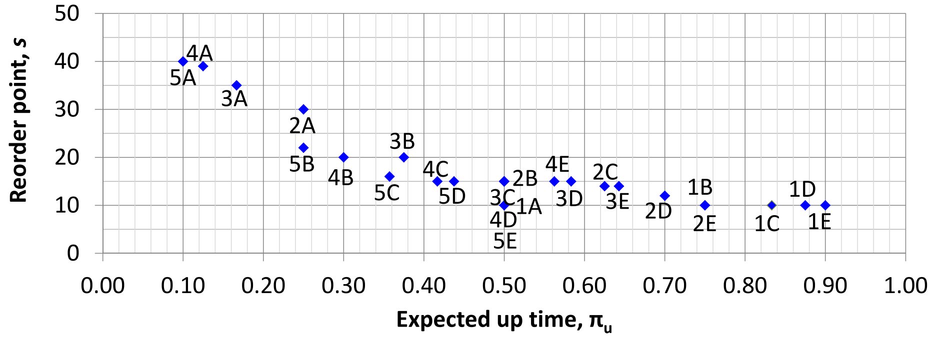


Figure 4.36. M2*InfCons*, state u: The relationship between the expected up time and reorder point.

If supplier F is in state w, from figure 4.37, in each α case, the ordered quantity from supplier N decreases with an increase in β. As we expected, the firm will only place an order if there is an immediate shortage (i.e., $i \le D$) and then order a quantity that decreases as β decreases because the expected length of disruption is less. We also can see that the order quantity from supplier N is independent of the probability of supply disruption, α. This is logical because the firm will have at least one opportunity to order from the offshore supplier after this supplier recovers from the disruption regardless of the value of supply disruption probability.

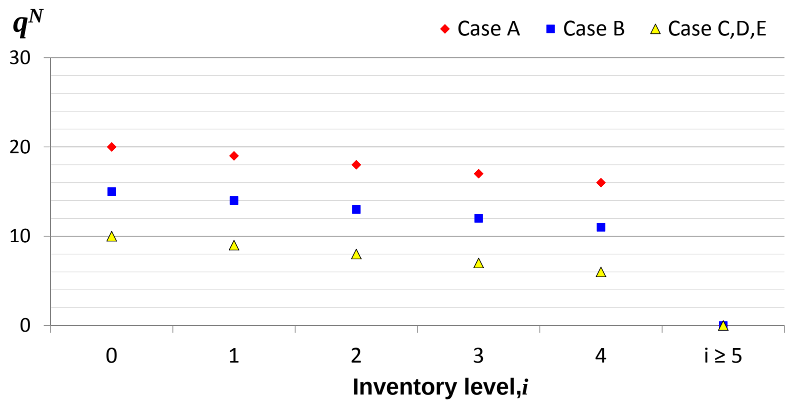


Figure 4.37. M2*InfCons*, state w: The optimal order from supplier N in each disruption recovery probabilities, β.

The optimal order of M2InfSto

If supplier F is in state u, the firm only orders from supplier N if the inventory level, i, is relatively low, ($i \leq 5$) in all cases, as as illustrated in figure 4.33. The form of the optimal order policy from the onshore supplier in M2*InfSto* is similar to M2*InfCons*, but the order quantity with the stochastic demand model is bigger than with the constant demand model due to variance in demand. However, in contrast, the ordering decision from supplier F does vary from case to case. From figure 4.38, in case 5 ($\alpha = 0.9$), the firm reduces the quantity ordered from supplier F as β increases if i is less than 59% of the maximum of the inventory level, I ($i \leq 41$). We can see from figure 4.38 that the optimal ordering policy from the offshore supplier is effectively an (s, S) policy. When the inventory level is relatively low, the order quantity is slightly less than would be expected by the (s, S) policy due to the items ordered from the onshore supplier. For higher inventory levels, when the firm no longer orders from the onshore supplier, the order quantity from the offshore supplier follows the (s, S) policy. The optimal ordering policy for supplier F has this form in all cases.

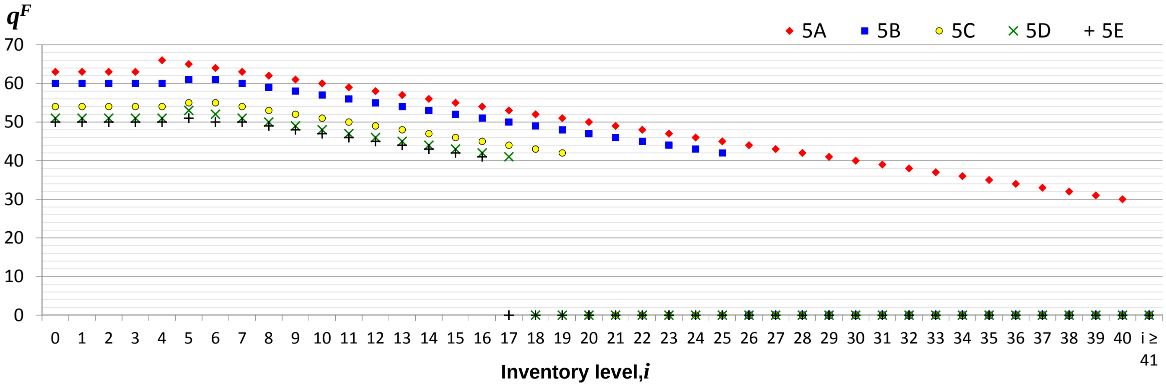


Figure 4.38. M2*InfSto*, state u: The optimal order from supplier F when $\alpha = 0.9$.

Table 4.13 shows how the parameters vary in the 25 cases considered. From table 4.13 we see that the incentive to order from supplier F increases with increases in α and decreases in β, especially in case A, ($\beta = 0.1$).

Table 4.13. M2*InfSto*, state u: Optimal ordering policy for supplier F.

	β α	**A** 0.1	**B** 0.3	**C** 0.5	**D** 0.7	**E** 0.9
1	0.1	(15,70)	(14,61)	(13,57)	(13,57)	(13,57)
2	0.3	(31,70)	(19,65)	(16,60)	(14,57)	(14,56)
3	0.5	(37,70)	(22,66)	(18,61)	(16,58)	(15,56)
4	0.7	(39,70)	(24,67)	(19,61)	(17,58)	(16,56)
5	0.9	(40,70)	(25,67)	(19,61)	(17,58)	(16,56)

If we look at the relationship between the proportion of time for which supplier F is either up or down and the optimal policy, from figure 4.39, we see that order up to level, S, increases with an increase in expected down time, π_w. However, from figure 4.40, reorder point, s, decreases with an increase in expected up time, π_u. From figures 4.39 and 4.40, we can see that, there is a relationship between the proportion of time and the optimal ordering from the offshore supplier. We can conclude that, the firm will increase the order quantity from the offshore supplier if the expected down time increases and the point of inventory level at which the firm places an order with this supplier will be reduced if the expected up time increases. As with M2$InfCons$, the relationship between down time and order-up-to level is approximately linear while the relationship between up time and reorder point is non-linear.

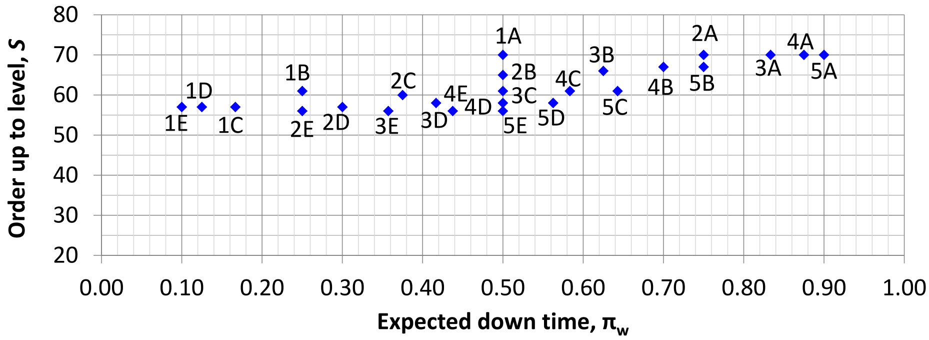


Figure 4.39. M2*InfSto*, state u: The relationship between the expected down time and order up to level.

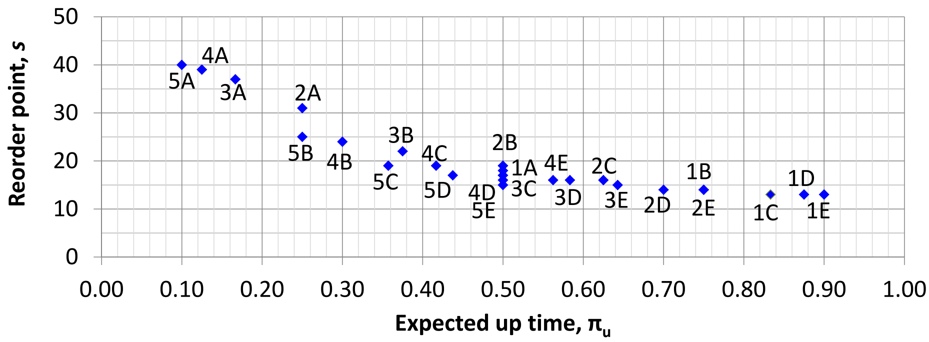


Figure 4.40. M2*InfSto*, state u: The relationship between the expected up time and reorder point.

From figures 4.41 and 4.42, if supplier F is in state w, the firm will only order from supplier N if there is a perceived immediate shortage (i.e., $i \leq 6$) and then order a quantity that decreases as β decreases because the expected length of disruption is less. However, from figures 4.41 and 4.42, the quantity ordered from supplier N is also affected slightly by the α values. We can see that, the order quantity increases as supply disruption probability increases. As opposed to the constant demand model, the quantity ordered from supplier N when supplier F is down is also affected by the α values. We could say, under the crisis event and stochastic demand condition, the quantity ordered from the onshore supplier will be affected by the probabilities of supply disruption and disruption recovery. However, the main effect remains that the longer the expected disruption length, the more the firm needs to order from the onshore supplier.

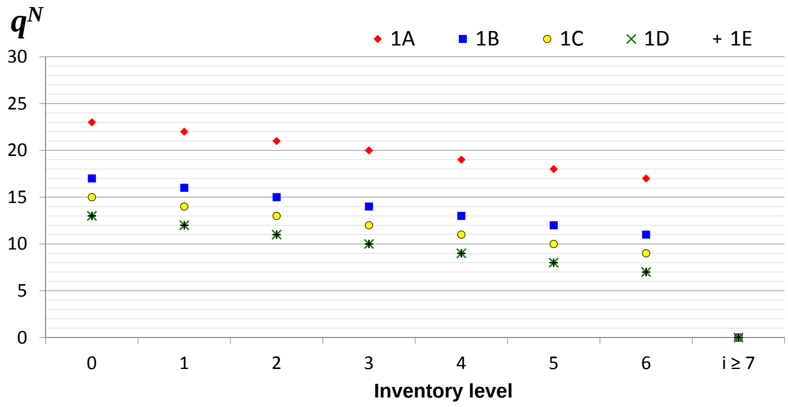


Figure 4.41. M2*InfSto*, state w: The optimal order from supplier N in Case 1.

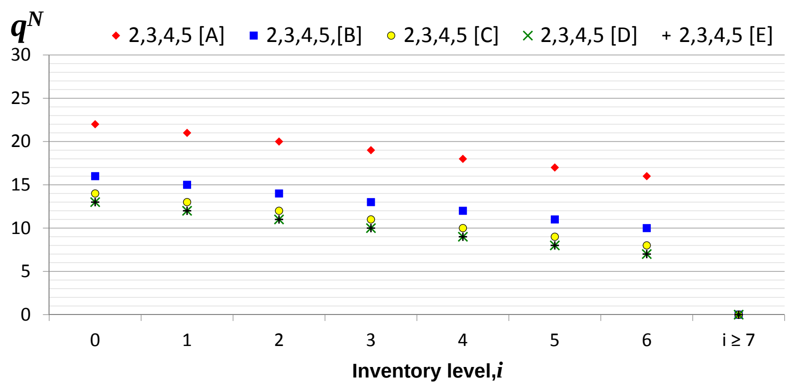


Figure 4.42. M2*InfSto*, state w: The optimal order from supplier N in Cases 2, 3, 4 and 5.

Discussion

From the findings, when the offshore supplier is up, we can see that the risks of supply disruption and disruption recovery (α and β), have more impact on the optimal order placed with the offshore supplier than the onshore supplier. The firm will only place orders with the onshore supplier if the inventory level is so low that there is a high risk of immediate shortages and this applies in both constant and stochastic demand models. In these situations the order quantity is just enough to satisfy the immediate shortage. The findings show that the onshore supplier is only needed as backup if the firm does not have enough stock in inventory, which is similar to the properties of the ordering policy from the onshore supplier in Model 1. When the offshore supplier is down, the firm also only orders from the onshore supplier when it is faced with a high risk of an immediate shortage. However, in this situation, the order quantity is bigger than in the situation when the offshore supplier is up. The order is not only to satisfy the immediate shortage, but also to satisfy demand during subsequent periods of disruption. The firm will order less from this supplier when the offshore supplier has a better chance to recover from a disruption. However, the frequency of occurrence of disruptions has little effect on the order policy for the onshore supplier.

The properties of the order policy with the offshore supplier are very different from those of the order policy with the onshore supplier. In both constant and stochastic demand models, the firm will increase the order quantity from the offshore supplier if this supplier has a higher risk of disruption and decrease the order if the chance of recovery from the disruption is higher. It is quite interesting to see the pattern of ordering from the offshore supplier under the condition that this supplier has a low risk of disruption and a low chance to recover from disruption. From our observations, we can see that the firm ordered more from the offshore supplier under this condition. We might think that the firm will reduce the quantity ordered from the offshore supplier due to the low risk of this supplier facing the disruption. However, our findings appear to show that the low chance of recovery has more impact on the optimal order quantity for the offshore supplier than the risk of disruption. In our opinion, the firm should pay more attention to situations where the offshore supplier has a low chance to recover from disruption. In such situations, the firm should keep a stock of cheaper items from the supplier even though the firm believes that the chance of this supplier being down (or unreliable) at the next period is very slim.

4.3.6 The Impacts of the Transition Probabilities on the Properties of the Minimum Costs

In this section, we explain how the risks of supply disruption, α, and disruption recovery, β, can affect the firm's ordering costs. We first discuss the result of the minimum cost under the finite-horizon setting in section 4.3.6a covering M2*FinCons* and M2*FinSto*. Then, we report the long-run average cost under the infinite-horizon setting in section 4.3.6b covering M2*InfCons* and M2*InfSto*.

The minimum costs with the finite-horizon model

Under the finite-horizon setting, we discover that the expected minimum costs for every inventory level and every period when supplier F is in state w are higher than those when supplier F is in state supplier N in both analyses of M2*FinCons* and M2*FinSto*. If we focus on the minimum cost when supplier F is in state w, in both constant and stochastic demand models, at most inventory levels and time periods, the minimum costs are high in case A ($\beta = 0.1$) and the highest expected minimum costs are in case 5A ($\alpha = 0.9, \beta = 0.1$). It is as expected as one would expect higher cost when disruption is more frequent (i.e., $\alpha = 0.9$) and the chance of recovery is low (i.e., $\beta = 0.1$). Lower minimum costs mostly fall in case E and the lowest minimum cost is in case 1E ($\alpha = 0.1, \beta = 0.9$). Again, one would expect that the cost should be lower when disruption is less frequent (i.e., $\alpha = 0.1$) and the chance of recovery is high (i.e., $\beta = 0.9$).

The optimal costs with the infinite-horizon model

From figure 4.43, we can see that the pattern of the long-run average costs, g, across the cases is the same with both constant and stochastic demand. In addition, the pattern of g is also the same for each case of α. Most higher g are in case A ($\beta = 0.1$) for all α values and overall the highest g are in cases 4A and 5A. The long-run average cost decreases gradually as β increases with the lowest g in case E ($\beta = 0.9$). Overall, the lowest g is in case 1E. Based on the pattern of g across the cases, we can conclude that the disruption recovery probability has more impact on the optimal long-run average cost than supply disruption probability. From the findings, we see the optimal cost is high when the risk of the offshore supplier facing a disruption is high ($\alpha \geq 0.5$) and the chance of the offshore supplier recovering from disruption is low ($\beta = 0.1$). However, it is quite surprising to see the cost under the case of lowest risk for the offshore supplier faces the disruption ($\alpha = 0.1$) can be high too if the disruption recovery probability is low. We might expect that optimal cost under this case (i.e., 1A) should be lower. Therefore, based on the findings, the firm should give more attention to the situation where the offshore supplier has a low chance to recover from the disruption.

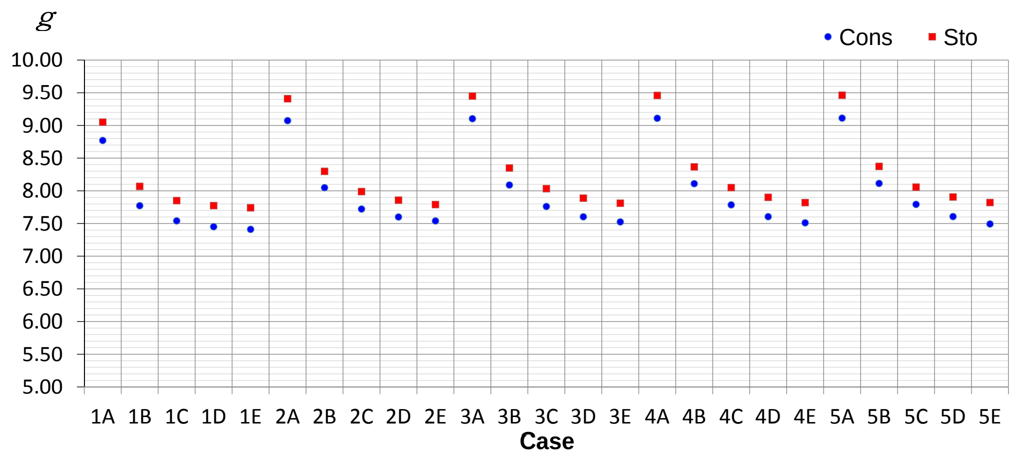
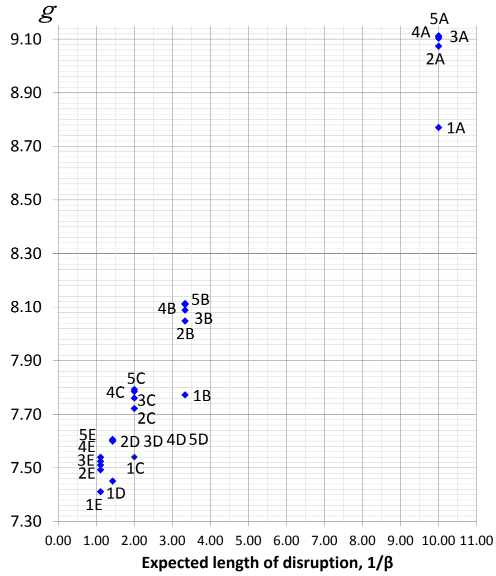


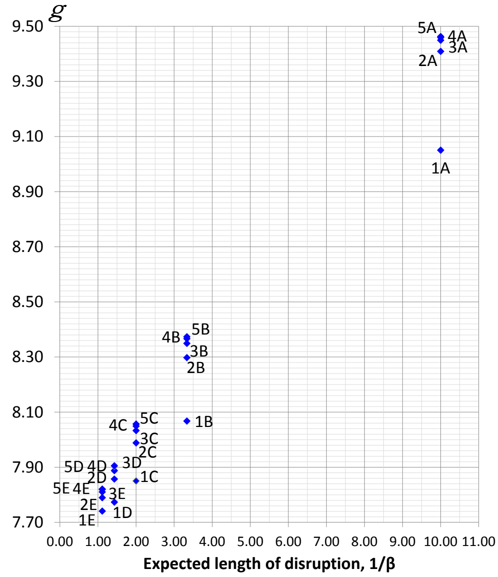
Figure 4.43. M2*Inf*: Optimal long-run average cost, g in different cases for constant and stochastic demand models.

If we look from the aspect of the expected length of disruption, $1/\beta$, from figure 4.44, we see that there is a relationship between the long-run average cost, g and $1/\beta$, which there looks an almost linear relationship between g and $1/\beta$. This is illustrated in figures 4.44a and 4.44b. From both figures, in both constant and stochastic demand models, g increases as $1/\beta$ increases. As we expected, the firm will face higher cost if the length of disruption at the offshore supplier is expected to be longer.

From the aspect of the expected down time, π_w, the long-run average cost, g also has a relationship with π_w, From figure 4.45, in both constant and stochastic demand models, (see figures 4.45a and 4.45b), g increases as π_w increases. Similar to the relationship with $1/\beta$, as we expected, the firm will face higher cost if down time at the offshore supplier is expected to be longer.

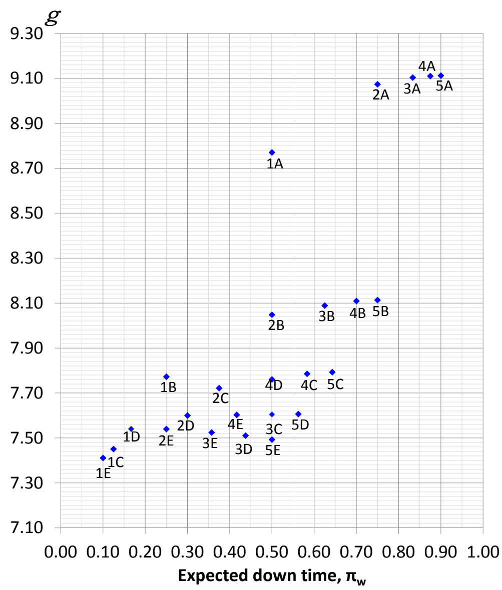


(a) M2*InfCons*

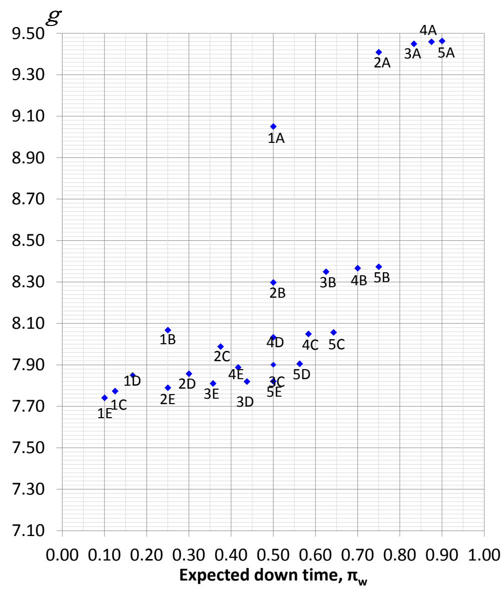


(b) M2*InfSto*

Figure 4.44. M2: The relationship between the long-run average cost and the expected length of intervals of normal service and disruption



(a) M2*InfCons*



(b) M2*InfSto*

Figure 4.45. M2: The relationship between the long-run average cost and the expected down time

Discussion

A comparison of the results of Model 1 and Model 2 confirms that the cost for the firm to operate under the crisis event is higher than the cost under an environment without supply disruption. Comparing the expected minimum cost among the different cases of the values of supply disruption probability, α, and disruption recovery probability, β, the highest cost occurs in the case with the highest value of supply disruption probability and the lowest value of disruption recovery probability, and the lowest cost occurs in the case with the lowest value of supply disruption probability and the highest value of disruption recovery probability. For the findings, we can see that both supply disruption and disruption recovery probabilities have influence on the minimum cost of the optimal ordering policy. As we expected, the firm will face higher cost if down time and the disruption length at the offshore supplier is expected to be longer.

4.3.7 The Impacts of the Transition Probabilities on the Performance of the Policies

In this section, we discuss the performance of the ordering policy under the infinite horizon plan and stochastic demand model (M2*InfSto*), focussing on the performance of the fill rate (section 4.3.7a) and the average inventory level (section 4.3.7b).

Fill rate

From figure 4.46, the percentage of demand satisfied from stock in hand in all cases are estimated to lie between 99.67% and 99.83% with 95% confidence interval. From findings, we see that even though there is a high disruption probability and a small recovery probability, the capability for the firm to satisfy demand is still high since it has a backup source from the onshore supplier. It is quite noticeable that the fill rate in case A (β = 0.1) for every α case is lower than other β cases. We could say the firm does not perform well under the condition that the offshore supplier has a low chance to recover from disruption. The variation in the fill rates for other values of β is within the range of the confidence intervals of the simulation results. In general, we might expect the fill rate to improve as beta increases.

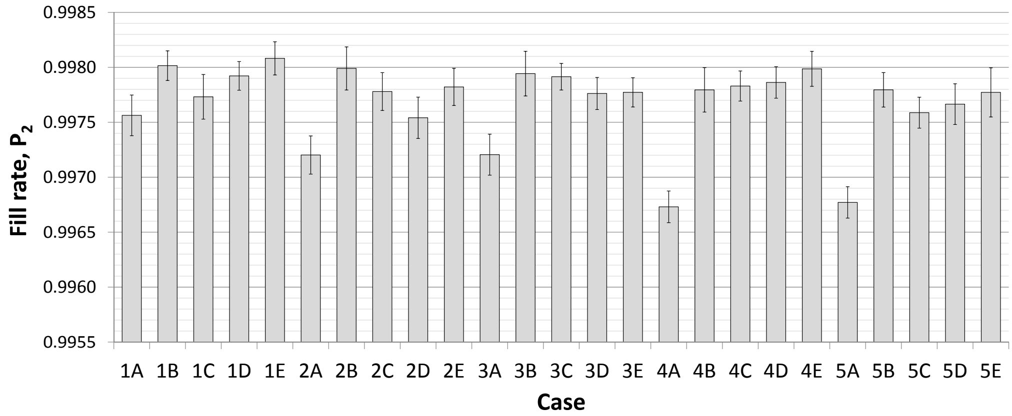
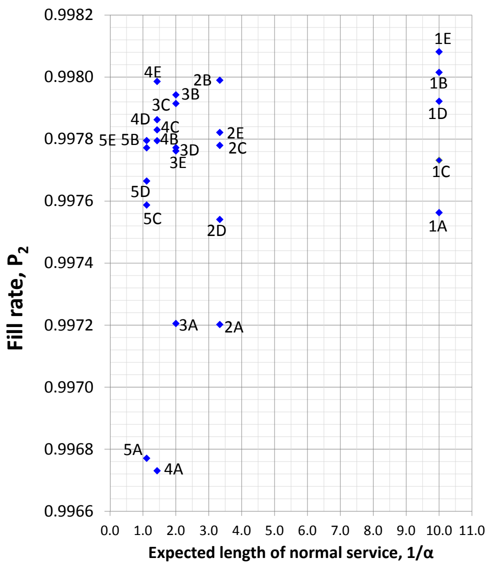


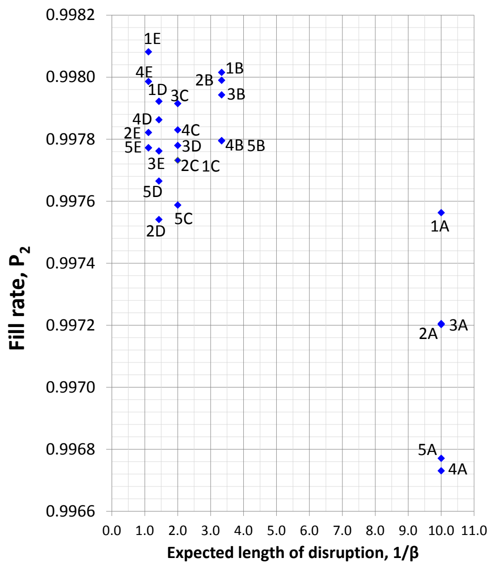
Figure 4.46. M2: Fill rate in each case of α and β

From the aspect of the expected length of normal service and the expected length of disruption, $1/\alpha$ and $1/\beta$ respectively, there are no clear relationship with the fill rate, P_2, as illustrated in figure 4.47. From figure 4.47a, there are cases of relatively low and relatively high fill rates across the range of values for $1/\alpha$. Similarly from figure 4.47b, there is no clear trend in fill rate across the range of values for $1/\beta$. However, it should be noted that the firm is able to meet a very high proportion of demand even when expected length of normal service is very low and the expected length of disruption is high due to having a backup source from the onshore supplier.

However, from figure 4.48, we see a weak positive correlation between fill rate, P_2, and proportion of up time, π_u. P_2 increases as π_u increases. From these findings, we could say the capability of the firm to satisfy demand will increase if the intervals for which the offshore supplier operates normally last a little longer.



(a) P_2 *vs.* $1/\alpha$



(b) P_2 *vs.* $1/\beta$

Figure 4.47. M2: The relationship between fill rate and the expected length of intervals of normal service and disruption

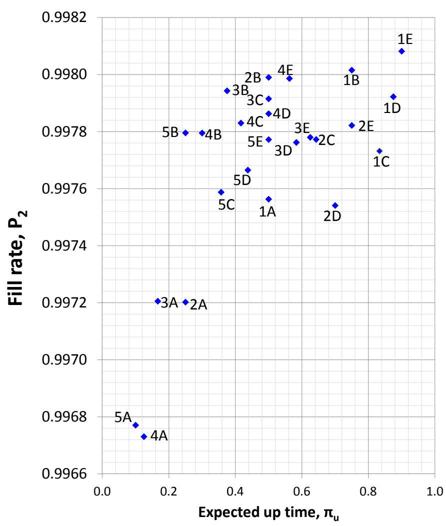


Figure 4.48. M2: The relationship between fill rate and the expected up time

Average inventory level

From figure 4.49, the average inventory level, I_A, in all cases are estimated to lie between 30.38 and 36.94 with 95% confidence interval, which is roughly one half the maximum inventory level. We also see the highest I_A is in case 5B (35.95 ± 0.090) and the lowest is in case 1E (30.41 ± 0.037). This result is a little surprising as we might have expected the average inventory level to be highest in case A because the expected length of disruption is highest in this case. Perhaps this effect is due to the fact that the firm has fewer opportunities to place large orders with the offshore supplier due to the high frequency of disruption and, in case 5A, the long length of disruption.

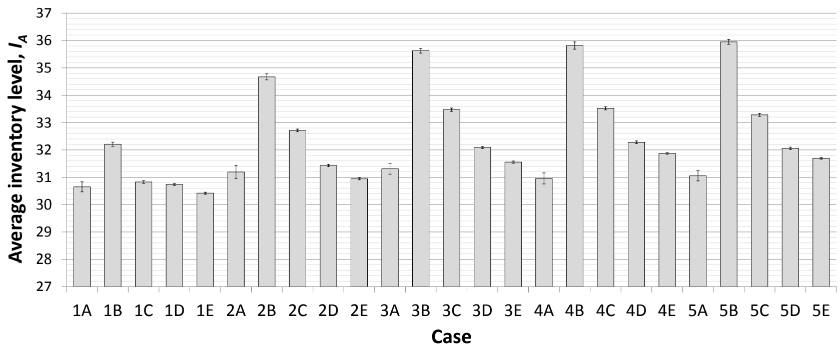
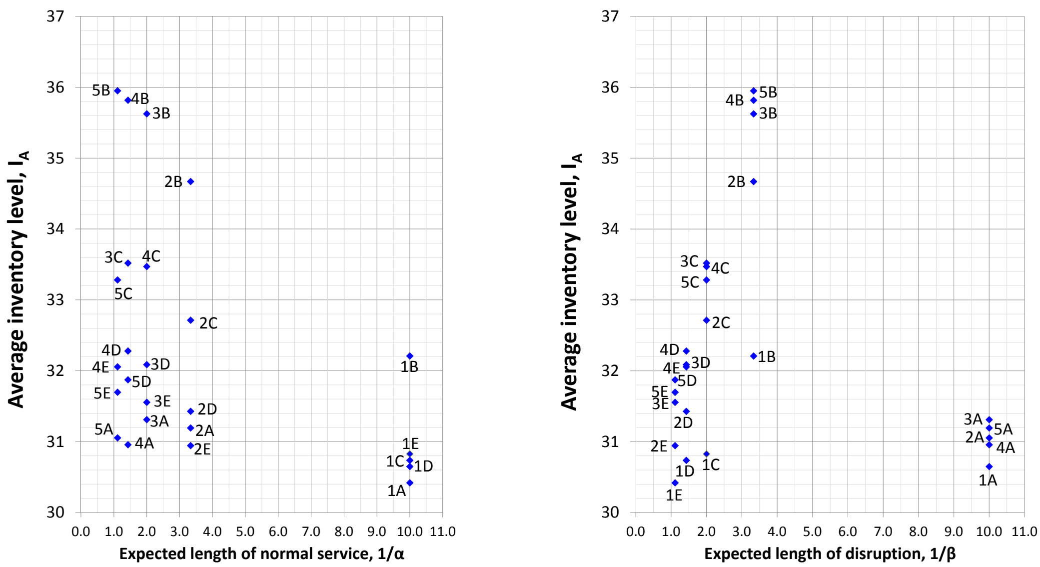


Figure 4.49. M2*Inf*: Average inventory level in each case α and β

From the aspect of the expected length of periods of normal service and the expected length of disruptions, $1/\alpha$ and $1/\beta$ respectively, there are no clear relationship with average inventory level, I_A, which is illustrated in figure 4.50. For case B in figure 4.50a, I_A is quite high ($34 \leq i \leq 36$) when the length of normal service is expected to be low. Perhaps due to low expected normal service length from the offshore supplier, it is optimal to carry more inventory to avoid ordering more from the expensive onshore supplier during disruptions. From figure 4.50b, we see that the average inventory level can be low when the expected length of disruptions is low (i.e., case E) or high (i.e., case A). However, we could say the average inventory level, I_A, has a relationship with the expected up time, π_u. From figure 4.51, we see a negative weak correlation between I_A and π_u. The average inventory level decreases as π_u increases. From these findings, it is optimal to carry less inventory level when the offshore supplier is expected to be operating a little longer. From figures 4.50b and 4.51, it is again apparent that case A is distinct from the other cases as the average inventory is lower than one would expect from the trends apparent in the other cases.



(a) Average inventory level versus $1/\alpha$

(b) Average inventory level versus $1/\beta$

Figure 4.50. M2: The relationship between average inventory level and the expected length of intervals of normal service and disruption

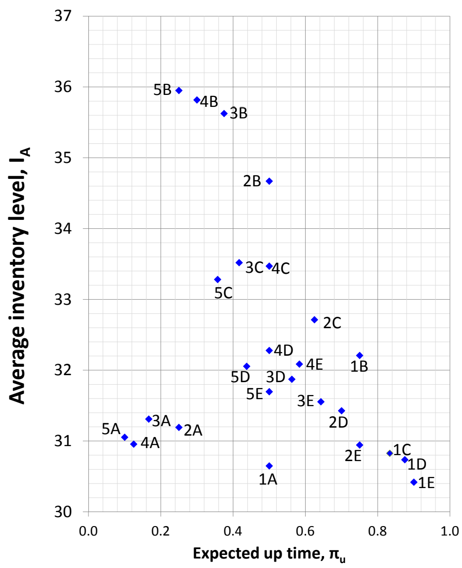


Figure 4.51. M2: The relationship between average inventory level and the expected up time

Discussion

In Model 2 analyses, the performance of the firm's ordering policy has been measured with the values of fill rate and average inventory level. Based on fill rate, the firm still has high capability to satisfy demand from the customer under the optimal policy determined by the model. The percentage of demand satisfied from stock in hand is still high even when there is a high supply disruption probability and a small disruption recovery probability. From the average inventory level perspective, it is optimal to carry more inventory if the disruption recovery probability is relatively small ($\beta = 0.3$) to avoid having to order from the expensive onshore supplier during disruptions. However, the average inventory level decreases when the disruption recovery probability is increased. In the extreme situation of case A ($\beta = 0.1$), the policy would like to order more.

4.3.8 Conclusion

In Model 2, we addressed how the firm's optimal ordering policies can be affected by the risk of disruptive supply events at the offshore supplier. From the analyses of Model 2, we demonstrate how the properties and performance of the optimal ordering policy depend on the values of the transition probabilities of the Markov chain model of disruption to the offshore supplier. We show that the uncertain state of the offshore supplier can affect the firm's ordering decision from both the onshore and the offshore supplier.

Having established that the uncertain state of the offshore supplier can affect the firm's ordering decisions, we will examine the impact, case by case. The firm increases the order quantity from the offshore supplier if the risk of disruption increases and decreases the order quantity from this supplier if the risk of recovery increases. We discovered that the disruption recovery probability has more impact on the optimal ordering policy than the supply disruption probability. The most interesting discovery is the firm does not perform well under the condition that the offshore supplier has a very low chance to recover from disruption across the range of supply disruption probabilities, resulting in high minimum expected cost and low fill rate.

The findings from Model 2 can help the firm to quantitatively measure the impact of disruptive events at the firm's offshore supplier, but the formulation in Model 2 cannot easily be applied in practice since the the definition of the risk probabilities are too general. Therefore, we would like to further investigate the disruption process at the offshore supplier by investigating the value of additional information on the disruption and exploring an appropriate disruption recovery strategy that could be implemented to hedge the supply disruption. This study will be introduced in Model 3 and Model 4 in the next chapter.

4.4 Conclusion

In this chapter, we presented the analyses of the routine ordering policy model (Model 1) and the crisis ordering policy model (Model 2). We analysed these two models in four different settings, the model with constant demand and finite planning horizon, the model with stochastic demand and finite planning horizon, the model with constant demand and infinite planning horizon and, the model with stochastic demand and infinite planning horizon. In this thesis, Model 1 is a preliminary work that has been developed with minimal restrictions. This model has purposely been designed in that way with the intention of examining the policies of ordering for the firm with two non-identical suppliers in a simple supply chain setting (these two suppliers are distinguished by their lead-times, fixed ordering costs and unit ordering costs). Model 2 introduces a simple model to explore the effect of supply disruption on the ordering process in a similar supply chain setting.

The findings from Model 2 have provided us with some basic understanding on how the firm who has implemented global dual-sourcing strategy can manage the inventory system in the event of supply disruption at one of its suppliers. When both suppliers are available, the firm increases the quantity order from the offshore if the risk of disruption is increased and decreases the quantity order from the supplier if the chance of recovery is increased. When only the offshore supplier is down, the firm will only place an order with the onshore supplier if the inventory level is critically low. In this case, the firm will increase the order quantity from the onshore supplier with a decrease in the disruption recovery probability.

In Models 1 and 2, we can see that the costs related to the ordering process are the dominant factor when making a sourcing decision from the onshore and offshore suppliers. Since the onshore supplier sells the items/semi-items with a higher cost when there is no disruption in the supply chain, the firm would never sole-source from it even though the delivery period is shorter than for the offshore supplier. Under the crisis operation, the firm needs to trade off the costs versus the state of the offshore supplier. The interesting discovery in Model 2 is on the properties of the optimal ordering from the offshore supplier. If the estimated chance of disruption recovery by the offshore supplier is low, the firm should try to keep a higher volume of stock from this supplier to avoid the high shortage cost during disruptions. However, if the chance of disruption recovery by the offshore supplier is too low, then it seems that it is not possible to do this.

Managing the ordering decisions during the disruption discovery and recovery requires the firm to have accurate and precise information about the disruption to the offshore supplier. Model 2 is just a simple disruption model, in which information on the disruption is based on a single parameter, the constant probability that the supplier recovers from disruption by the start of the next period. We expect additional information on the disruption can help the firm to improve the ordering policy in terms of the expected minimum costs and fill rate values. Therefore, in the next chapter, we present the analyses on the value of supply disruption information.

5. On the Value of Supply Disruption Information

5.1 Introduction

In the previous chapter, under the analyses of Model 2, we demonstrated how supply disruptions at the offshore supplier can affect the firm's ordering policies. The findings from Model 2 however have been explained in general without considering any specific characteristics of the disruption process at the supplier. In this chapter, we will therefore further investigate the measurement of the disruption processes, focusing on the visibility of information in the event of supply disruption. Supply disruptions are typically rare but the effects of this type of disruption can be catastrophic for inventory management. Advanced information on the disruptions is very useful in mitigating their effect on inventory management. Furthermore, inventory is an effective safeguard only if it can be isolated from the disruptive event (Wang et al., 2010). Zsidisin and Wagner (2010) reveal that the greatest benefit to create resilience with disruption information appears to come from the risk of offshoring the supply. In addition, Handfield et al. (2007) reports that firms with a high exposure to global supply chain risk invest more to improve the capability of inventory and capacity visibility systems in detecting disruptions.

This chapter presents an inventory model of the firm and two non-identical suppliers with risk of disruption to one supplier. The aim of the model is to explore the significance of disruption information for the firm's mitigation planning and consider circumstances under which the firm is not advised to apply the dual-sourcing policy. We analyse two models namely Model 3 and Model 4 using the DMDP modelling framework to explore different aspects of the disruption processes. Notably, most of the disruption-inventory models in the literature have defined more than one parameter estimation (e.g., velocity, impact and loss) in formulating supply disruption processes (Schmitt, 2008; Tomlin, 2006). For example, these can represent the estimated frequency and length of disruptions (i.e., rare disruption event that persists for long periods or regular disruption events lasting for but occur in short periods). In this chapter, the frequency of disruption is modelled by a single parameter which represents the constant risk of disruption. We also assume that the length of disruption follows a known probability distribution. Models 3 and 4 differ in the disruption information available when disruption occurs. In Model 3, it is assumed that the length of the disruption is known to the firm, while in Model 4, it is assumed that the firm only knows the probability distribution of the length of disruption. We can say that the severity (length) of a disruption determines how much inventory the firm would need to fully protect itself against any supply disruption (Wang et al., 2010).

Those firms who offshore their sourcing activities have disadvantages in term of distance to supply source and delivery performance (Sting and Huchzermeier, 2010). We believe that these two issues have a strong correlation with the lead-time for delivery and the lead-time is among the most important criteria in managing global supply chains (Sting and Huchzermeier, 2010; Handfield et al., 2007; Parthasarathy et al., 2007). A long distance to supply source without good supply management can cause long delivery and affect the delivery performance. If the duration of the disruption on the order delivery from these suppliers is too long, then it will affect the lead-time, as well as the firms' inventory management (i.e., not enough stock in inventory and delay in order delivery) (Xia and Tang, 2011). Furthermore, from inventory management perspective, even-though there are always other suppliers as backup, sometimes it may still not be sufficient due to a capacity constraint on a high volume order from backup suppliers if the disruptions are too long. For this reason, we believe that, the frequency and length of disruptions are the main issues for planning disruption risk management in the supply chain network.

This chapter is structured in four sections. We present the analysis of Models 3 and 4 in section 5.2 and section 5.3 respectively. Then, the relationships between Model 3 and Model 2 and, Model 4 and Model 2 are discussed in section 5.4 and finally the conclusion for this chapter is presented in section 5.5.

5.2 Full Information at Start of Disruption

Model 3 presents a model involving the acquisition of advance disruption information to be used in the firm's mitigation planning. We assume that the firm has information on the duration of the supply disruption as soon as the disruption takes place at the offshore supplier. The information may be obtained from the supplier based on an assessment of the disruption event at their production plants and the delay to the order delivery process. As an example of a real catastrophic event, consider the fire at Nokia and Ericsson's supplier semiconductor plant in 2000. The firm were informed by their supplier that there would be a one week delay in shipment (Schmitt et al., 2015). We call this type of information as *'true information at start of disruption'* and assume that the information is completely accurate.

In Model 3, the true information spectrum is characterised by the exact number of periods of disruption at the offshore supplier. The disruption is modelled as a Markov chain and we investigate how the firm's inventory policy will be affected by the state of the Markov chain model of the disruption process. We examine the impact of the rate of transition between states of the disruption process on the ordering decision and the minimum expected inventory cost. The rates of transition are assumed to be known and fixed. Following the occurrence of a disruption, there is certainty as to the path and direction of transition that will be taken by the offshore supplier during recovery to the up state. All other factors being equal, we expect the inventory cost in Model 3 to be higher than the cost during the normal operation in Model 1, but lower than in Model 2 due to the additional information available to help with inventory management planning during disruption. In addition, similar to Model 2, if the offshore supplier is in the down state, the firm will increase the quantity to be ordered from the onshore supplier to manage the inventory shortage during the supply disruption.

The structure of this section is as follows. We describe Model 3 and its assumptions in sections 5.2.1 and 5.2.2, followed by the formulation of the ordering decision problem under supply disruption via the DMDP in section 5.2.3. Then, in section 5.2.4, we present the transition probability values used when conducting the numerical experiment. The results and findings are reported in sections 5.2.5, 5.2.6 and 5.2.7. Finally, the conclusion for Model 3 is presented in section 5.2.8.

5.2.1 Model Description

The firm seeks to split the order between the onshore supplier (or supplier N) and the offshore supplier (or supplier F), based on the disruption process at supplier F. During normal operations of supplier F, the firm can order from both suppliers. However, during a disruption at supplier F, the firm can only order from supplier N. We assume that the firm has information on the exact length of the disruption to order delivery from supplier F, or in other words, the firm knows when the disruption at the offshore supplier is going to end.

The Markov model of the disruption process at supplier F in Model 3 is as follows. When supplier F is in the up state there is a constant risk α that a disruption event will occur. In other words, α is the probability that supplier F moves from the up state to the down state during any period. When a disruption event occurs, supplier F assesses the cause and formulates a recovery plan. Information about the length of the recovery process is shared with the firm. We assume that the length of the disruption follows a known probability distribution with finite support W. Let A denote a random variable representing the length of a disruption. We denote the probability that the disruption lasts for w periods as $P(A = w)$ for $w = 1, 2, \ldots, W$. During disruption, the state of the offshore supplier is represented by a positive integer representing the remaining length of the disruption. Hence, given that a disruption event has just occurred, the state of supplier F moves to state w with probability $P(A = w)$. It is convenient to represent the up state of supplier F by 0. During the recovery process the state of supplier F moves from w to $w - 1$ with probability 1 and the recovery is complete when the recovery process returns to state 0. The durations of intervals between disruptions are modelled as independent geometric random variables, while the lengths of disruptions are modelled as independent identically distributed random variables with known distribution. For a better understanding, the transitions between normal operation and states of disruption for supplier F are illustrated in figure 5.1.

In figure 5.1, p_j represents the probability normal operations at supplier F during a period are disrupted resulting in an interruption to delivery for the next j periods. Hence $p_j = \alpha \times P(A = j)$. Whenever the disruption process is in state 0, the process either remains in state 0, with probability $1 - \alpha$, or moves to states j, with probability p_j, where $1 \leq j \leq W$. From state $j > 0$, the state of supplier F moves through states $j - 1, j - 2, \ldots, 0$ as the recovery process progresses. In this way, after j periods the recovery is complete and the supplier's operation is back to normal.

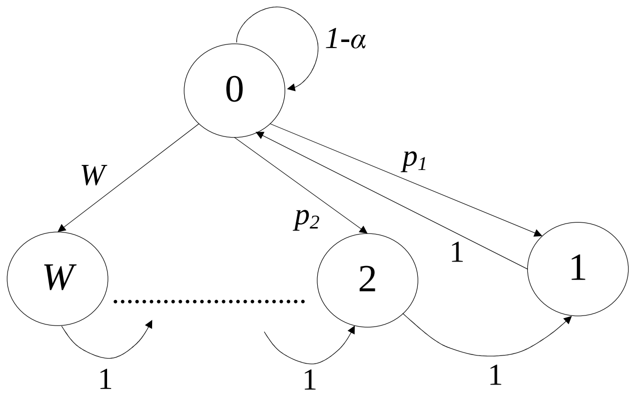


Figure 5.1. The transition structure for disruption process in Model 3

5.2.2 Model Assumptions

The assumptions of Model 3 are as follows:

a. The option of sourcing from supplier F is at risk of disruptions. The order from supplier F arrives either in full or not at all. However, the status of supplier N is perfectly reliable.

b. The firm has information about the exact length of the disruption. The durations of intervals between disruptions are modelled as independent geometric random variables, while the lengths of disruptions are modelled as independent identically distributed random variables with known distribution.

c. The firm's inventory planning horizon is discrete.

d. Demand is either deterministic or stochastic. The stochastic demand distribution follows the truncated Poisson distribution, $P(d_t) \sim Pois(\lambda, K)$.

e. Customers do not accept backorders, thus the firm encounters lost sales. The firm is charged with a penalty cost for each unit of demand that cannot be satisfied in a period, $PNLTY$.

f. The firm incurs a holding cost for inventory held during period t, $HOLD$.

5.2.3 Model Formulation

In this section, we explain the formulation of the problem analysed in Model 3 as a DMDP model and present the optimality equation.

Components of the DMDP for Model 3

The DMDP components in Model 3 are as follows:

Decision epoch

A decision is taken at the beginning of each period. Let t denote the decision epoch when there are t periods remaining in the planning horizon, $t = 1, 2, \ldots, T$.

States

At each decision epoch, the firm reviews the inventory level, i, and the state of supplier F, a. The parameters i and a comprise the state of the process y, such that $y = (i, a)$. The state space, Y, of Model 3 is given by:

$$Y = \Big\{(i, a) : i \in 0, 1, \ldots, I \;\;\&\;\; a \in 0, 1, \ldots, W\Big\}.$$

Actions

Based on the current state, the firm then decides on the quantity to be ordered from supplier N and supplier F. The action is denoted by $b = (q^N, q^F)$ and the set of admissible action, $B(y)$ is given by:

$$B(i,0) = \left\{(q^F, q^N) : \quad q^F, q^N \geq 0 \quad \& \quad q^F + q^N \leq I - i\right\} \quad \text{for} \quad 0 \leq i \leq I.$$

$$B(i,j) = \left\{(0, q^N) : \quad q^N \in \{0, ..., I - i\}\right\} \quad \text{for} \quad 0 \leq i \leq I \quad \text{and} \quad 1 \leq j \leq W.$$

Under the admissible action set of $B(i,0)$, the firm can choose to order up to $I - i$ items either from supplier N only or from supplier F only or from both the suppliers. Whilst during recovery of supplier F from disruption, under the admissible action set of $B(i, j)$, the firm can place an order for up to $I - i$ items with supplier N only.

Transition probabilities

We model changes in the inventory level and changes in the state of supplier F, separately. The transition matrix describing changes in the inventory level depends on the order quantities and is the same as in previous models. See section 4.2.3.a for a full description. The transition matrix describing changes in the state of supplier F follows from figure 5.1 above. The transition matrix is denoted by X and is formally presented below.

$$X = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ W \end{array} \begin{array}{c} \begin{array}{ccccc} 0 & 1 & \cdots & W-1 & W \end{array} \\ \begin{pmatrix} 1-\alpha & \alpha P(A=1) & \cdots & \alpha P(A=W-1) & \alpha P(A=W) \\ 1 & 0 & \cdots & 0 & 0 \\ 0 & 1 & \cdots & 0 & 0 \\ \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & \cdots & 1 & 0 \end{pmatrix} \end{array}$$

One-step costs:

As in Models 1 and 2, the one-step cost function, as a result of choosing action b in state y consists of the ordering cost, $ORDER$, the holding cost, $HOLD$ and the penalty cost, $PNLTY$. In the one-step cost function with the stochastic demand, the values of $HOLD$ and $PNLTY$ depend on the random demand during the period. The one-step costs for Model 3 with constant and stochastic demand settings are based on the same assumptions as the one-step costs for the models in previous chapters. See section 3.2.1 for a detailed explanation.

The one-step cost when action b is chosen in state y at decision epoch y is denoted by $C_t^y(b)$. Under a constant demand setting, this cost is given by:

$$\begin{aligned}
C_t^y(b) &= ORDER + HOLD + PNLTY \\
&= \sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \; h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\
&\quad + \; m\Big(\max(D_t - i - q^N, 0)\Big)
\end{aligned}$$

and under a stochastic demand setting it is given by:

$$\begin{aligned}
C_t^y(b) &= ORDER + \Big(E_{D_t}\Big(HOLD + PNLTY\Big)\Big) \\
&= \sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t)\Big\{h\Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) \\
&\quad + \; m\Big(\max(d_t - i - q^N, 0)\Big)\Big\}
\end{aligned}$$

Optimality equation

Let $V_t(i, a)$ denote the minimum cost over the last t periods of the planning horizon when the inventory level is i and the state of supplier F is a at decision epoch t. The optimality equation for Model 3 with constant demand is given by:

$$\begin{aligned}V_t(i,a) = \min_{b\in B(i,a)}\Bigg\{ &\sum_{sp\,\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + m\Big(\max(D_t - i - q^N, 0)\Big) \\ &+ h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\ &+ \sum_{k=0}^{W} X_{a,k}\Big(V_{t-1}\Big(\max(i + q^N - D_t, 0) + q^F, k\Big)\Big)\Bigg\}\end{aligned}$$

The optimality equation with stochastic demand is given by:

$$\begin{aligned}V_t(i,a) = \min_{b\in B(i,a)}\Bigg\{ &\sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d=0}^{\infty} P(D_t = d)\Big\{m\Big(\max(d - i - q^N, 0)\Big) \\ &+ h\Big(\frac{1}{2}(i + \max(i + q^N - d, 0))\Big) \\ &+ \sum_{k=0}^{W} X_{a,k}\Big(V_{t-1}\Big(\max(i + q^N - d, 0) + q^F, k\Big)\Big)\Big\}\Bigg\}\end{aligned}$$

Using these optimality equations, we seek to minimise $V_t(i,a)$ and find the optimal quantities to order from supplier N and supplier F, depending on the values in the transition matrix X. We are interested to investigate numerically how the values in the transition matrix, and therefore the characteristics of the disruption, can affect the firm's ordering policy.

An equilibrium distribution of the Markov chain model

The expected length of a disruption is denoted by $\bar{A}$ and can be calculated as follows.

$$\bar{A} = \sum_{j=1}^{W} jP(A = j).$$

The proportion of time for which the offshore supplier is up or down can be deduced from the equilibrium distribution of the Markov chain model of the state of the offshore supplier. Let $\pi = (\pi_0, \pi_1, \cdots, \pi_W)$ denote this equilibrium distribution. Using standard methods (see Tijms (2003) for example), it is easy to show that the equilibrium distribution is unique and satisfies the following equations.

$$\pi_0 = (1-\alpha)\pi_0 + \pi_1 \tag{5.1}$$

$$\pi_j = \alpha P(A=j)\pi_0 + \pi_{j+1} \ \text{ for } \ 1 \le j \le W \tag{5.2}$$

$$\pi_w = \alpha P(A=W)\pi_0 \tag{5.3}$$

$$1 = \sum_{j=1}^{W} \pi_j \tag{5.4}$$

We claim that $\pi_j = \alpha\pi_0 \sum_{j=1}^{W} P(A=j)$ for $1 \le i \le W$. The proof is by induction. The result is true for $i = W$ for equation 5.3. Assume the result is true for some $i > 1$. From equation 5.3:

$$\begin{aligned}\pi_{i-1} &= \alpha P(A=i-1)\pi_0 + \pi_i \\ &= \alpha P(A=i-1)\pi_0 + \alpha\pi_0 \sum_{j=i}^{W} P(A=j)\end{aligned}$$

since result holds for i,

$$\begin{aligned}&= \alpha\pi_0\Big(P(A=i-1) + \sum_{j=i}^{W} P(A=j)\Big) \\ &= \alpha\pi_0 \sum_{j=i-1}^{W} P(A=j)\end{aligned}$$

Hence, result holds for $i-1$.

By the principle of mathematical induction, it follows that the result is true for $i \le i \le W$.

Finally, from equation 5.4:

$$
\begin{aligned}
1 &= \pi_0 + \sum_{i=1}^{W} \pi_i \\
&= \pi_0 + \sum_{i=1}^{W} \alpha\pi_0 \sum_{j=i}^{W} P(A = j) \\
&= \pi_0\Big(1 + \alpha \sum_{i=1}^{W} \sum_{j=1}^{W} P(A = j)\Big) \\
&= \alpha_0\Big(1 + \alpha \sum_{i=1}^{W} jP(A = j)\Big) \\
&= \pi_0(1 + \alpha\bar{A})
\end{aligned}
$$

Hence $\pi_0 = \frac{1}{1+\alpha\bar{A}}$. Therefore, the proportion of the for which the offshore supplier is up is $\pi_u = \frac{1}{1+\alpha\bar{A}}$ and the proportion of time for which the offshore supplier is down is $\pi_w = 1 - \pi_u = \frac{\alpha\bar{A}}{1+\alpha\bar{A}}$.

Note that, this argument makes no assumption about the characteristics of the probability distribution of the length of disruption and so generalises to any probability distribution. For example, in Model 2, the expected length of disruption is $1/\beta$ and the proportion of time for which the offshore supplier is up is $\frac{1}{1+\alpha/\beta} = \frac{\beta}{\alpha+\beta}$ as before. This result will be used to calculate the proportion of time for which the offshore supplier is up or down in Model 4.

5.2.4 Choice of Parameters Values

In this section, we present various transition probability values used for the numerical analysis. Our objective is to analyses how the optimal policy changes with different transition probabilities. In this numerical study, we consider the offshore supplier will be down for at most 5 periods, which represents the maximum length of disruption, W, thus $W = 5$. We combine a few values of α and $P(A = w)$, and generate 20 cases, as shown in table 5.1. For other parameters in the optimality equation, we use the base values that have been presented in section 3.6.1, as shown in table 5.2.

We number the cases according to the value of α and $P(A = w)$. For example for case 1A, number **1** is used to represent the corresponding sets of α values and letter **A** is used to represent the corresponding sets of $P(A = w)$ values. The α value considered in this study is use to represent the frequency of disruption to supplier F. In each instance of case 1 in table 5.1, supplier F has a high probability to stay operating when in the up state ($j = 0$) and this probability decreases for instances of case 2, case 3 and so on. The higher the numbers, the lower the probability for the offshore supplier to remain up. Cases with letter represent various conditions of the disruption length for each disruption period. We set case A as a base value. For case A, $P(A = w)$ values are calculated based on the discrete uniform distribution, such that $P(A = w) \sim Uniform(w)$ for $w = 1, 2, 3, 4, 5$. The probability values of $P(A = w)$ for $w = 1, 2, 3, 4, 5$ are equal, which shows that the risk of supplier F to move to any state w from state u are equal. The discrete uniform probability mass function (p.m.f) is given by: $P(A = w) = \frac{1}{W}$ for $W = 1, 2, 3, 4, 5$. For case B, this case is set to represent the condition where there is a high probability that the disruption lasts for 1 period. Case C represents the case of high probability that the disruption lasts for 3 periods and case D represents the case of high probability that the disruption lasts for 5 periods. The first two scenarios (i.e., cases A and B) would then have same average length of disruption but the variance of the length of disruption would be different, while the second two scenarios (i.e., cases C and D) have different average lengths but the same variance. For a better understanding, the values of $P(A = w)$, the average of disruption length, $\bar{A}$, and the variance of disruption length, σ^2, in each case are tabulated in table 5.3.

We are also interested to examine the relationship between the optimal ordering policy and the expected length of an interval of normal service, $1/\alpha$ and the expected length of disruption, $\bar{A}$. The expected length of the disruption, $\bar{A}$, is given by the probability distribution:

$$\bar{A} = \sum_{w=1}^{W} wP(A = w)$$

Table 5.1. 20 sets of α and $P(A = w)$ values.

Case	α	$P(A = 1)$	$P(A = 2)$	$P(A = 3)$	$P(A = 4)$	$P(A = 5)$	$1/\alpha$	$\bar{A}$	π_u	π_w
1A	0.1	0.2	0.2	0.2	0.2	0.2	10.00	3.00	0.77	0.23
2A	0.3	0.2	0.2	0.2	0.2	0.2	3.33	3.00	0.53	0.47
3A	0.5	0.2	0.2	0.2	0.2	0.2	2.00	3.00	0.40	0.60
4A	0.7	0.2	0.2	0.2	0.2	0.2	1.43	3.00	0.32	0.68
5A	0.9	0.2	0.2	0.2	0.2	0.2	1.11	3.00	0.27	0.73
1B	0.1	0.1	0.2	0.4	0.2	0.1	10.00	3.00	0.77	0.23
2B	0.3	0.1	0.2	0.4	0.2	0.1	3.33	3.00	0.53	0.47
3B	0.5	0.1	0.2	0.4	0.2	0.1	2.00	3.00	0.40	0.60
4B	0.7	0.1	0.2	0.4	0.2	0.1	1.43	3.00	0.32	0.68
5B	0.9	0.1	0.2	0.4	0.2	0.1	1.11	3.00	0.27	0.73
1C	0.1	0.4	0.2	0.2	0.1	0.1	10.00	2.30	0.81	0.19
2C	0.3	0.4	0.2	0.2	0.1	0.1	3.33	2.30	0.59	0.41
3C	0.5	0.4	0.2	0.2	0.1	0.1	2.00	2.30	0.47	0.53
4C	0.7	0.4	0.2	0.2	0.1	0.1	1.43	2.30	0.38	0.62
5C	0.9	0.4	0.2	0.2	0.1	0.1	1.11	2.30	0.33	0.67
1D	0.1	0.1	0.1	0.2	0.2	0.4	10.00	3.70	0.73	0.27
2D	0.3	0.1	0.1	0.2	0.2	0.4	3.33	3.70	0.47	0.53
3D	0.5	0.1	0.1	0.2	0.2	0.4	2.00	3.70	0.35	0.65
4D	0.7	0.1	0.1	0.2	0.2	0.4	1.43	3.70	0.28	0.72
5D	0.9	0.1	0.1	0.2	0.2	0.4	1.11	3.70	0.23	0.77

Table 5.2. The values of the base set of parameters.

Parameters	I	D_t	λ	m	h	c^N	c^F	v^N	v^F
Values	70	5	5	8	$\frac{0.35*v^N}{13}$	5	10	2	1

Table 5.3. The values of $P(A = w)$, $\bar{A}$ and σ^2 in each case.

Case	W					$\sum P(A = w)$	$\bar{A}$	σ^2
	1	2	3	4	5			
A	0.2	0.2	0.2	0.2	0.2	1	3.0	2.0
B	0.1	0.2	0.4	0.2	0.1	1	3.0	1.2
C	0.4	0.2	0.2	0.1	0.1	1	2.3	1.81
D	0.1	0.1	0.2	0.2	0.4	1	3.7	1.81

In addition, we are also interested to examine the relationship between the optimal policy and the proportion of time for which the offshore supplier is up or down, π_u and π_w respectively. The values of π_u and π_w can be obtained from the equilibrium distribution of the Markov chain, as shown is section 5.2.2c. The values of $1/\alpha$, $\bar{A}$, π_u and π_w are tabulated in table 5.1.

From this numerical study, we illustrate the effects of the transition probabilities, case by case, on the three areas namely the firm's optimal ordering decisions, the cost of optimal policies and the performance of the optimal policy under stochastic demand model (i.e., fill rate and average inventory level). To do the experiment, we analyse Model 3 with the combination of α and $P(A = w)$ values, case by case, as in table 5.1.

In what follows, we first results on the effect of the cases on the properties of the ordering decisions, then results relating of effects on the properties of the costs of policies, and finally the results of the effects on the properties of the fill rate and the average inventory under the stochastic demand model analysis.

5.2.5 The Impact of the Transition Probability Values on the Ordering Decisions

In this section, we explain how the transition probability values can affect the firm's ordering decision. We discuss the result under the infinite-horizon setting, covering the infinite-horizon Model 3 with constant demand (later known as M3*InfCons*) and stochastic demand (later known as M3*InfSto*).

The ordering policies in the infinite-horizon Model 3

The optimal order decisions with the infinite-horizon planning under constant and stochastic demands are as follows.

The ordering policy of M3InfCons

If supplier F is up, the firm only order from supplier N if the inventory level, i, is relatively low, ($i < 5$) in all cases, as illustrated in figure 5.2. However, the ordering decision from supplier F does vary from case to case. For an example, from figure 5.3, in case 4 ($\alpha = 0.7$), the quantity order from supplier F in base case (case A) is higher than other cases. In other cases, even though the ordered quantity from this supplier are the same, but the inventory level point to which the firm places order are vary. The optimum inventory level at which to order when supplier F has higher probability to be down for 1 period is when $i = 19$. The point at which optimal for the firm to place order decreases when supplier F has higher probability to be down increase to 3 and 5 periods ($i = 15$). Comparing the ordering policy from this supplier from the equal average of disruption length scenario, there is a difference in optimal ordering between case A and case B, which the firm will order more in case A. However, there is no difference in optimal ordering between case C and case D under the scenario of equal variance of disruption length.

We can see from figure 5.3 that the optimal ordering policy from the offshore supplier is effectively an (s, S) policy. When $i < D$, the items in inventory and the order from supplier N are used to meet demand during the period exactly and no items are carried forward. The order from supplier F then ensures that there are S items in inventory at the beginning of the next period. When $D \leq i < s$, demand this period is satisfied from inventory and $i - D$ items of inventory are carried forward to the next period. The firm also orders $S - i + D$ items from supplier F to bring the inventory level up to S at the beginning of the next period. The optimal ordering policy for supplier F has this form in all cases. Table 5.4 shows how the parameters vary in the 20 cases considered.

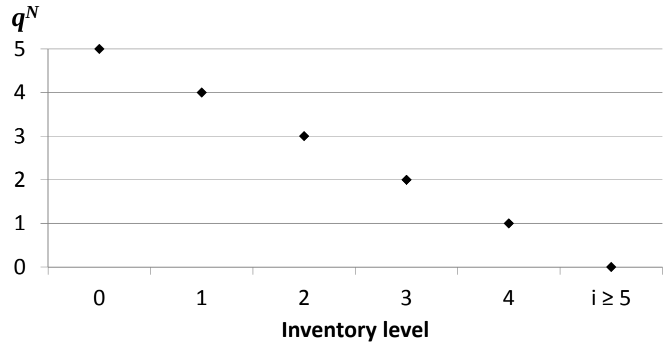


Figure 5.2. M3*InfCons*, state u: The optimal order from supplier N.

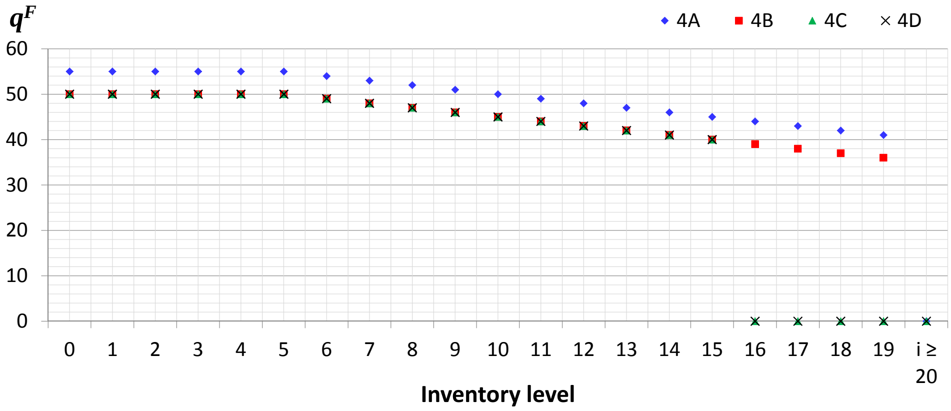


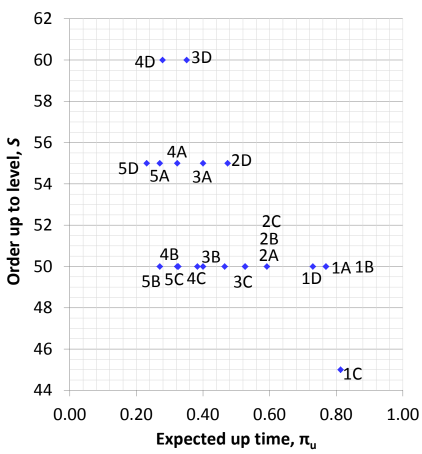
Figure 5.3. M3*InfCons*, state u: The optimal order from supplier F when $\alpha = 0.7$.

From table 5.4 we see that the incentive to order from supplier F increase with increases in α. However, from the scenario of disruption period perceptive, there are differences in optimal ordering policy in cases of equal average and variance of disruption length. In equal average of disruption length scenario, in case 3 ($\alpha = 0.5$), the firm will order more in case A ($S = 55$) as compare to case B ($S = 50$) and the inventory level point at which the firm will first placing order to the offshore supplier is higher in case A ($s = 19$) as compare to case B ($s = 17$). However, optimal ordering policy in other cases are the same for both cases A and B. In equal variance of disruption length scenario, the firm will order more in case D in each α case. In addition, except in case 1 ($\alpha = 0.1$), the inventory level point at which the firm will start placing order with the offshore supplier is higher in case D too. From the findings, we can conclude that policy in case B is better than policy in case A under higher supply disruption probability condition ($\alpha > 0.5$), and the policy in case C is better than policy in case D.

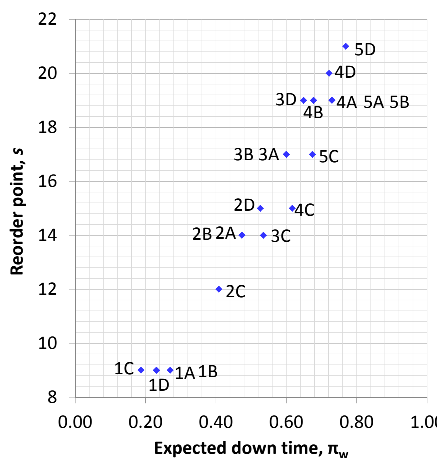
Table 5.4. M3*InfCons*, state u: Optimal ordering policy for supplier F.

Case	A	B	C	D
1	(9,50)	(9,50)	(9,45)	(9,50)
2	(14,50)	(14,50)	(12,50)	(15,55)
3	(17,55)	(17,50)	(14,50)	(19,60)
4	(19,55)	(19,50)	(15,50)	(20,60)
5	(19,50)	(17,50)	(19,50)	(21,55)

Figure 5.4 shows the relationship between the proportion of time for which supplier F is either up or down and the optimal policy. From figure 5.4a, we can see that order up to level, S approximately decreases with an increase in the expected up time, π_u. However, from figure 5.4b, we can see that reorder point, s increases with an increase in the expected down time, π_w. We can conclude that, the firm will increase the quantity ordered from the offshore supplier if the expected down time increases and the point of inventory level at which the firm to place an order with this supplier will be reduced if the expected up time increases. The relationship between the order up to level and the proportion of time supplier F is down is non-linear. The decrease in reorder point with proportion of time supplier F is up is approximately linear.



(a) S $vs.$ π_u



(b) s $vs.$ π_w

Figure 5.4. M3*InfCons*: The relationship between the optimal order and the expected up or down time

If supplier F is down, from figure 5.5, when $i \leq 5$, ordered quantity from supplier N increases by 5 unit when the number of down periods of supplier F increases in each α case and disruption period scenario. As we expected, it is optimal to increase the quantity of order from the onshore supplier because the disruption is expected to be longer. Under this situation, we can see that supply disruption probability and the expected length of disruption at each disruption period do not influence the optimal ordering policy from the onshore supplier, but the number of disruption periods does affect the optimal ordering from this supplier.

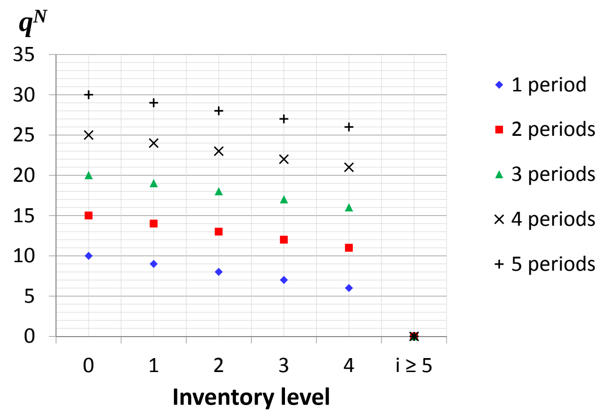


Figure 5.5. M3*InfCons*, state w: The optimal order from supplier N in all cases.

The ordering policy of M3InfSto

If supplier F is in the up state, the firm only order from supplier N if the inventory level, i, is relatively low, ($i < 7$) in all cases, as illustrated in figure 5.6. However, the ordering decision from supplier F does vary from case to case. For example, from figure 5.3, in case 4 ($\alpha = 0.7$), the quantity order from supplier F in cases A is higher than other cases. In other cases, even though the ordered quantity from this supplier are the same, but the inventory level point at which the firm places order are vary. The optimum inventory level at which to order when supplier F has higher probability to be down for 1 period is when $i = 22$. The point to which optimal to place order decreases ($i = 20$) when the probability for supplier F to be down increase to 3 and 5 periods. Comparing the ordering policy from this supplier from the equal average of disruption length scenario, there is a difference in optimal ordering between case A and case B, which the firm will order more in case A. However, there is no difference in optimal ordering between case C and case D under the scenario of equal variance of disruption length. The property of this optimal ordering policy is as the same of the constant demand model.

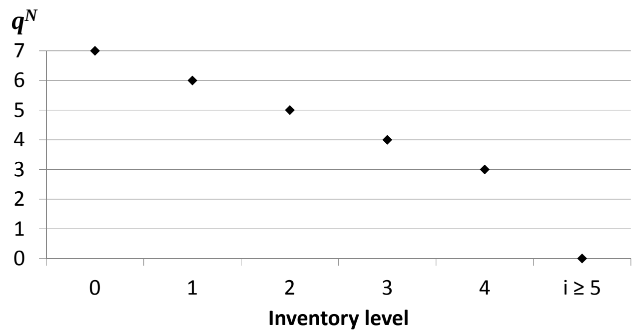


Figure 5.6. M3*InfSto*, state u: The optimal order supplier N.

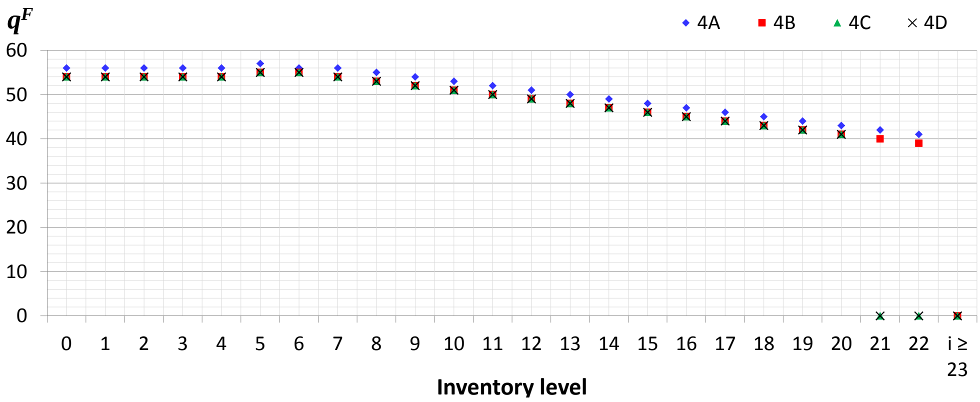


Figure 5.7. M3*InfSto*, state u: The optimal order supplier N when $\alpha = 0.7$.

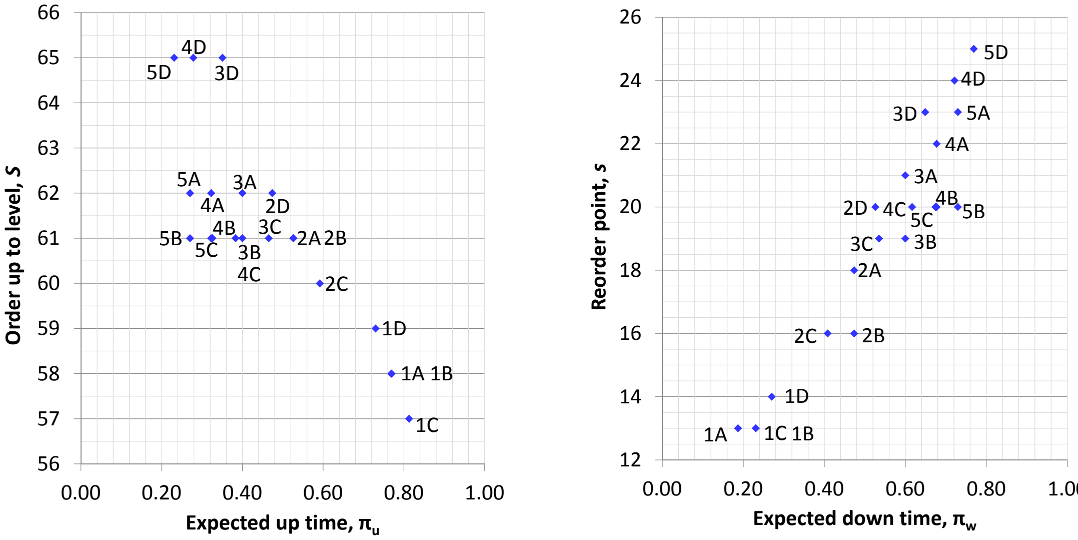
We can see from figure 5.7 that the optimal ordering policy from the offshore supplier is effectively an (s, S) policy. Similar to the constant demand model, when $i < d$, the items in inventory and the order from supplier N are used to meet demand during the period exactly and no items are carried forward. The order from supplier F then ensures that there are S items in inventory at the beginning of the next period. When $d \leq i < s$, demand this period is satisfied from inventory and $i - d$ items of inventory are carried forward to the next period. The firm also orders $S - i + d$ items from supplier F to bring the inventory level up to S at the beginning of the next period. The optimal ordering policy for supplier F has this form in all cases. Table 5.5 shows how the parameters vary in the 20 cases considered.

From table 5.5 we see that the incentive to order from supplier F increases with an increase in α. However, from the scenario of disruption period perspective, there are differences in optimal ordering policy in cases of equal average and variance disruption lengths. In equal average disruption length scenario, except in case 1 ($\alpha = 0.1$), the inventory level point at which the firm will first placing order to the offshore supplier is higher in case A as compare to case B, and the firm will order more in case A as compare to case B too except in cases 1 and 2 ($\alpha = 0.1, 0.2$). In equal variance of disruption length scenario, the firm will order more in case D in each α case. In addition, except in case 1 ($\alpha = 0.1$), the inventory level point at which the firm will start placing order with the offshore supplier is higher in case D too. From the findings, we can conclude that policy in case B is better than policy in case A and, the policy in case C is better than policy in case D.

Table 5.5. M3*InfSto*, state u: Optimal ordering policy for supplier F.

Case	**A**	**B**	**C**	**D**
1	(13,58)	(13,58)	(13,57)	(14,59)
2	(18,61)	(16,61)	(17,60)	(20,62)
3	(21,62)	(19,61)	(20,61)	(23,65)
4	(22,62)	(20,61)	(21,61)	(24,65)
5	(23,62)	(20,60)	(22,61)	(25,65)

Figure 5.8 shows the relationship between the proportion of time for which supplier F is either up or down and the optimal policy. From figure 5.8a, order up to level, S decreases with an increase in the expected up time, π_u increases. However, from figure 5.8b, reorder point, s increases with an increase in the expected down time, π_w. From the findings, we can see negative and positive relationships between the optimal order and the expected up and down time respectively. It is logical since it is optimal for the firm to carry less inventory when the offshore is expected to operating longer and start placing order earlier when the disruption is expected to be longer.



(a) S $vs.$ π_u (b) s $vs.$ π_w

Figure 5.8. M3*InfSto*: The relationship between the optimal order and the expected up or down time

If supplier F is down, from figure 5.9, when i is relatively low ($i \leq 8$), the order quantity from supplier N increases by 5 unit when the number of disruption periods increases in each α case and disruption period scenario. It is optimal to increase the quantity of order from the onshore supplier because the length of disruption is expected to be longer. However, the inventory level point at which the firm places order to the offshore supplier is lower in case the offshore supplier has high risk to be down for 1 period and 5 periods than other cases. Under this situation, similar with constant demand model, we can see that the number of disruption periods does affect the optimal order from the onshore supplier, but not supply disruption probability and the expected length of disruption at each disruption period.

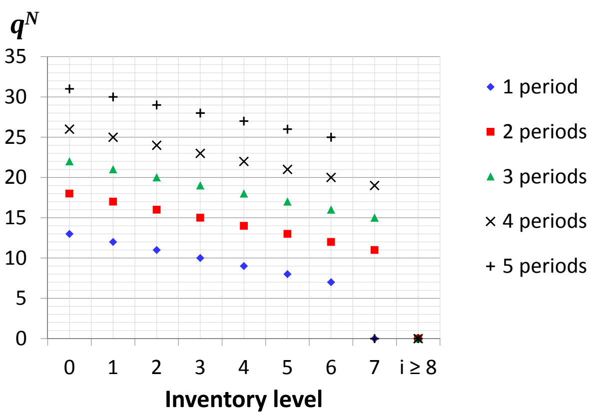


Figure 5.9. M3*InfSto*, state w: The optimal order from supplier N in all cases.

Discussion

Under the infinite-horizon plan, in both constant and stochastic demand settings, in all cases, the firm will only place order with the onshore supplier if the inventory level is so low that there is a high risk of immediate shortages. The findings show that the order quantity from the onshore supplier is only needed as backup if the firm does not have enough stock in inventory. This ordering policy to the onshore supplier is also apply to the situation when the offshore supplier is under a crisis operation. However, in this situation, the order quantity is bigger than in the situation when the offshore supplier is under a routine operation. The order is not only to satisfy the immediate shortage, but also to satisfy demand during subsequent periods of disruption. The firm will order more from this supplier when the offshore supplier is expected to face a disruption longer. The properties of the order policy with the offshore supplier are very different from those of the order policy with the onshore supplier. In both constant and stochastic demand models, the firm will increase the order quantity from the offshore supplier if this supplier have higher risks of disruption and expected disruption length of each disruption period.

5.2.6 The Impact of Different Transition Probabilities on the Long-run Average Costs

In this section, we discuss the result on how the values of transition probability can affect the properties of the optimal policies costs under the infinite-horizons Model 3, which the model covered the experiments with the constant and stochastic demand settings.

The optimal policies costs in the infinite-horizon Model 3

From figure 5.10, we can see that the pattern of the long-run average cost, g, across the cases is the same with both constant and stochastic demand. In addition, the pattern of g is also the same for each case of α. Most higher g are in cases 3 and 4 ($\alpha = 0.5, 0.7$) for all the expected disruption periods cases and overall the highest g is in case 3D. The long-run average cost increases gradually as the number of disruption periods that the offshore supplier to be down increases. Overall, lower g is in most case 1. From equal average and variance of disruption length scenarios, in each α case, g in case B is lower that in case A, and g cost C is lower than in case D. Based on the pattern of g across the cases, we can conclude that the number of disruption periods and the expected disruption length at each disruption period have more impact on the optimal long-run average cost than supply disruption probability. From the disruption period scenario perspective, the optimal policy in case B is better than in case A and, case C is better than case D.

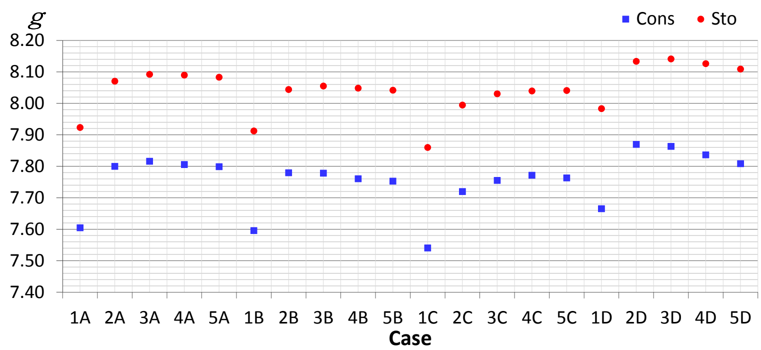
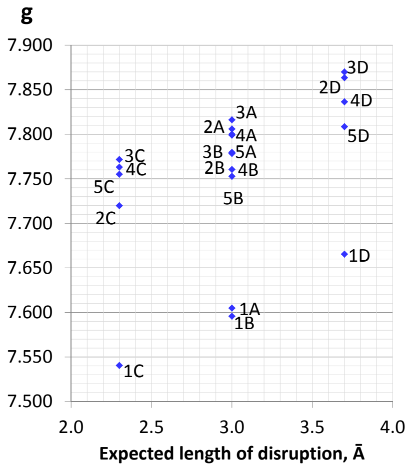


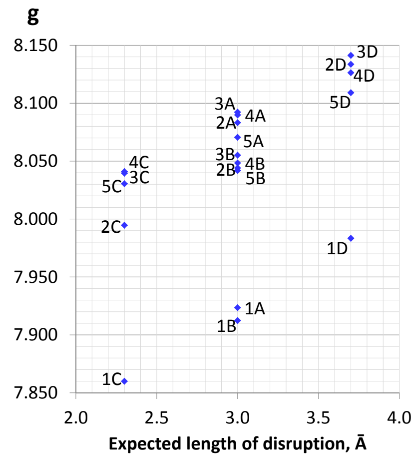
Figure 5.10. M3*Inf*: Optimal long-run average cost, g, in different cases for constant and stochastic demand models

From the findings, we see the optimal cost is high when the risk of the offshore supplier facing a disruption is high ($\alpha \geq 0.5$) and the offshore supplier is expected to has high probability to be down more than 3 periods (cases C and D). Therefore, the firm should give more attention to the situation where the offshore supplier has a high chance to face a disruption and longer disruption periods. In addition, even though the average and variance of the disruption length are the same for each disruption period scenario, the firm cannot expect to have similar long-run average costs because the number of disruption periods does effect the long-run average costs.

If we look from the aspect of the expected length of disruption, $\bar{A}$, from figure 5.11, we see that there is no relationship between the long-run average cost, g and $\bar{A}$, in both constant and stochastic demand models, as illustrated in figures 5.11a and 5.11b. However, if we exclude case 1 ($\alpha = 0.1$) from the both plots, we can see that the long-run average cost increases with an increase in expected length of disruption. From the findings, we can conclude that the firm will incur higher cost if the disruption length is expected to be longer except under low risk of supply disruption at the offshore supplier.



(a) M3*InfCons*



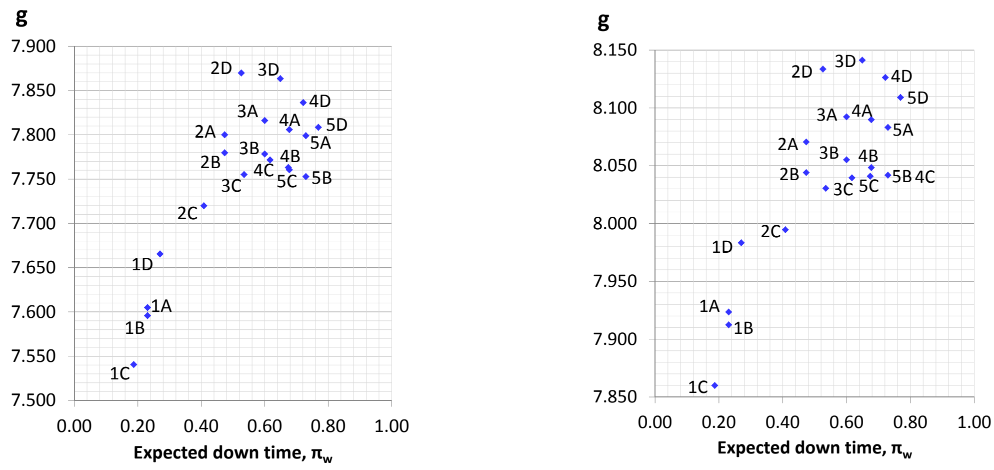
(b) M3*InfSto*

Figure 5.11. M3*Inf*: The relationship between the long-run average cost and the expected length of disruption

From the aspect of the expected down time, π_w, the long-run average cost, g has a relationship with π_w, From figure 5.12, in both constant and stochastic demand models, (see figures 5.12a and 5.12b), g increases as π_w increases. As we expected, the firm will face higher cost if down time at the offshore supplier is expected to be longer. The relationship between the long-run average cost and the proportion of time supplier F is down in both demand models are non-linear.

Discussion

From the findings, we can see that α, the number of disruption period and the expected disruption length at each disruption period will affect the long-run average cost of the optimal ordering policies. As we expected, the firm will face higher cost if the offshore supplier has high risk to face the disruption and the number of down period is expected to be longer.



(a) M3*InfCons* (b) M3*InfSto*

Figure 5.12. M3*Inf*: The relationship between the long-run average cost and the expected down time

5.2.7 The Impact of the Transition Probability Values on the Performance of the Policies

In this section, we discuss the performance of the ordering policy under the infinite horizon plan and stochastic demand model (M3*InfSto*), focussing on the performance of the fill rate (section 5.2.7a) and the average inventory level (section 5.2.7b).

Fill rate

From figure 5.13, the percentage of demand satisfied from stock in hand in all cases are estimated to lie between 99.80% and 99.76% with 95% of confidence interval. From findings, we see that even though there is a high disruption probability and a small number of disruption periods, the capability for the firm to satisfy demand still high since it has a backup source from the onshore supplier. It is quite noticeable that the fill rate in case A and case D are higher than case B and case C, respectively, except in higher supply probability condition ($\alpha > 0.5$). The variation in the fill rates for other values of α is within the range of the confidence intervals of the simulation results. From the findings, we can see that, the values of fill rate are vary under the situation where the average and variance of disruption length are equal, thus we can conclude that supply disruption probability and the expected disruption length at each disruption period can affect the fill rate.

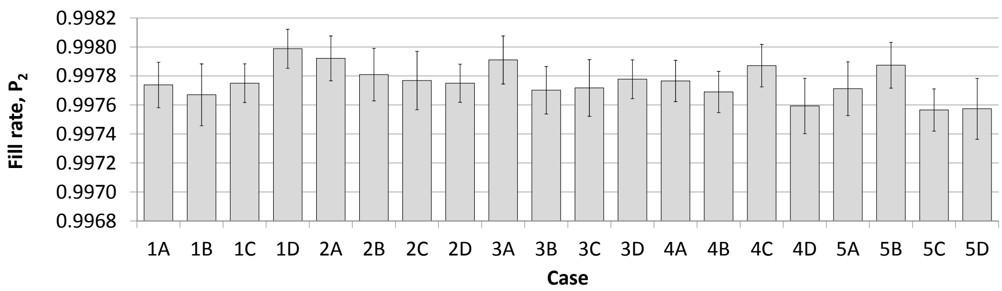
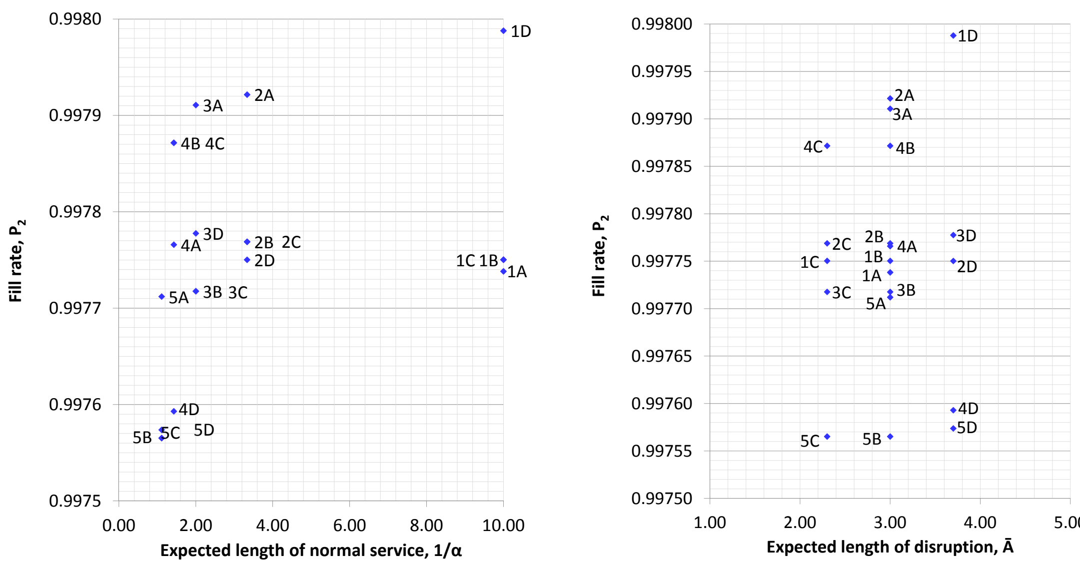


Figure 5.13. M3: Fill rate in each case

Looking at the performance of fill rate from the expected normal service and the expected disruption length at the offshore supplier, from figure 5.14, we can see that the fill rate, P_2 has relationships with $1/\alpha$ and $\bar{A}$. From figure 5.14a, if case 1 is excluded from the plot, P_2 increases with an increase in expected length of normal service. The fill rate, P_2, also increases with an increase in the expected length of disruption, $\bar{A}$, as illustrated in figure 5.14b. The relationship between the fill rate and the expected length of normal service (if we exclude the case of low supply disruption probability ($\alpha = 0.1$)) is approximately linear. The increases of fill rate with an increase of the expected length of disruption is non-linear.



(a) P_2 *vs.* $1/\alpha$

(b) P_2 *vs.* $\bar{A}$

Figure 5.14. M3: The relationships between fill rate and the expected normal service and the down time

From figure 5.15, we see two non-linear relationships between the fill rate, P_2, and the expected up time, π_u. P_2 increases as π_u increases.

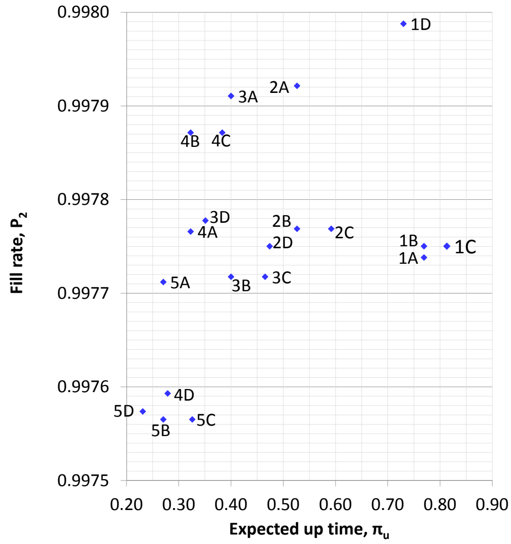


Figure 5.15. M3: The relationship between fill rate and the expected up time.

Average inventory level

From figure 5.16, the average inventory level, I_A, in all cases are estimated to lie between 31.16 and 36.27 with 95% of the confidence interval, which is roughly one half the maximum inventory level. We also see the highest I_A is in case 4D (36.27 ± 0.065) and the lowest is in case 1C (31.16 ± 0.042). This result is as we expected. Due to high frequency of disruption, it is optimal to carry more inventory to avoid expensive onshore supplier during disruptions. In addition, it is quite noticeable that the average inventory level in cases A and C are higher than in respect of cases B and D. Therefore, we can conclude that, even though the average and variance of the disruption length are equal, the average inventory level is vary, depending on the disruption period scenario.

From the aspect of the expected normal service, $1/\alpha$, and the expected disruption length, $\bar{A}$, the average inventory level, I_A has relationships with $1/\alpha$ and $\bar{A}$, which illustrated in figure 5.17. From figure 5.17a, I_A decreases with an increase in $1/\alpha$. However, I_A increases with an increase in $\bar{A}$, as illustrated in figure 5.17b. The relationship between the average inventory level and the expected length of normal service is approximately linear. The increases of average inventory level with an increase of the expected length of disruption is non-linear.

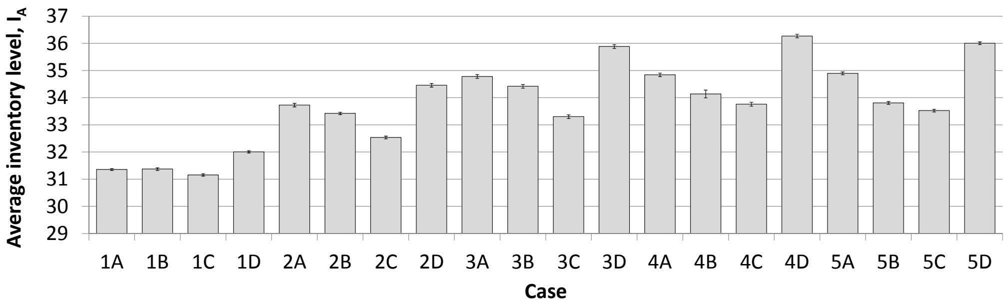
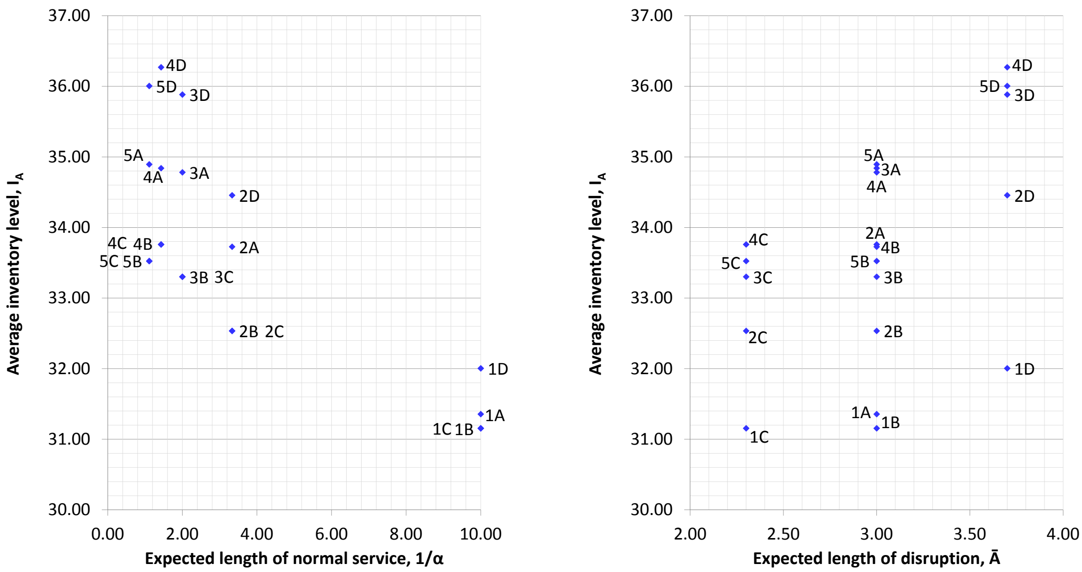


Figure 5.16. M3: Average inventory level in each case α



(a) Average inventory level $vs.\ 1/\alpha$

(b) Average inventory level $vs.\ \bar{A}$

Figure 5.17. M3: The relationships between average inventory level and the expected normal service and the down time

From figure 5.18, we see a negative correlation between the average inventory level and the expected up time, π_u. The average inventory level decreases as π_u increases. From findings, it is optimal to carry less inventory level when the expectation for the offshore supplier to operate is high.

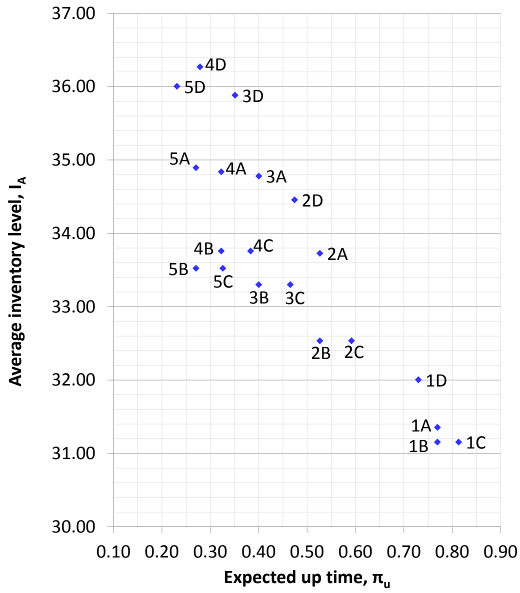


Figure 5.18. M3: The relationship between average inventory level and the expected up time

Discussion

In Model 3 analyses, similar to the analyses of Model 2, the performance of the firm's ordering policy has been measured with the values of fill rate and average inventory level. Based on fill rate, the firm still has high capability to satisfy demand from the customer when it has chose to use the findings policy. The percentage of demand satisfied from stock in hand is still high even though there are high supply disruption probability and expected periods of disruption. From the average inventory level perspective, having less stock in the inventory is more favourable if the supply disruption probability is small and the offshore supplier is expected to be down less than 3 periods.

5.2.8 Conclusion

In Model 3, we addressed how the firm's optimal ordering policies can be affected by the risk of disruptive supply events and the expected periods of disruption at the offshore supplier. From the analyses of Model 3, with advance information of supply disruption, we demonstrate how the properties and the performance of the optimal ordering policy depend on the values of the transition probabilities of the Markov chain model of disruption to the offshore supplier.

We show that advance supply disruption information has gave more detail ordering policies, as compare to the ordering policy in Model 2.

Similar to Model 2, the firm increases the quantity order from the offshore supplier if the risk of disruption and the expected periods of disruption increase. The most interesting discovery is the minimum expected cost in Model 3 is lower than the cost in Model 2. Consistent with our theory, advance information on supply disruption at the offshore supplier has helped the firm to have ordering policies that performed well than the policies with general supply disruption information.

The findings from Model 3 can help the firm to quantitatively measure the impact of disruptive events at the firm's offshore supplier. The recovery process that has been facilitated with advance supply disruption information helps the firm to have better ordering policies. We plan to further investigate the additional information on the disruption, which examine the firm's ordering policy under disruptive supply event without information on a disruption. The study will be introduce in Model 4 in a next section.

5.3 Partial Information on the Length of Disruption

Model 4 presents a model involving the availability of disruption information that are limited at the time of the disruption. During the disruption, the firm learns more about its nature and updates its belief about the length of the disruption. This situation might arise if the cause of disruption is unknown or unprecedented or if suppliers are reluctant to share information on the condition of its business operation in the event of the disruption (Kurano et al., 2014). As an example of such a situation consider the real catastrophic event happened in Japan on April, 2011 (BBC, 2012). The earthquake in Japan was a primary disruption that led to secondary disruption due to the nuclear reactor leakage. It caused most of the agriculture and food processing companies all over the world to stop placing orders with Japanese suppliers. This action was taken as a precaution due to the risk of nuclear contamination in the products. What made things even worse was the fact that the suppliers did not know how long the disruption would last and this event can be considered as a new disruption event caused by the earthquake. We call the type of information used in Model 4 *'observed' information*.

In Model 4, the observed information spectrum is characterised by the length for which the current disruption to the offshore supplier has lasted. The disruption process for Model 4 is modelled as a Markov chain and we investigate how the parameters of the Markov chain model of the disruption process affects the firm's inventory policy. As in Model 3, we focus on the risk of disruption occurring when the offshore supplier is in the up state and on the probability distribution of the length of disruptions. The impact of the rate of transition between states of the disruption process on the ordering decision and the minimum expected inventory cost are examined. Following the occurrence of a disruption, the length and path of transition that will be taken by the offshore supplier to recover normal operation is uncertain, but the firm can observe with certainty the outset of a disruption event at the supplier.

The structure of this section is as follows. We describe Model 4 and its assumptions in sections 5.3.1 and 5.3.2, followed by the formulation of the ordering decision problem under supply disruption via the DMDP in section 5.3.3. Then, in section 5.3.4, we present the parameter values used when conducting the numerical experiment. The results and findings are reported in sections 5.3.5, 5.3.6 and 5.3.7. Finally, the conclusion for Model 4 is presented in section 5.3.8.

5.3.1 Model Description

The firm seeks to split the order between the onshore supplier (or supplier N) and the offshore supplier (or supplier F), to minimise cost over the planning horizon. During normal operations of supplier F, the firm can order from both suppliers. However, during disruption at supplier F, the firm can only order from supplier N. We assume that the firm knows the probability distribution of the length of a disruption, but does not know how long the disruption is going to last. The firm is able to observe when disruption starts and for how long a disruption has lasted.

The Markov model of the disruption process at supplier F is as follows. The state of the disruption process is denoted by j, where $j = 0, 1, \cdots, W$. State 0 represents the condition where supplier F is up (or at normal status) and able to deliver a complete order. Following the occurrence of a disruption, supplier F enters state 1, indicating that supply will be disrupted for at least one period. From state 1, the state of supplier F either returns to normal status, in which case the duration of the disruption was 1 period, or moves to state 2 indicating that the disruption will last for at least 2 periods. This process continues so that, in general, from state j the state of supplier F either returns to normal status, in which case the duration of the disruption was j periods, or moves to state $j + 1$ indicating that the disruption will last for at least $j + 1$ periods. Finally, if the process enters state W, we know that the disruption will definitely end this period. In other words, during a disruption, the firm observes information about the nature of the disruption (the number of periods for which it has lasted) and updates its belief about the total length of the disruption. We also assume that the length of a period of normal operation of supplier F also follows a geometric distribution. For a better understanding, the transition between normal operation and states of disruption for supplier F are illustrated in figure 5.19.

In figure 5.19, α represents the constant risk that normal operations of supplier F are subject to a disruption event and hence the probability that the state of supplier F moves from 0 to 1 during a period. Therefore, α represents the probability normal operations continue. The function $h(j)$ represents the probability that the disruption to supplier F ends after j periods given that it has already lasted for $j - 1$ periods. When supplier F is in state $j > 0$, the probability that the disruption event ends, and the supplier returns to normal operation, is $h(j)$. Hence, the probability that supplier F makes a transition from state j to state 0 is $h(j)$ and otherwise it makes a transition to state $j + 1$.

For theoretical developments, we can consider the case where W approaches ∞. However, for numerical calculation, we require W to be finite, but possibly large. As explained by Tijms (2003)., to examine a DMDP model with the infinite state space, a *truncated model* designed such that the probability of visiting any of the deleted states under an optimal policy is very small. Nonetheless, the numerical solution of the truncated model can be time-consuming, if the process of truncation leaves a very large system of linear equations to be solved. In such cases, other methods than can be used to solve the infinite state space problem. For example, geometric tail behaviour may be used to reduce the infinite system of state equations to a finite set of linear equations (Tijms, 2003).

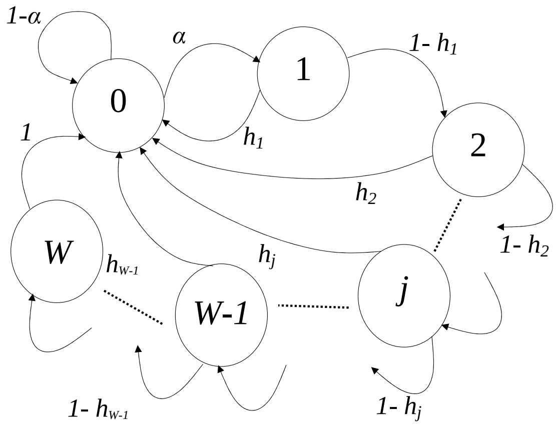


Figure 5.19. The transition structure for the disruption process in Model 4

5.3.2 Model Assumptions

The assumptions of Model 4 are as follows:

a. The option of sourcing from supplier F is at risk of disruptions. The order from supplier F arrives either in full or not at all. However, the status of supplier N is perfectly reliable.

b. The firm has no information about the exact length of the disruption. Therefore, the length of periods of normal operation and disruption of supplier F are assumed have followed the geometric distributions.

c. The firm's inventory planning horizon is discrete.

d. Demand is either deterministic or stochastic. The stochastic demand distribution follows the truncated Poisson distribution, $P(d_t) \sim Pois(\lambda, K)$.

e. Customers do not accept backorders, thus the firm encounters lost sales. The firm is charged with a penalty cost for each unit of demand that cannot be satisfied in a period, $PNLTY$.

f. The firm incurs a holding cost for inventory held during period t, $HOLD$.

5.3.3 Model Formulation

In this section, we explain the formulation of the problem analysed in Model 4 as a DMDP model and present the optimality equation.

Components of the DMDP for Model 4

The components of the DMDP for Model 4 are as follows:

Decision epoch

A decision is taken at the beginning of each period. Let t denote the decision epoch when there are t periods remaining in the planning horizon, $t = 1, 2, \ldots, T$.

States

At each decision epoch, the firm reviews the inventory level, i, and the state of supplier F, j. The parameters i and j comprise the state of the process y, such that $y = (i, j)$. The state space, Y, of Model 4 is given by:

$$Y = \Big\{(i, j) : i \in \{0, 1, \cdots, I\} \quad \& \quad j \in \{0, 1, \cdots, W\}\Big\}$$

Actions

Based on the current state, the firm then decides on the quantities to order from supplier N and supplier F. The action is denoted by $b = (q^N, q^F)$ and the set of admissible action, $B(y)$ is given by:

$$B(i,0) = \left\{(q^F, q^N) : \quad q^F, q^N \geq 0 \quad \& \quad q^F + q^N \leq I - i\right\} \quad \text{for} \quad 0 \leq i < I.$$

$$B(i,j) = \left\{(0, q^N) : \quad q^N \in \{0, ..., I - i\}\right\} \quad \text{for} \quad 0 \leq i \leq I \quad \text{and} \quad 0 \leq j \leq W.$$

The firm can choose to order up to $I - i$ items either from supplier N only or from supplier F only or from both the suppliers under $B(i,0)$. Whilst, the decision is to order up to $I - i$ items with supplier N only under $B(i, j)$.

Transition probabilities

Changes in the inventory level and changes in the state of supplier F, α, are modelled separately. The transition in the state of the inventory level depends on the order quantities and is the same as in previous models. See section 4.2.3a for a full description. However, the transition in the state of supplier F follows from figure 5.19 above. The transition matrix, X, in this model is formally presented below.

$$X = \begin{array}{c} \\ 0 \\ 1 \\ 2 \\ \vdots \\ W-2 \\ W-1 \\ W \end{array} \begin{array}{c} \begin{array}{ccccccc} 0 & 1 & 2 & 3 & \cdots & W-1 & W \end{array} \\ \begin{pmatrix} 1-\alpha & \alpha & 0 & 0 & \cdots & 0 & 0 \\ h(1) & 0 & 1-h(1) & 0 & \cdots & 0 & 0 \\ h(2) & 0 & 0 & 1-h(2) & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ h(W-2) & 0 & 0 & 0 & \cdots & 1-h(W-2) & 0 \\ h(W-1) & 0 & 0 & 0 & \cdots & 0 & 1-h(W-1) \\ 1 & 0 & 0 & 0 & \cdots & 0 & 0 \end{pmatrix} \end{array}$$

The elements of X are obtained from the problem parameters, specifically the risk that normal operations at supplier F are disrupted (i.e., α) and the probability distribution of the length if a disruption. The recovery rate in state j, denoted by $h(j)$, is the probability that normal operations at supplier F are recovered after exactly j periods given that the disruption will last for at least j periods in total. Let A denote a random variable representing the length of a disruption. The value of $h(j)$ is given by:

$$\begin{aligned} h(j) &= \frac{Pr(A = j)}{Pr(A \geq j)} \\ &= \frac{Pr(A = j)}{1 - \sum_{\ell=1}^{j-1} Pr(A = \ell)} \end{aligned}$$

One-step costs:

As in Models 1, 2 and 3, the one-step cost function, as a result of choosing action b in state y consists of the ordering cost, $ORDER$, the holding cost, $HOLD$ and the penalty cost, $PNLTY$. In the one-step cost function with stochastic demand, the values of $HOLD$ and $PNLTY$ depend on the random demand during the period. The one-step costs for Model 4 with constant and stochastic demand settings are based on the same assumptions as the one-step costs for the models in previous chapters. See section 3.2.1 for a detailed explanation.

The one-step cost when action b is chosen in state y at decision epoch y is denoted by $C_t^y(b)$. Under constant demand setting, this cost is given by:

$$\begin{aligned} C_t^y(b) &= ORDER + HOLD + PNLTY \\ &= \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \ h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\ &\quad + \ m\Big(\max(D_t - i - q^N, 0)\Big) \end{aligned}$$

and under stochastic demand setting, it is given by:

$$C_t^y(b) = ORDER + \Big(E_{D_t}\Big(HOLD + PNLTY\Big)\Big)$$
$$= \sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t)\Big\{h\Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big)$$
$$+ \ m\Big(\max(d_t - i - q_t^N, 0)\Big)\Big\}$$

Optimality equation

Let $V_t(i, j)$ be the minimum cost over the last t periods of the planning horizon when, at decision epoch t, the inventory level is i and the state of supplier F is j. The optimality equation for Model 4 with constant demand is given by:

$$V_t(i, j) = \min_{b\in B(y)}\Big\{ \sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + m\Big(\max(D_t - i - q^N, 0)\Big)$$
$$+ h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big)$$
$$+ \sum_{k=0}^{W} X_{j,k}\Big(V_{t-1}\Big(\max(i + q^N - D_t, 0) + q^F, k\Big)\Big)\Big\}$$

The optimality equation with stochastic demand is given by:

$$V_t(i, j) = \min_{b\in B(y)}\Big\{ \sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d=0}^{\infty} P(D_t = d)\Big\{m\Big(\max(d - i - q^N, 0)\Big)$$
$$+ h\Big(\frac{1}{2}(i + \max(i + q^N - d, 0))\Big)$$
$$+ \sum_{k=0}^{W} X_{j,k}\Big(V_{t-1}\Big(\max(i + q^N - d, 0) + q^F, k\Big)\Big)\Big\}\Big\}$$

Using these optimality equations, we seek to minimise $V_t(i, j)$ and find the optimal quantities to order from supplier N and supplier F, depending on the values in the transition matrix, X. We are interested to investigate by numerically how the values in this transition matrix can affect the firm's ordering policy.

5.3.4 Choice of Parameters Values

The objective of the numerical study for Model 4 are to analyse how the optimal policy changes with different transition probabilities. The experimental design in this numerical study is the same as in Model 3, thus a set of 20 cases based on various combinations of α values (i.e., the frequency of disruption to supplier F) and the cases of $P(w = A)$ (i.e., the disruption period scenarios) are considered in this numerical study. See section 5.2.4 for a detailed explanation. The difference between Model 4 and Model 3 is in the computation of transition probabilities due to different design of transition state in the Markov chain process. In Model 4, the firm knows that the disruption will definitely end at the offshore supplier, but this firm does not know how long the disruption will last. Hence, the transition from state 0 to state w depends on the observe duration of the disruption (the number of period for which it has lasted). Transition probability values in this model are calculated by using a formula in section 5.3.3a(iv).

In what follows, we first present results on the effects of the cases on the properties of the ordering decisions, then results relating to the effects on the properties of the costs of policies, and finally the results on the effects of the fill rate and the average inventory under the stochastic demand model analysis.

5.3.5 The Impact of the transition probability values on the Ordering Decisions

In this section, we explain how various transition probabilities values in each case can affect the properties of firm's ordering decision. The discussion covers the infinite-horizon model under the constant and stochastic demand settings (later known as M4*InfCons* and M4*InfSto*, respectively). The analyses of the constant and stochastic demand models are reported in sections 5.3.5a and 5.3.5b, respectively.

The optimal ordering policy of M4*InfCons*

If supplier F is in state u, the firm only order from supplier N if the inventory level, i, is relatively low, ($i < 5$) in all cases, as illustrated in figure 5.20. However, the ordering decision from supplier F does vary from case to case. For an example, from figure 5.21, in case 4 ($\alpha = 0.7$), the quantity order from supplier F is higher in case B than other cases. In other cases, the quantity ordered from this supplier decreases and the same for each case, but the inventory level point to which the firm places order are vary. The point at which optimal for the firm to place order decreases when supplier F has higher probability to be down increase to 3 and 5 periods ($i = 17$). Comparing the ordering policy from this supplier from the equal average of disruption length scenario, there is a difference in optimal order between case A and case B, which the firm will order more in case A. However, there is no difference in optimal ordering between case C and case D under the scenario of equal variance of disruption length.

Similar to Model 3 with infinite horizon and constant demand settings, we consider the optimal ordering policy from the offshore supplier as an (s, S) policy. Table 5.6 shows how the parameters vary in the 20 cases considered. From table 5.6, we see that the incentive to order from supplier F increase with increases in α. However, there are differences in optimal ordering policy in cases of equal average and variance of disruption length (i.e., cases A, B, C and D). In equal average of disruption length scenario, in cases 3 to 5 ($\alpha \geq 0.5$), the firm will order more in case A ($S = 50$) as compare to case B ($S = 55$), but the inventory level points at which the firm will first placing order to the offshore supplier are the same in both cases. In equal variance of disruption length scenario, the firm will order more in case D in each case of $\alpha \geq 0.3$. However, the firm will wait longer in case C before placing order with supplier F than in case D. From the findings, we can conclude that policy in case B is better in case A under higher supplier disruption probability condition ($\alpha > 0.1$), and the policy in case C is better than in case D.

Figure 5.22 shows the relationship between the proportion of time for which supplier F

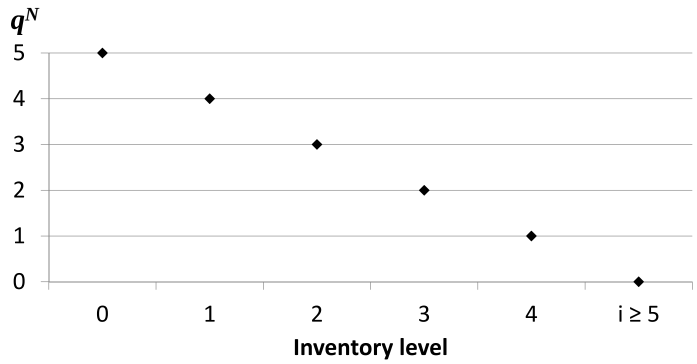


Figure 5.20. M4*InfCons*, state u: The optimal order from supplier N.

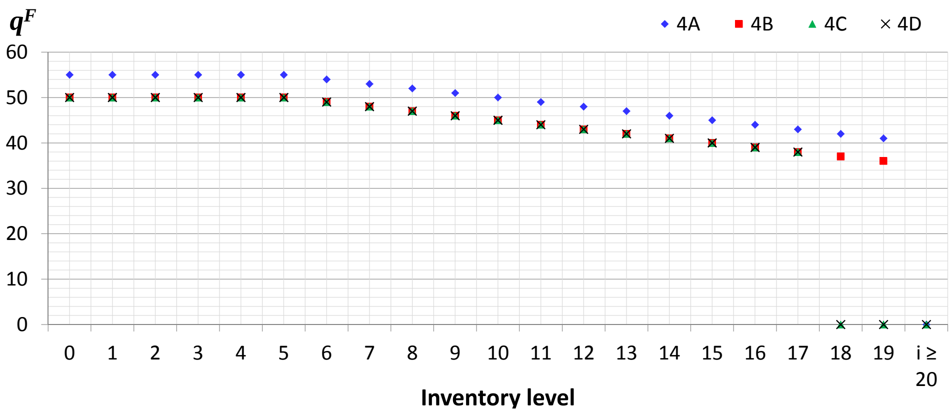
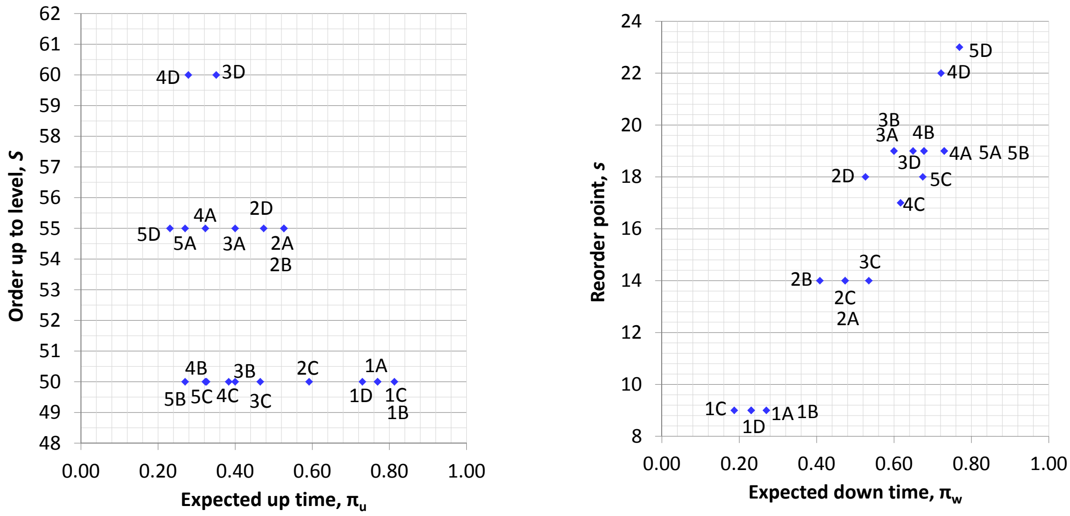


Figure 5.21. M4*InfCons*, state u: The optimal order from supplier F.

Table 5.6. M4*InfCons*, state u: Optimal ordering policy for supplier F.

Case	**A**	**B**	**C**	**D**
1	(9,50)	(9,50)	(9,50)	(9,50)
2	(14,50)	(14,50)	(14,50)	(18,55)
3	(19,55)	(19,50)	(14,50)	(19,60)
4	(19,55)	(19,50)	(17,50)	(22,60)
5	(19,55)	(19,50)	(18,50)	(23,55)

is either up or down and the optimal policy. From figure 5.22a, we can see that order up-to level, S approximately decreases with an increase in the expected up time, π_u. However, from figure 5.22b, we can see that reorder point, s increases with an increase in the expected down time, π_w. From the findings, the firm will increase the order quantity places with the offshore supplier if the expected down time increases and the point of inventory level at which the firm to place an order with this supplier will be reduced if the expected up time increases. The relationship between the order up to level and the proportion of time supplier F is down is non-linear. The decrease in reorder point with proportion of time supplier F is up is approximately linear.



(a) S *vs.* π_u

(b) s *vs.* π_w

Figure 5.22. M4*InfCons*: The relationships between the optimal order and the expected up and down time.

If supplier F is down, from figure 5.23, when $i \leq 5$, an order from supplier N in each α case and disruption period scenario is vary depending on the number of down period of supplier F has lasted. The quantity ordered from this supplier in periods 1, 2 and 4 is bigger than the order quantity in periods 3 and 5. We can see that the order quantity decreases if the expected down period increases to higher period at the first half of the duration of down period has lasted. However, at another half of the duration period, the order quantity increases with an increase in the expected down period. At the end of the disruption period, lower quantity will be ordered from this supplier. Under this situation, we can see that supply disruption probability and the expected length of disruption at each disruption period do not influence the optimal ordering policy from the onshore supplier, but the number of disruption periods has lasted does affect the optimal ordering from this supplier.

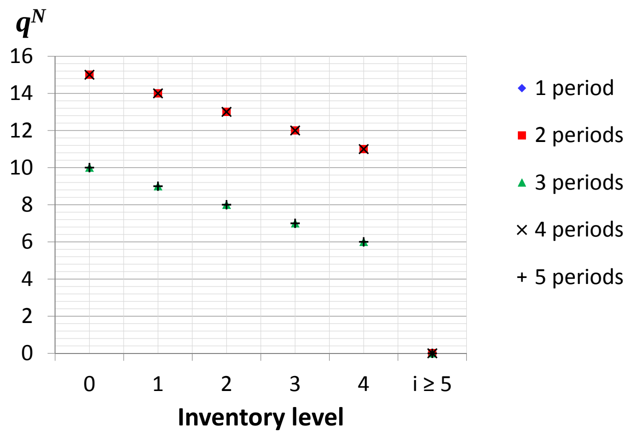


Figure 5.23. M4*InfCons*, state w: The optimal order from supplier N in each case.

The optimal ordering policy of M4*InfSto*

If supplier F is in state u, the firm only order from supplier N if the inventory level, i, is relatively low, ($i < 7$) in all cases, as illustrated in figure 5.24. However, the ordering decision from supplier F does vary from case to case. From figure 5.25, in case 4, the quantity ordered from this supplier in base case (case A) is higher than the quantities in other cases. The order quantity in cases B, C and D are the same, but the level point at which the firm will start placing order is vary. The firm start placing order with supplier F later in cases C and D than in case B.

We consider the optimal ordering policy from the offshore supplier as an (s, S) policy. Table 5.7 shows how the parameters vary in the 20 cases considered. From table 5.7, we see that the incentive to order from supplier F decrease with increases in α. However, from cases of equal average and variance disruption lengths, the optimal ordering policy is different in each case. In equal average disruption length scenario, the firm will order more in case A as compare to case B, but the inventory level point at which the firm will first placing order to the offshore supplier in both cases is equal (except in case 2 and 4 (α = 0.3 and 0.7). In equal variance of disruption length scenario, the firm will order more in case D in each α case. In addition, the inventory level point at which the firm will start placing order with the offshore supplier is higher in case D too. From the findings, we can conclude that policy in case B is better than policy in case A and, the policy in case C is better than policy in case D.

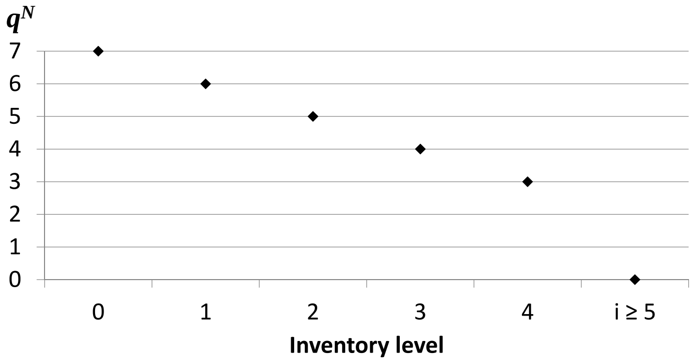


Figure 5.24. M4*InfSto*, state u: The optimal order from supplier N.

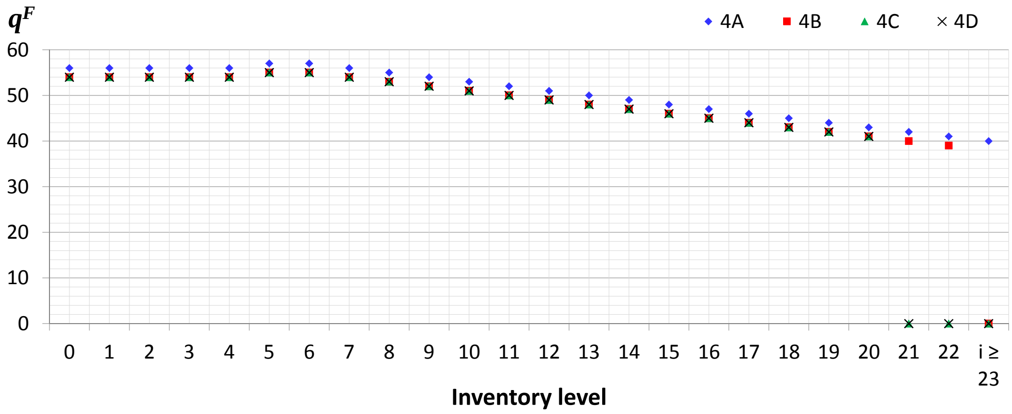
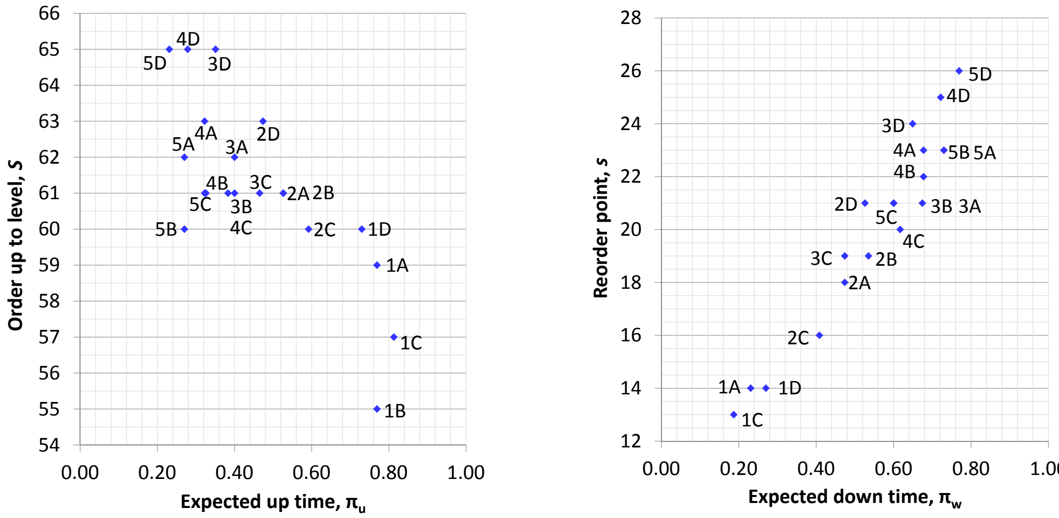


Figure 5.25. M4*InfSto*, state u: The optimal order supplier F.

Table 5.7. M4*InfSto*, state u: Optimal ordering policy for supplier F.

Case	**A**	**B**	**C**	**D**
1	(14,59)	(9,55)	(13,57)	(14,60)
2	(18,61)	(19,61)	(16,60)	(21,63)
3	(21,62)	(21,61)	(19,61)	(24,65)
4	(23,62)	(22,61)	(20,61)	(25,65)
5	(23,62)	(23,60)	(21,61)	(26,65)

We also examine the relationship between the proportion of time for which supplier F is either up or down and the optimal policy as shown in figure 5.26. From figure 5.26a, order up to level, S decreases when the expected up time, π_u increases. However, from figure 5.26b, reorder point, s increases when the expected down time, π_w increases. We can see that there are negative and positive relationships between the optimal order and the expected up and down time, respectively. As we expected, the firm will carry less stock when the offshore supplier is expected to be operating longer and start placing order earlier when the disruption is expected to be longer.



(a) The expected up time, π_u (b) The expected down time, π_w

Figure 5.26. M4*InfSto*: The relationships between the optimal order and the expected up and down time.

If supplier F is in state w, from figure 5.27, when i is relatively low ($i < 8$), the order quantity from supplier N increases when the expected number of disruption periods increases in each α case and disruption period scenario. It is optimal to increase the quantity of order from the onshore supplier because the expected length of disruption to the offshore supplier is expected to be longer. Under this situation, we can see that the expected number of disruption periods does affect the optimal order from the onshore supplier, but not supply disruption probability and the expected length of disruption at each disruption period.

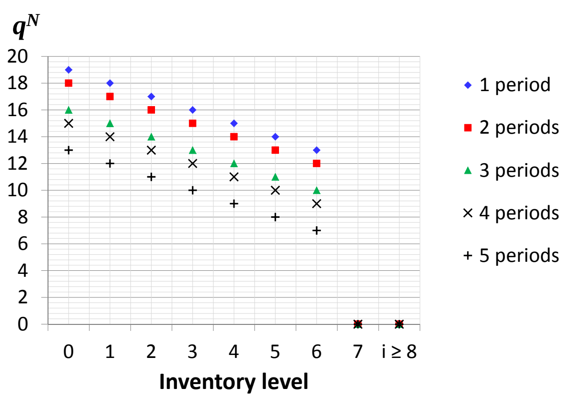


Figure 5.27. M4*InfSto*, state w: The optimal order from supplier N in each case.

Discussion

Under the infinite-horizon plan, in both constant and stochastic demand settings, in all cases, the firm will only place order with the onshore supplier if the inventory level is so low that there is a high risk of immediate shortages. The findings show that the order quantity from the onshore supplier is only needed as backup if the firm does not have enough stock in inventory. This ordering policy to the onshore supplier is also apply to the situation when the offshore supplier is under a crisis operation. However, in this situation, the order quantity is bigger than in the situation when the offshore supplier is under a routine operation. The order is not only to satisfy the immediate shortage, but also to satisfy demand during subsequent periods of disruption. The firm will order more from this supplier when the offshore supplier is expected to face a disruption longer.

5.3.6 The Impact of Different Transition Probabilities on the Long-run Average Costs

In this section, we explain how various transition probabilities values in each case can affect the properties of the long-run average costs under the infinite-horizon Model 6, which covered the experiment with the constant and stochastic demand settings.

From figure 5.28, we can see that the pattern of the long-run average cost, g, across the cases is the same with both constant and stochastic demand. In addition, the pattern of g is also the same for each case of α. Most lower g are in case 1 (of $\alpha = 0.1$) for all the expected disruption periods cases and overall the highest g is in case 3D. The long-run average cost increases gradually as the expected number of disruption periods lasted at the offshore supplier increases. From equal average and variance of disruption length scenarios, in each α case, g in case B is lower that in case A, and g cost C is lower than in case D. Based on the pattern of g across the cases, we can conclude that the number of disruption periods and the expected disruption length at each disruption period have more impact on the optimal long-run average cost than supply disruption probability. We also can conclude that the optimal policy in case B is better than in case A and, case C is better than case D.

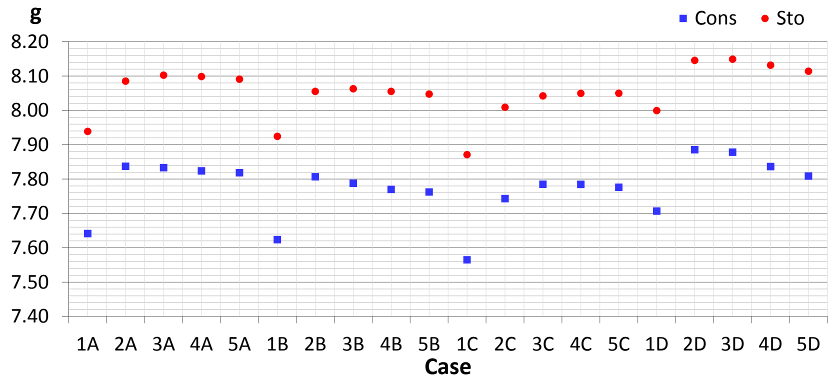
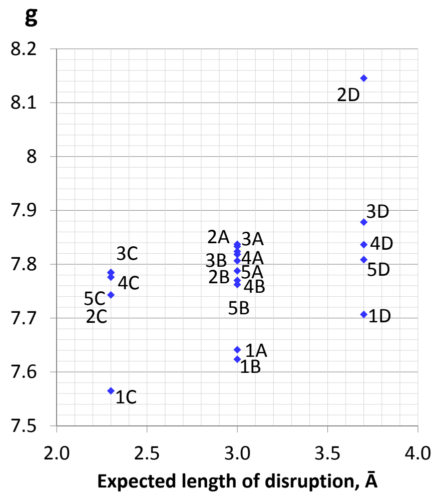
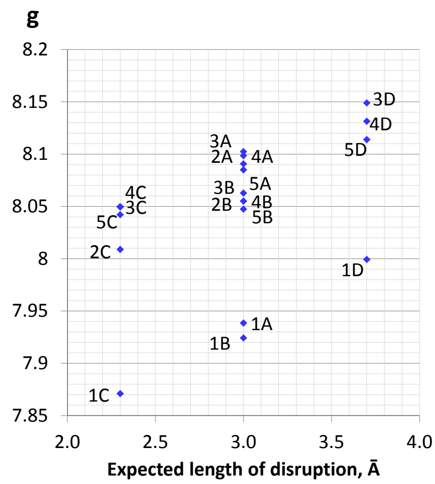


Figure 5.28. M4*Inf*: Optimal long-run average cost, g, in different cases for constant and stochastic demand models

From figure 5.29, we illustrate a relationship between the long-run average cost, g, and the expected length of disruption, $\bar{A}$. From figures 5.29a and 5.29b, in both constant and stochastic demand models, g increases as $\bar{A}$ increases. From the findings, we can conclude that the firm will incur higher cost if the disruption length is expected to be longer.



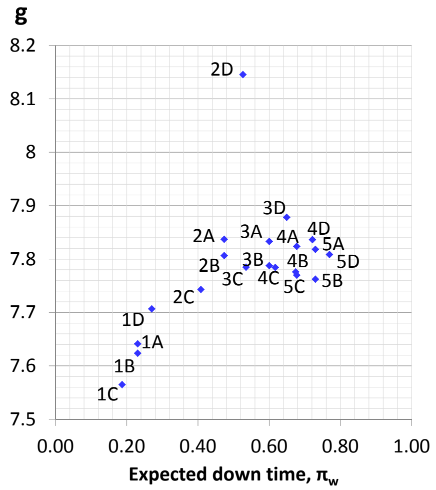
(a) M4*InfCons*



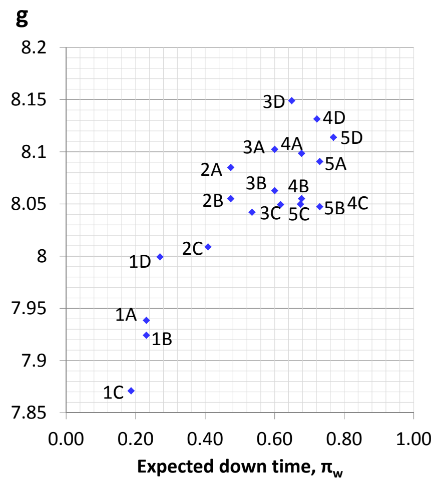
(b) M4*InfSto*

Figure 5.29. M4*Inf*: The relationship between the long-run average cost and the expected length of disruption

From the aspect of the expected down time, π_w, we also can see a relationship between the long-run average cost, g, and π_w, as illustrated in figure 5.30. From figures 5.30a and 5.30b, in both constant and stochastic demand models, g increases as π_w increases. We can conclude that the firm will face higher cost if the period that the offshore supplier will be down is expected to be longer.



(a) M4*InfCons*



(b) M4*InfSto*

Figure 5.30. M4*Inf*: The relationship between the long-run average cost and the expected down time

Discussion

Comparing the expected minimum cost among the different cases, the lowest cost occurs in the case with the lowest value of supply disruption probability of each expected periods of disruption, and the highest cost occurs in the case with bigger number of disruption periods (i.e., the offshore supplier is expected to be down for 3 and 5 periods). From the findings, we can see that α, the number of disruption period and the expected disruption length at each disruption period will affect the minimum cost and the long-run average cost of the optimal ordering policies. As we expected, the firm will face higher cost if the offshore supplier has high risk to face the disruption and the number of down period is expected to be longer.

5.3.7 The Impact of the Transition Probability Values on the Performance of the Policies

In this section, we discuss the performance of the ordering policy under the infinite horizon plan and stochastic demand model (M4*InfSto*, focussing on the performance of the fill rate (section 5.3.7a) and the average inventory level (section 5.3.7b).

Fill rate

From figure 5.31, the percentage of demand satisfied from stock in hand in all cases are estimated to lie between 99.77% and 99.79% with 95%of confidence interval. From findings, we see that even though there is a high disruption probability and a small number of disruption periods, the capability for the firm to satisfy demand still high since it has a backup source from the onshore supplier. It is quite noticeable that the fill rate in case A and case D are higher than case B and case C, respectively. The variation in the fill rates for other values of α is within the range of the confidence intervals of the simulation results. From the findings, we can see that, the values of fill rate are vary under the situation where the average and variance of disruption length are equal, thus we can conclude that supply disruption probability and the expected disruption length at each disruption period can affect the fill rate.

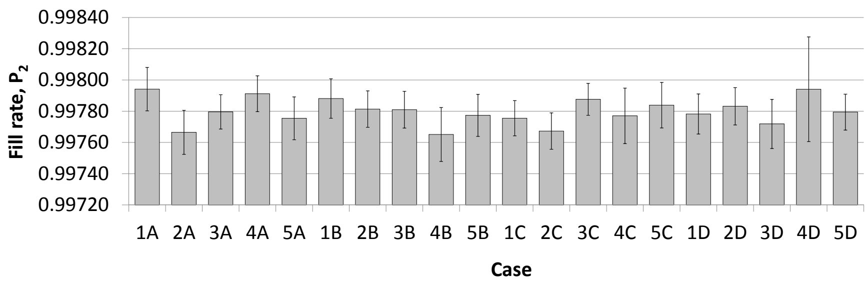
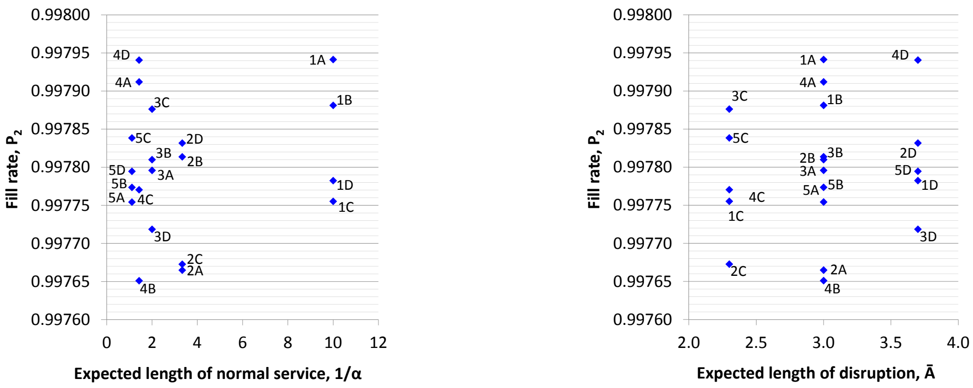


Figure 5.31. M4: Fill rate in each case

Figure 5.32 illustrates the relationships between the fill rate, P_2, and the expected normal service, $1/\alpha$, and the expected disruption length, $\bar{A}$. From figure 5.32a, if we exclude case 1 ($\alpha = 0.1$) from the plot, P_2 approximately increases with an increase in $1/\alpha$ (for $1/\alpha < 4$). We can see a same pattern between P_2 and $\bar{A}$, as illustrated in figure 5.32b. P_2 approximately increases with an increase in $1/\alpha$ (for $1/\alpha < 4$).



(a) P_2 *vs.* $1/\alpha$

(b) P_2 *vs.* $\bar{A}$

Figure 5.32. M4: The relationships between fill rate and the expected normal service and the down time.

The fill rate, P_2, also has a relationship with the expected up time, π_u. The pattern of the relationship is illustrated in figure 5.33.

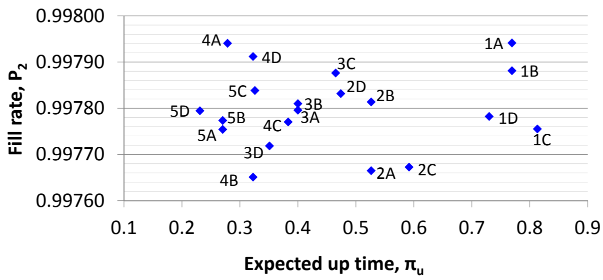


Figure 5.33. M4: The relationship between fill rate and the expected up time

Average inventory level

From figure 5.16, the average inventory level, I_A in all cases are estimated to lie between 31.22 and 36.78 with 95% of confidence interval, which is roughly one half the maximum inventory level. We also see the highest I_A is in case 4D (36.78 ± 0.068) and the lowest is in case 1C (31.22 ± 0.208). This result is as we expected. It is optimal to carry more inventory due to high frequency of disruption. The action taken as to avoid expensive onshore supplier during disruptions. In addition, it is quite noticeable that the average inventory level in cases A and C are higher than in respect of cases B and D. Therefore, we can conclude that, even though the average and variance of the disruption length are equal, the average inventory level is vary, depending on the disruption period scenario.

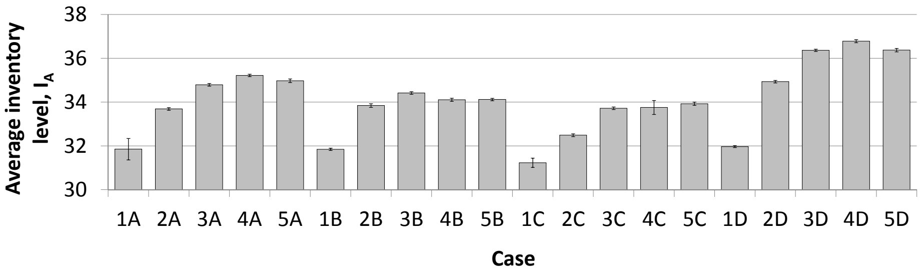
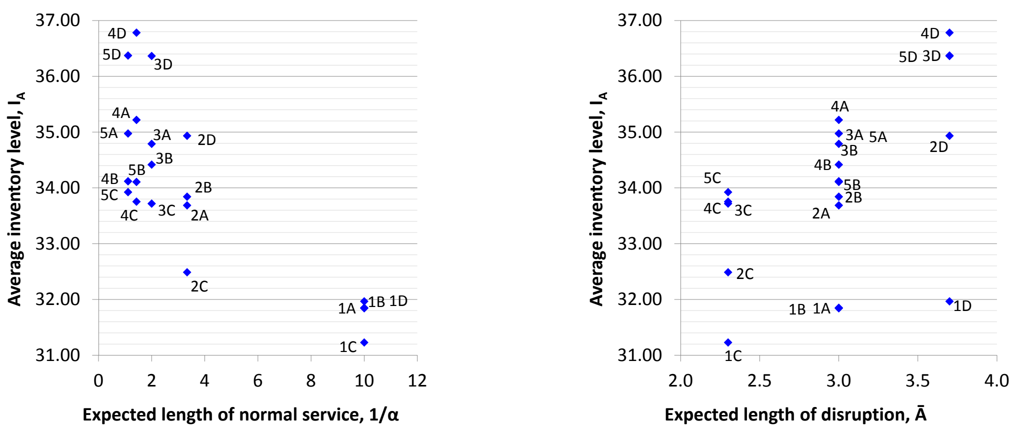


Figure 5.34. M4: Average inventory level in each case α.

From figure 5.35, we can see that the average inventory level, I_A has relationships with the expected length of normal service, $1/\alpha$, and the expected length of disruption, $\bar{A}$. From figure 5.35a, I_A has a negative correlation with $1/\alpha$ and from figure 5.35b, I_A has a positive correlation with $\bar{A}$. As we expected, it is optimal for the firm to carry more item if the frequency of normal service to the offshore supplier to be higher and the frequency of disruption period to be lower.



(a) I_A *vs.* $1/\alpha$

(b) I_A *vs.* $\bar{A}$

Figure 5.35. M4: The relationships between average inventory level and the expected normal service and the down time

The average inventory level, I_A, also has a relationship with the expected up time, π_u. From figure 5.36, we can see a negative correlation between I_A and π_u. The average inventory level decreases as the expected up time increases. From findings, it is optimal to carry more inventory when the normal operation of the offshore supplier is expected to be longer.

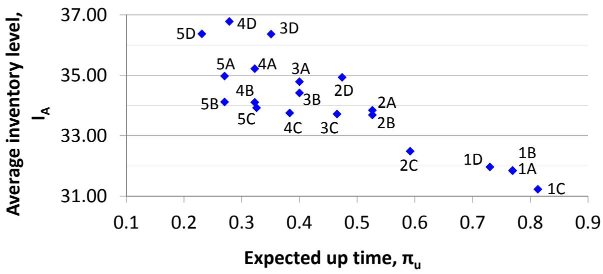


Figure 5.36. M4: The relationship between average inventory level and the expected up time

Discussion

In Model 4 analyses, similar to the analyses of Model 3, the performance of the firm's ordering policy has been measured with the values of fill rate and average inventory level. Based on fill rate, the firm still has high capability to satisfy demand from the customer when it has chose to use the findings policy. The percentage of demand satisfied from stock in hand is still high even though there are high supply disruption probability and expected periods of disruption. From the average inventory level perspective, having less stock in the inventory is more favourable if the supply disruption probability is small and the offshore supplier is expected to be down at most 3 periods.

5.3.8 Conclusion

In Model 4, we addressed how the firm's ordering policies can be affected by the risk of disruptive supply event at the offshore supplier. From the analyses of Model 4, without information of supply disruption, on the belief of the firm, we demonstrate how the properties and the performance of the optimal ordering policy depend on the values of the transition probabilities of the Markov chain model of disruption to the offshore supplier. Similar to Model 3, the firm will place higher order from the offshore supplier if the risk of disruption is increased. The firm will also place higher order from this supplier if it's belief that the expected periods of disruption will increase.

Even though without supply disruption information, the firm have a belief on the performance of the onshore and offshore suppliers before and during the disruption. Like Model 3, the ordering policies produce in Model 4 still have a capability to satisfy customer demand, based on the performance of the fill rate and the average inventory level. The analysis of Model 4 is significant where the firm is able to check the performance of the supplier (i.e., case for long term relationship with the supplier). It is crucial for the firm with a high profile product, for an example like Airbus, which important for a level of strategic decision making.

5.4 Special Case on the Property of the Length of Disruption

In previous section, the analyses on the value of additional supply disruption information for the firm have been conducted with advance full information on disruption length in Model 3 (section 5.2) and partial information on disruption length in Model 4 (section 5.3). From the findings, we discover that the additional disruption information in both models have improved the policies with reductions in the expected minimum costs (under the finite planning horizon model) and long-run average costs (under the infinite planning horizon model). To verify the findings, these two models are compared with the outcomes of Model 2. The comparison between Model 2 and Models 3 and 4 are presented in section 5.4.1 and 5.4.2, respectively.

5.4.1 The comparison between Model 2 and Model 3

A comparison between Model 2 and Model 3 is examined with another numerical analysis. In this numerical study, new scenarios are added in Model 3 analysis, which the length of disruption, A, now follows a truncated probability distribution that approximate the geometric distribution. The truncated probability mass function (p.m.f) is given by:

$$P(A = j) = P(A = j - 1)(1 - \beta), \quad j = 1, 2, \cdots$$

and the cumulative distribution function (c.d.f) is given by:

$$\sum_{j=1}^{W} P(A = j) = 1$$

For the calculation of $P(A = j)$ purpose, we assume that the probability that the disruption to the offshore supplier has lasted for 1 period, $P(A = 1)$, is equal to β values used in Model 2. This assumption has been made to make sure that these new scenarios are consistent with case of β in Model 2, thus five truncated probability distributions with various maximum expected disruption lengths, W, are considered in this numerical study. The values of α, β and W used in this study are tabulated in table 5.8.

Table 5.8. The values of α, W and $P(A = 1)$ in each case.

Case	1	2	3	4	5
α	0.1	0.3	0.5	0.7	0.9
Case	a	b	c	d	e
$P(A = 1)$	0.1	0.3	0.5	0.7	0.9
W	52	19	10	7	4

To be consistent with Model 2, we combine α and $P(A = 1)$ values and generate 25 cases. Similar to Model 3, the cases are numbered according to the values of α and $P(A = 1)$ and the interpretation on each α value is similar to Model 3 too. In each instance of case 1a in table 5.8, supplier F has a high probability to stay operating when in the up state ($j = 0$) and if this supplier is in the down state ($j > 0$), it is expected to be down for at most 52 periods with various expected disruption length at each disruption period. To do the experiment, we analyse Model 3 with the combination of α and $P(A = j)$ values, case by case, focusing on the analyses of the infinite planning horizon with constant and stochastic demand settings.

In what follows, we first results on the differences of the case on the properties of the ordering decisions, the results relating of differences on the properties of the long-run average cost of policies, and finally the results of differences of the effects on the properties of the fill rate and the average inventory level under the stochastic demand model analysis.

The comparison on the ordering decision

In this section, the performance of Model 3 as compared to Model 2 is analysed by examining the differences of reorder point and order up-to-level values in Model 3 and Model 2 (i.e., sM3-sM2 and SM3-SM2) in both the constant and stochastic demand models, as illustrated in figures 5.37 and 5.38. In each case of Model 3, from figure 5.37, reorder point, s is smaller than in Model 2. It shows that, in each case, it is optimal to order earlier in Model 3 than in Model 2. However, from figure 5.38, in some cases, order up-to-level is smaller in case Model 3 than in Model 2, thus, we can say that it is optimal to carry less inventory in Model 3 than in Model 2. From the findings, we can conclude that the assumption for Model 3 about additional full information at the outset of disruption event at the offshore supplier that can improves the firm's ordering policy is approximately accurate.

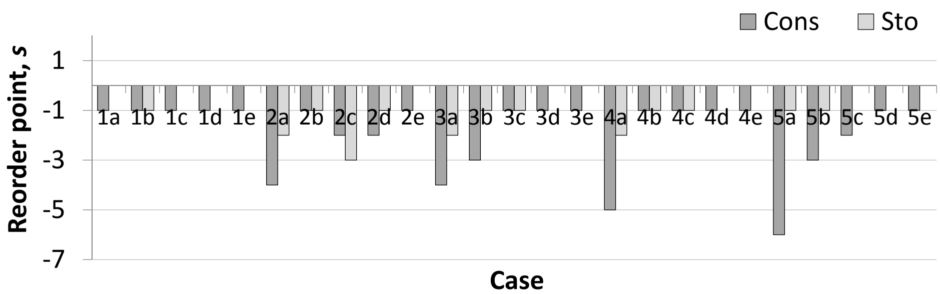


Figure 5.37. The difference of reorder point, s in Models 3 and 2 in each case.

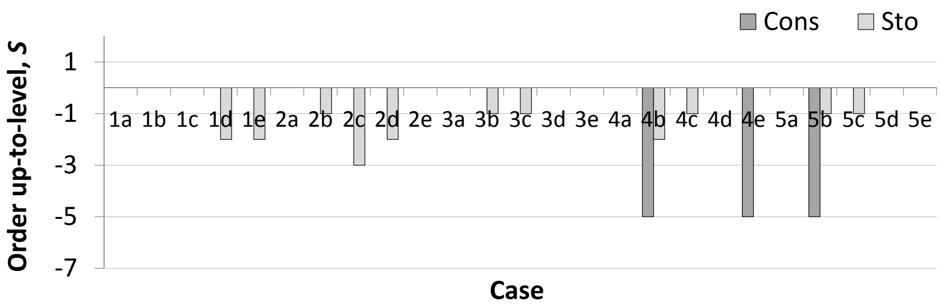


Figure 5.38. The difference of order up-to-level, S, in Models 3 and 2 in each case.

The comparison on the long-run average cost

In this section, cost reduction in each case is examined to compare the difference of long-run average cost, g, in Model 3 and Model 2, such that gM3-gM2. If the value of gM3-gM2 is positive (gM3-gM2 > 0), then we can say that Model 2 is better than Model 3 and vice versa if the value of gM3-gM2 is negative (gM3-gM2 ≤ 0). The value of gM3-gM2 in each case is illustrated in figure 5.39. In both the constant and stochastic demand models, in each case, we can see that the values of gM3-gM2 all are negative and the pattern of gM3-gM2 across the cases are the same. The highest cost reduction occurs in case a and the values of gM3-gM2 decreases gradually as the supply disruption probability increases. For the findings, we can conclude that Model 3 performs better than Model 2. Therefore, the assumption about additional full information at start of disruption to the offshore supplier that can improves the firm's ordering policy in Model 3 is approximately accurate.

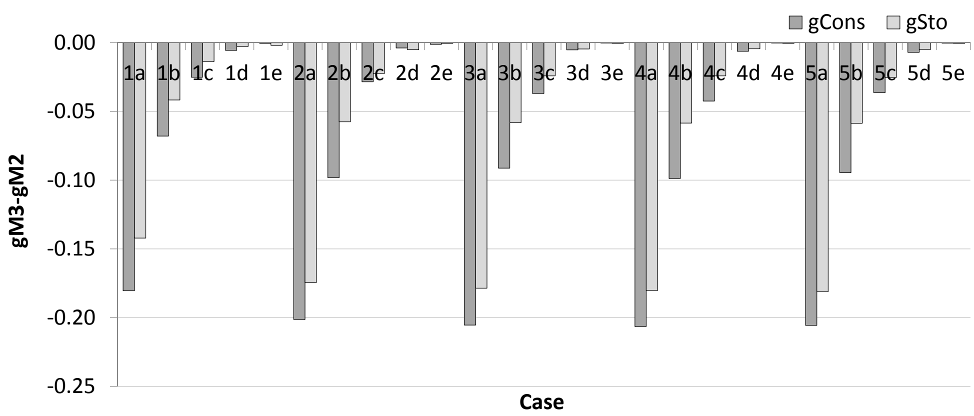


Figure 5.39. The difference of long-run average cost in Models 3 and 2 in each case.

The comparison on the performance of the policies

In this section, similar to the previous section, the performance of Model 3 as compared to Model 2 is analysed by examining the differences of fill rate and average inventory level values in Model 3 and Model 2 (i.e., FillrateM3-FillrateM2 and AvgLvlM3-AvgLvlM2), as illustrated in figures 5.40 and 5.41. From figure 5.40, we can see that in some cases, Model 3 performs better that Model 2 and in some case, Model 2 performs better than Model 3. The similar condition also occurs in the difference of average inventory level values as shown in figure 5.41. In both fill rate and average inventory level values, in each $\beta = 0.1$ case, Model 2 always performs better than Model 3.

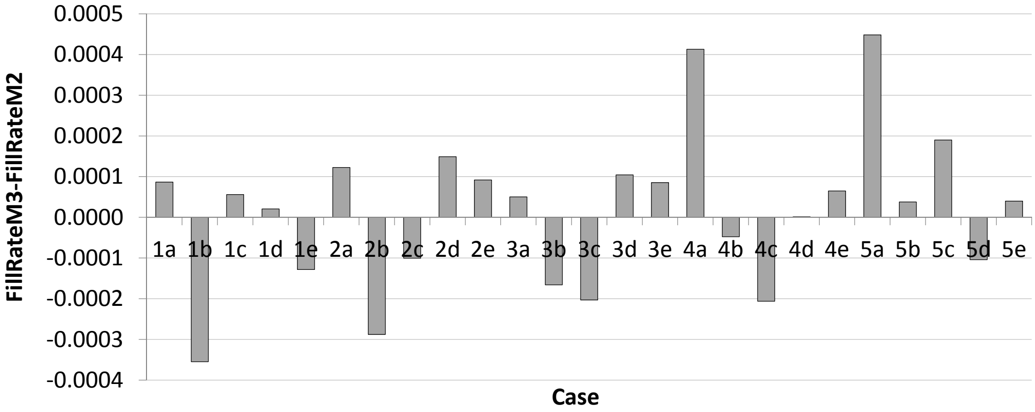


Figure 5.40. The difference of fill rate values in Models 3 and 2 in each case.

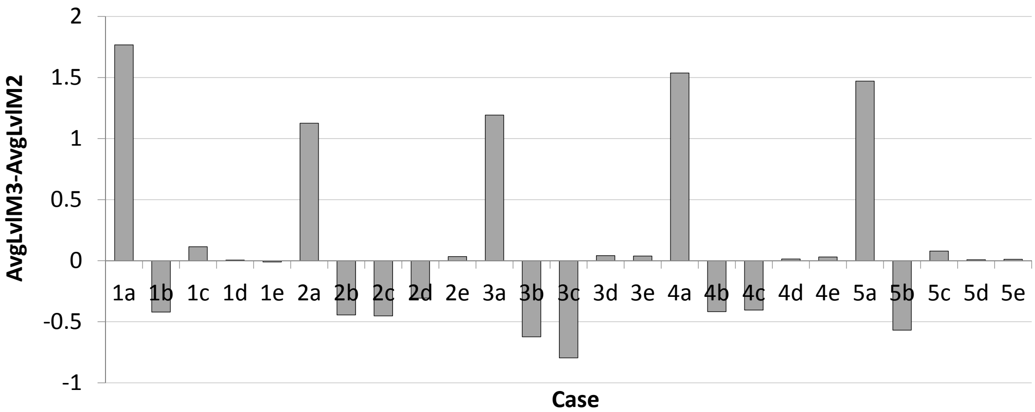


Figure 5.41. The difference of average inventory level values in Models 3 and 2 in each case.

The comparisons on the fill rate, P_2, and the average inventory level, A_I, between Model 3 and Model 2 are also examined with the statistical inference tests of two samples, t-test in each cases. In these analyses, the null hypothesis states that there are no difference between the averages of P_2 and I_A in Model 3 and averages of P_2 and I_A in Model 2. These tests have been conducted at a significance level of 0.05 for all cases. Table 5.9 tabulates *p-values* in each case. From table 5.9, most of *p-values* > 0.05 in each case of the fill rate, thus we fail to reject the null hypothesis except in case $2b$, $3c$, and $5c$. Therefore, from the test, we can infer that in most cases, there is no difference between the averages of P_2 in Model 3 and Model

2. In each case of the average inventory level, I_A, most of *p-values* < 0.05, thus we reject the null hypothesis except in case $1d$, $3d$, $4d$, $5c$, $5d$, and all e cases ($\beta = 0.9$). Therefore, from the test, we can infer that in most cases, there is difference between the averages of I_A in Model 3 and Model 2.

Table 5.9. The values of *p-values* in each case for the fill rate and the average inventory level t-tests.

Case	*p-values*	
	P_2	I_A
1a	0.3449	0.0000
1b	0.0178	0.0000
1c	0.5750	0.0008
1d	0.9589	0.7742
1e	0.1059	0.7242
2a	0.2433	0.0000
2b	0.0031	0.0000
2c	0.7577	0.0000
2d	0.1813	0.0000
2e	0.7126	0.0892
3a	0.8973	0.0000
3b	0.2537	0.0000
3c	0.0350	0.0000
3d	0.3129	0.0254
3e	0.5517	0.1409
4a	0.0008	0.0000
4b	0.6303	0.0000
4c	0.0621	0.0000
4d	0.6428	0.9191
4e	0.7705	0.2585
5a	0.0006	0.0000
5b	0.2569	0.0000
5c	0.0442	0.0985
5d	0.2456	0.6120
5e	0.9369	0.9208

5.4.2 The comparison between Model 2 and Model 4

During the process of formulating the transition probabilities for Model 4, we discovered a similarity between Model 4 and Model 2. Note that, the analysis of Model 2 has been discussed in Chapter 4 and the analysis of Model 4 is discussed in the previous section 5.3. Before the proof is presented in this section, we will recall the states and the transition probabilities of the DMDP components for Models 2 and 4 to be used in the proof statement.

Recall the states of supplier F, a, in Model 4. The set of $j \in \{0, 1, \cdots, W\}$ where W can approaches ∞. If the initial state of supplier F is at state w, the period of supplier F has been down for j periods is represented by the length of disruption. Let A denote a geometric random variable and $Pr(A = j)$ as the probability of the disruption length that supplier F takes to become fully operational after a minimum of j disruption periods which follows the geometric distribution, such that $A \sim Geo(p)$ for $j = 1, 2, 3, \ldots$ with a constant p where the value of p is the probability that supplier F's normal operation continues.

Recall the transition probabilities value in Model 4. The transition probability to move from state w to state u, $h(j)$ is calculated by using a recovery rate formula as shown below:

$$\begin{aligned} h(j) =& \frac{Pr(A = j)}{Pr(A \geq j)} \\ =& \frac{Pr(A = j)}{\sum_{\ell=1}^{j-1} Pr(A = \ell)} \end{aligned}$$

Recall the supplier F's states and the transition probabilities in Model 2. The member of set of supplier F state, a, is u and w, such that $a = \{u, w\}$. The transition probability values to move from state u to state w and from state w to state u are $p_{u,w}$ and $p_{w,u}$ respectively. If we make an assumption and formulate the transition states of supplier F in Model 2 as the assumption in Model 4, then we discover that the recovery rate in Model 4 does not depend on the length of disruption, thus we can conclude that Model 2 and Model 4 are equivalent. The proof of our statement on the relation between these two models is shown in the following section.

Proof Statement

Based on Model 2, we can assume that the period of normal operation (i.e., supplier F is in the up state) and the length of the disruption at the supplier F (i.e., supplier F is in the down state) are unknown but the period and the length can be predicted. Thus, time spent in the up and down states can be modelled as geometric random variables. Now, let:

$$A' = \text{length of disruption}$$

and the probability of A' is defined as:

$$Pr(A' = \ell).$$

If we assume that A' in Model 2 is similar to A in Model 4 where A is a geometric random variable with a constant p where p is a probability that supplier F's operation is back to normal after a minimum of ℓ periods, then the probability of A', $Pr(A' = \ell)$ follows a geometric distribution, such that $A' \sim Geo(p)$ for $j = 1, 2, \cdots$.

The probability of the disruption length is given by:

$$Pr(A' = \ell) = \acute{p}_{w,w}^{\ell-1}\ \acute{p}_{w,u} \qquad \text{for} \quad \ell = 1, 2, ..., \infty.$$

where

$$\acute{p}_{w,w} = \text{transition probability to stay at the same state,}$$

$$\acute{p}_{w,u} = \text{transition probability from state } w \text{ to state } u.$$

If the value of ℓ starts from 1 until it approaches ∞, then, the value of recovery rate, $Pr(A' = \ell)$

is approximately equals to 1. We prove this statement with the following equation:

$$\begin{aligned}
\sum_{\ell=1}^{\infty} {p_{w,w}}^{\ell-1} p_{w,u} &= p_{w,u} \sum_{0}^{\infty} {p_{w,w}}^{\ell} \\
&= \frac{p_{w,u}}{1 - p_{w,w}} \\
&= 1.
\end{aligned}$$

To check the relation between Model 2 and Model 4, we reformulate the formula of Model 4 recovery rate. For a new recovery rate Model 4, such that $p_{w_j,u}$ is shown below:

$$\begin{aligned}
p_{w_j,u} =& \frac{p_{w,w}^{j-1}\, p_{w,u}}{\sum_{\ell=n}^{\infty} p_{w,w}^{\ell-1}\, p_{w,u}} \\
=& \frac{p_{w,w}^{j-1}}{p_{w,w} \sum_{\ell=n}^{\infty} p_{w,w}^{\ell-j}} \\
=& \frac{1}{\sum_{\ell=0}^{\infty} p_{w,w}^{\ell}} \\
=& \frac{1}{\frac{1}{1-p_{w,w}}} \\
=& 1 - p_{w,w} \\
=& p_{w,u}
\end{aligned}$$

Based on the above equation, the new recovery rate, $p_{w_n,u}$ is equal to $p_{w,u}$, thus the recovery rate in Model 4 does not depend on the length of disruption. In the equation, if the value of ℓ approaching infinity, ∞, then the value of $Pr(A' = \ell)$ is approximately equals to 1. In other words, no matter how long the disruption length is (or how bigger the value of ℓ is), when ℓ approaching ∞, the value of $Pr(A' = \ell)$ is always approximately equals to 1, thus $Pr(A' = \ell)$ is equal to $Pr(A = \ell)$. Therefore, we can conclude that Model 2 and Model 4 are equivalent.

5.5 Conclusion

In this chapter, we presented the analyses of the ordering policy model with advance disruption information (Model 3) and the ordering policy model without disruption information but has a belief from the firm (Model 4).

The findings from Models 3 and 4 have provided us with a detail understanding on how the firm who has implemented global dual-sourcing strategy can manage the inventory system in the event of supply disruption at one of its suppliers. In Model 3, when both suppliers are available, the firm will carry more order from the offshore if the risk of disruption and the expected disruption periods are increased. When only the offshore supplier is down, the firm will only place an order with the onshore supplier if the inventory level is critically low. In this case, the firm will increase the order quantity from the onshore supplier with an increase in the expected disruption periods. In model 4, no matter how long the disruption is, it just has a small effect to the firm's ordering process. When only the offshore supplier is down, if the expected disruption length is too long, the firm will decrease the order quantity from the onshore supplier. In this situation, most probably at the firm's side, it got other strategy to mitigate the supply disruption, such as inventory, backup supply etc. We also discover that optimal policy in Model 3 is better than Model 2. While theoretically, we found that the similarity between Model 4 and Model 2.

This chapter has presented the value of supply disruption information for the firm in managing the risk of supply disruption in the process to discover the disruption. Therefore, in the next chapter, we present the analyses on the value of recovery planning to recover from the disruptions.

6. Phased Disruption Recovery Process

6.1 Introduction

Disruption recovery is a key aspect in the firms' operational planning for risk mitigation. Under the SCRM program, each and every firm that faces disruption is required to have a recovery plan. The recovery plan is usually a flexible process, depending on mitigation strategies, which aids the firm in recovering from disruptions (Hishamuddin et al., 2014; Handfield et al., 2007). The objective of recovery is to bring the firm back to a complete or near normal status of regular operations. The recovery plan aims to facilitate a fast and efficient recovery thus reducing the loss due to disaster. In addition, the recovery plan includes assessment of the disruption which provides vital information as to when and to what degree the firm can restore operations to a normal status.

This chapter presents a model involving an unreliable supplier with a disruption recovery plan consisting of a series of phases. As in earlier chapters, the focus is on a simple two-echelon supply chain with one firm and two suppliers in a single product setting. The aim of this recovery model is to use the DMDP modelling framework to explore the impact of a recovery plan with a quantitative assessment of progress. We analyse three models, namely Model 5, Model 6 and Model 7. Model 5 uses a basic model of a phased recovery process, which is similar to Model 2 in Chapter 4 and Models 6 and 7 focus on the information available on the length of each phase of the process, in a similar way to Model 3 and Model 4 in Chapter 5. We believe that the phase of a recovery plan often provides a good indication of the remaining length of a disruption. This information can be used to improve the efficiency of inventory management policies. Verification of this statement is provided in the analysis of Model 6 and Model 7. From a strategic decision-making perspective, we hope that these models can help decision makers to appreciate the value of a measurable recovery plan that charts the path of the disrupted supplier back to normal operations. The recovery plan is assumed to be a flexible process, depending on organisational mitigation strategies, with the objective to minimise the disruption to the fulfilment of orders placed with the supplier. For example, the recovery plan may involve using backup inventory such as safety stock or calling upon backup production facilities or orders with a third-party (Wang, 2008). The ordering decision in the optimal inventory policy usually changes according to the selected recovery plan (Allen and Toder, 2004).

This chapter is structured in four sections. We present the analyses of Model 5, Model 6 and Model 7 in sections 6.2, 6.3 and 6.4. Finally the conclusion for this chapter is presented in section 6.5.

6.2 A Basic Model of Phased Recovery

Model 5 is a basic model with the aim of exploring how to employ a quantifiable measurement of recovery in inventory policies. In this model, we develop an exploratory model with recovery assessment via the DMDP technique. The recovery of the offshore supplier is modelled as a Markov chain and we investigate how the firm's optimal inventory policy will be affected by the state of the Markov chain model of the recovery process. The DMDP modelling framework of Model 5 is similar to Model 2, with the main difference being that the recovery process consists of several phases rather than just one phase. We assume that during recovery, the offshore supplier goes through a number of distinct recovery phases. Each and every time the supplier encounters disruptions, it implements the same recovery plan and so is required to go through the same recovery phases. Each phase may have a different expected duration and the total length of the disruption is given by the sum of the durations of the phases of the recovery process. It is common for the recovery process to be conducted in a phase approach (Hishamuddin et al., 2014; Hishamuddin, 2013; Chen et al., 2009; Allen and Toder, 2004). We examine the impact of the rate of transition between phases of the recovery process on the ordering decision and the minimum expected inventory cost. The length of each phase is modelled by a constant hazard rate.

The structure of this section is as follows. We describe Model 5 and its assumptions in sections 6.2.1 and 6.2.2, followed by the formulation of the ordering decision problem under supply disruption via the DMDP in section 6.2.3. Then, in section 6.2.4, we present the transition probability values used when conducting the numerical experiment. The results and findings are reported in section 6.2.5 and section 6.2.6. Finally, the conclusion for Model 5 is presented in section 6.2.8.

6.2.1 Model Description

The firm seeks to split the order between the onshore supplier (or supplier N) and the offshore supplier (or supplier F), under the assumption of a phased recovery process at supplier F. Similar to the models in the previous chapter, supplier N is always reliable and supplier F is at risk of disruption. During normal operations of supplier F, the firm can order from both suppliers. However, during the disruption at supplier F, the firm can only order from supplier N. The length of disruption is unknown when a disruption occurs, but the firm has knowledge of a disruption from the outset of the occurrence of the disruption.

The Markov model of the recovery process at supplier F is as follows. The recovery process is assumed to consist of R phases, with different expected length. The phase of recovery is denoted by j where $j = \{0, 1, \ldots, R\}$. The length of each phase is modelled by a constant hazard rate, (i.e., the probability that the phase will end in this period). Following the occurrence of a disruption, the state of supplier F enters phase 1 of the recovery plan. During recovery, supplier F progresses through phase 2, phase 3, and so on to phase R. On completion of phase R, supplier F's operation is back to normal status, denoted by 0. Hence, state 0 represents the condition where supplier F is up and able to deliver a 100% order and state $j > 0$ represents the condition where supplier F is down and in phase j of the recovery process. The durations of the recovery phases are modelled as independent geometric random variables. We also assume that the length of a period of normal operation of supplier F

follows a geometric distribution. For a better understanding, the transitions between normal operation and phases of recovery for supplier F are illustrated in figure 6.1.

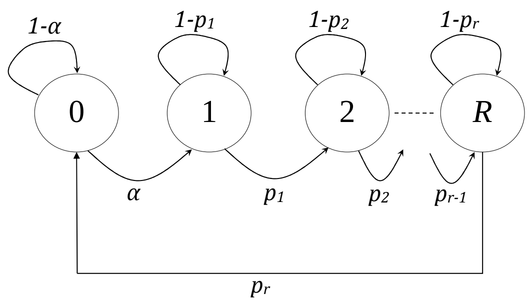


Figure 6.1. Markov chain model of phased recovery process for Model 5

In figure 6.1, α represents the probability of supplier F failing due to disruption and p_j represents the probability that phase j is completed during a period. Whenever the state of supplier F is in state 0, the process either remains in state 0, with probability $1 - \alpha$, or moves to state 1, with probability α. Whenever the recovery process is in state j for $j = 1, 2, \ldots, R - 1$, the process either remains in state j, with probability $1 - p_j$, or moves to the next phase of recovery (or state $j + 1$), with probability p_j. Whenever the recovery process is in state supplier F, the process either remains in state supplier F, with probability $1 - p_R$, or returns to normal operations (or state 0), with probability p_R. The formulation of the ordering decision problem is presented in the next section.

6.2.2 Model Assumptions

The assumptions of Model 5 are as follows:

a. The option of sourcing from supplier F is at risk of disruptions. The order from supplier F arrives either in full or not at all. However, the status of supplier N is perfectly reliable.

b. The length of disruption is unknown when a disruption occurs, but the firm has knowledge of a disruption from the outset of the occurrence of the disruption.

c. The recovery process is assumed to consist of R phases, with different expected length.

The durations of the recovery phases are modelled as independent geometric random variables.

d. The length of a period of normal operation of supplier F follows a geometric distribution.

e. The firm's inventory planning horizon is discrete.

f. Demand is either deterministic or stochastic. The stochastic demand distribution follows the truncated Poisson distribution, $P(d_t) \sim Pois(\lambda, K)$.

g. Customers do not accept backorders, thus the firm encounters lost sales. The firm is charged with a penalty cost for each unit of demand that cannot be satisfied in a period, $PNLTY$.

h. The firm incurs a holding cost for inventory held during period t, $HOLD$.

6.2.3 Model Formulation

In this section, we explain the formulation of the problem analysed in Model 5 as a DMDP model and present the optimality equation.

Components of the DMDP for Model 5

The DMDP components in Model 5 are as follows:

Decision Epochs

A decision is taken at the beginning of each period. Let t denote the decision epoch when there are t periods remaining in the planning horizon, $t = 1, 2, \ldots, T$.

States

At each decision epoch, the firm reviews the inventory level, i, and the recovery state of supplier F, j with $j \in \{0, 1, \ldots, R\}$. The parameters i and j comprise the state of the process y, such that $y = (i, j)$. The state space, Y, of Model 5 is given by:

$$Y = \{(i, j) : i \in \{0, 1, ..., I\},\ j \in \{0, 1, \ldots, R\}\}$$

Actions:

Based on the current state, the firm then decides on the quantity to be ordered from supplier N and supplier F. The action is denoted by $b = (q^N, q^F)$ and the set of admissible actions, $B(y)$ is given by:

$$B(i, 0) = \left\{(q^F, q^N) : \quad q^F, q^N \geq 0 \quad \& \quad q^F + q^N \leq I - i\right\} \quad \text{for} \quad 0 \leq i \leq I.$$

$$B(i, j) = \left\{(0, q^N) : \quad q^N \in \{0, ..., I - i\}\right\} \quad \text{for} \quad 0 \leq i \leq I, \quad 1 \leq j \leq F.$$

Under the admissible action set of $B(i, 0)$, the firm can choose to order up to $I - i$ items either from supplier N only or from supplier F only or from both the suppliers. Whilst under the admissible action set of $B(i, j)$ with $j > 0$, the decision is to place an order for up to $I - i$ items with supplier N only.

Transition probabilities:

We model changes in the inventory level, i, and changes in the recovery phases of the offshore supplier, j, separately. The transition matrix describing changes in the inventory level, i, depends on the order quantities and is the same as for previous models. See section 4.2.3a for a full description. The transition matrix describing changes in the state of supplier F follows from figure 6.1 above. The transition matrix, X, and is formally presented below.

$$X = \begin{array}{c c} & \begin{array}{cccccc} 0 & 1 & 2 & \cdots & r-1 & r \end{array} \\ \begin{array}{c} 0 \\ 1 \\ \vdots \\ r-1 \\ r \end{array} & \begin{pmatrix} 1-\alpha & \alpha & 0 & \cdots & 0 & 0 \\ 0 & 1-p_1 & p_1 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & 1-p_{r-1} & p_{r-1} \\ p_r & 0 & 0 & \cdots & 0 & 1-p_r \end{pmatrix} \end{array}$$

One-step costs:

The one-step cost as a result of action b in state y consists of the ordering cost, $ORDER$, the holding cost, $HOLD$ and the penalty cost, $PNLTY$. In the one-step cost with stochastic demand, the values of $HOLD$ and $PNLTY$ depend on the random demand during the period. The one-step costs for Model 2 with the constant and stochastic demand settings are based on the same assumptions as the one-step costs for the models in previous chapters. See section 3.2.1 for a detailed explanation.

The one-step cost is denoted by $C_t^y(b)$ and this cost under a constant demand setting is given by:

$$\begin{aligned} C_t^y(b) &= ORDER + HOLD + PNLTY \\ &= \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \; h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\ &\quad + \; m\Big(\max(D_t - i - q^N, 0)\Big) \end{aligned}$$

and under a stochastic demand setting is given by:

$$\begin{aligned} C_t^y(b) &= ORDER + \Big(E_{D_t}\Big(HOLD + PNLTY\Big)\Big) \\ &= \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t)\Big\{h\Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) \\ &\quad + \; m\Big(\max(d_t - i - q^N, 0)\Big)\Big\} \end{aligned}$$

Optimality equation

Let $V_t(y)$ be the minimum cost over the remainder of the planning horizon when the process is in state y at decision epoch t. The optimality equation for Model 5 with constant demand is given by:

$$V_t(i,j) = \min_{b\in B(y)}\Bigg\{\sum_{sp\in\{N,F\}}(\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + m\Big(\max(D_t - i - q^N, 0)\Big) + h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) + \sum_{k=0}^{F} X_{j,k}\Big(V_{t-1}\Big(\max(i + q^N - D_t, 0) + q^F, k\Big)\Big)\Bigg\}$$

Similarly, the optimality equation for Model 5 with stochastic demand is given by:

$$V_t(i,j) = \min_{b\in B(y)}\Bigg\{\sum_{sp\in\{N,F\}}(\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t)\Big\{m\Big(\max(d_t - i - q^N, 0)\Big) + \Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) + \sum_{g=0}^{F} X_{j,g}\Big(V_{t-1}\Big(\max(i + q^N - d_t, 0) + q^F, g\Big)\Big)\Big\}\Bigg\}$$

Using these two equations, we seek to minimise $V_t(y)$ and to find the optimal quantities to be ordered from supplier N and supplier F, depending on the recovery phases and the rates of transition between recovery phases in the transition matrix, X. We are particularly interested to investigate by numerical experiment how the values in this transition matrix can affect the firm's ordering policy.

An equilibrium distribution of the Markov chain model

Expected length of disruption is equal to the sum of the expected length of each phase of the recovery process, which denoted by $\bar{A}$ and can be calculated as follows.

$$\bar{A} = \sum_{j=1}^{R} \frac{1}{p_j}$$

From the previous results in the previous chapter, the proportion of time for which the offshore supplier is up is given by:

$$\begin{aligned}\pi_0 &= \frac{1}{1+\alpha\bar{A}} \\ &= \frac{1}{1+\alpha\sum_{j=1}^{R}\frac{1}{p_j}}\end{aligned}$$

Note that in this case, it is possible to go further due to the structure of the Markov chain model of the offshore supplier. The proportion of time for which the offshore supplier is up or down can be deduced from the equilibrium distribution of the Markov chain model of the state of the offshore supplier. Let

$$\pi = (\pi_0, \pi_1, \cdots, \pi_R)$$

denote an equilibrium distribution of the Markov chain model of the offshore supplier. Using standard methods (see for example Tijms (2003)), it is easy to show that the equilibrium distribution is unique and satisfies the following equations:

$$\begin{aligned}\pi_1 &= \alpha\pi_0 + (1-p)\pi_1 \Rightarrow p_1\pi_1 = \alpha\pi_0 \\ \pi_j &= p_{j-1}\pi_{j-1} + (1-p_j)\pi_j \Rightarrow p_j\pi_j = p_{j-1}\pi_{j-1} \quad 1 < j \leq R \\ 1 &= \pi_0 + \pi_1 + \cdots + \pi_R.\end{aligned}$$

It follows that $p_j\pi_j = \alpha\pi_0$ for $1 \leq j \leq R$. Using the above expression for π_0, the proportion of time for which the offshore supplier is in phase j of the recovery process is given by:

$$\pi_j = \frac{\alpha}{p_j(1 + \alpha \sum_{k=1}^{R} \frac{1}{p_k})}$$

Therefore, the proportion of the for which the offshore is up is $\pi_0 = \frac{1}{1+\alpha \sum_{j=1}^{R} \frac{1}{p_j}}$ and the proportion of time for which the offshore supplier is in recovery phase j (when the offshore supplier is down) is $\pi_j = \frac{\alpha}{p_j(1+\alpha \sum_{k=1}^{R} \frac{1}{p_k})}$.

Note that, this argument makes no assumption about the characteristic of the probability of the length of disruption recovery and so generalises to any probability distribution. Therefore, this result will be used to calculate the proportion of time for which the offshore supplier is up or at any recovery phase in Models 6 and 7.

6.2.4 Choice of Parameters Values

In this section, we present various transition probability values used for the numerical analysis. Our objective is to analyse how the optimal policy changes with different transition probabilities. In this numerical study, we consider four recovery states and one up state, thus $j = 5$. Four recovery states were chosen based on the life-cycle of recovery management suggested by Chen et al. (2009) and Allen and Toder (2004) and one up state to represent the normal status of supplier F. We consider a few different probabilities of supplier F failing due to disruption, α, and and a few different probabilities that recovery phase j is completed during a period, p_j, for $j = \{1, 2, 3, 4\}$ (also referred to as the hazard rate for phase j). A set of 25 cases based on various combinations of α and hazard rate, p_j, values are tabulated in table 6.1.

We number the cases according to α and hazard rate, p_j, values. For example for case 1A, number **1** is used to represent the corresponding values of α and letter **A** is used to represent the corresponding set of hazard rate, p_j, values. For the case of number, the enumerations of these numbers have been sorted into ascending α. The higher the numbers, the bigger the α values are and vice versa. For the case of letter, cases A, B and C assume that the hazard rate is constant for each phase. The constant hazard rate is increasing from case A to case B to case C which corresponds to the expected length of each phase decreasing. In case D, the hazard rate increases as the recovery process progresses through phases 1, 2, 3 and 4, while in case E, the hazard rate decreases as the recovery process progresses through phases 1, 2, 3 and 4.

We are also interested to examine the impacts of the values of the expected length of a disruption, the expected length of an interval of normal service and the proportion of time for which the offshore supplier is up on the optimal policy. These values are tabulated in table 6.1 for each of the 25 cases. The expected length of normal service is the mean of a geometric distribution with parameter α, which denoted by $1/\alpha$.

From this numerical study, we illustrate the effects of the transition probabilities, case by case, on the three areas namely the firm's optimal ordering decisions, the cost of optimal policies and the performance of the optimal policy under the stochastic demand model (i.e., fill rate and average inventory). To do the experiment, we analyse Model 5 with the combination of α and p_j values, case by case, as in table 6.1.

In what follows, we first present results on the effects of the cases on the properties of the ordering decisions, then results relating to the effects on the properties of the costs of policies, and finally the results on the effects of the fill rate and the average inventory under the stochastic demand model analysis.

Table 6.1. 25 cases based on various combination of α and p_j values

Case	α	p_1	p_2	p_3	p_4	$\frac{1}{\alpha}$	$\bar{A}$	π_u	π_1	π_2	π_3	π_4
1A	0.10	0.25	0.25	0.25	0.25	10.0	16.00	0.385	0.154	0.154	0.154	0.154
1B	0.10	0.45	0.45	0.45	0.45	10.0	8.89	0.529	0.118	0.118	0.118	0.118
1C	0.10	0.95	0.95	0.95	0.95	10.0	4.21	0.704	0.074	0.074	0.074	0.074
1D	0.10	0.10	0.30	0.70	0.90	10.0	15.87	0.387	0.387	0.129	0.055	0.043
1E	0.10	0.90	0.70	0.30	0.10	10.0	15.87	0.387	0.043	0.055	0.129	0.387
2A	0.30	0.25	0.25	0.25	0.25	3.33	16.00	0.172	0.207	0.207	0.207	0.207
2B	0.30	0.45	0.45	0.45	0.45	3.33	8.89	0.273	0.182	0.182	0.182	0.182
2C	0.30	0.95	0.95	0.95	0.95	3.33	4.21	0.442	0.140	0.140	0.140	0.140
2D	0.30	0.10	0.30	0.70	0.90	3.33	15.87	0.174	0.521	0.174	0.074	0.058
2E	0.30	0.90	0.70	0.30	0.10	3.33	15.87	0.174	0.058	0.074	0.174	0.521
3A	0.50	0.25	0.25	0.25	0.25	2.00	16.00	0.111	0.222	0.222	0.222	0.222
3B	0.50	0.45	0.45	0.45	0.45	2.00	8.89	0.184	0.204	0.204	0.204	0.204
3C	0.50	0.95	0.95	0.95	0.95	2.00	4.21	0.322	0.169	0.169	0.169	0.169
3D	0.50	0.10	0.30	0.70	0.90	2.00	15.87	0.112	0.560	0.187	0.080	0.062
3E	0.70	0.90	0.70	0.30	0.10	2.00	15.87	0.112	0.062	0.080	0.187	0.560
4A	0.70	0.25	0.25	0.25	0.25	1.43	16.00	0.082	0.230	0.230	0.230	0.230
4B	0.70	0.45	0.45	0.45	0.45	1.43	8.89	0.138	0.215	0.215	0.215	0.215
4C	0.70	0.95	0.95	0.95	0.95	1.43	4.21	0.253	0.187	0.187	0.187	0.187
4D	0.70	0.10	0.30	0.70	0.90	1.43	15.87	0.083	0.578	0.193	0.083	0.064
4E	0.70	0.90	0.70	0.30	0.10	1.43	15.87	0.083	0.064	0.083	0.193	0.578
5A	0.90	0.25	0.25	0.25	0.25	1.11	16.00	0.065	0.234	0.234	0.234	0.234
5B	0.90	0.45	0.45	0.45	0.45	1.11	8.89	0.111	0.222	0.222	0.222	0.222
5C	0.90	0.95	0.95	0.95	0.95	1.11	4.21	0.209	0.198	0.198	0.198	0.198
5D	0.90	0.10	0.30	0.70	0.90	1.11	15.87	0.065	0.589	0.196	0.084	0.065
5E	0.90	0.90	0.70	0.30	0.10	1.11	15.87	0.065	0.065	0.084	0.196	0.589

6.2.5 The Impact of Various Transition Probabilities on the Ordering Decisions

In this section, we explain how the transition probability values can affect the firm's ordering decision. We discuss the result under the infinite-horizon setting, covering the infinite-horizon Model 5 with constant demand (later known as M5*InfCons*) and stochastic demand (later known as M5*InfSto*).

The ordering policies in the infinite-horizon Model 5

The optimal ordering policies for a finite horizon under constant and stochastic demands are as follows.

The ordering policy of M5InfCons

If supplier F is in the up state (or state u), the firm will only place an order with supplier N if there is an immediate shortage (i.e., $i < D$). This applies at every decision epoch in all cases. In this situation, the quantity ordered from supplier N is just enough to meet the immediate shortage (i.e., $D - i$). This aspect of policy is the same as for models in the previous chapters. However, the optimal order placed with supplier F varies from case to case, depending on the supply disruption probability and the expected length of each phase of the recovery plan. From figure 6.2, in case 3 ($\alpha = 0.3$), cases B and C (i.e., the hazard rate in each phase is equal and relatively high) are different from cases A, D and E. In cases B and C, the firm orders a smaller quantity and waits until inventory level is lower before ordering. Cases A, D and E are very similar with only minor differences on the inventory level at which the firm starts to order. In general, the point at which the firm starts to order decreases as the expected length of disruption decreases.

Table 6.2 shows how the optimal ordering policy (i.e., an (s, S) policy) vary in the 25 cases considered. From table 6.2, we see that the incentive to order from supplier F decreases as α increases. In case of equal hazard rates in each phase, the incentive to order form this supplier decreases too as the hazard rate increases and so the expected length of each phase decreases. As we expected, the firm keeps a higher stock of cheap items from the offshore supplier when the chance of the offshore supplier remaining up is getting slim and when the expected length of disruption is increasing. The reorder point is slightly higher in case E compared to case D while the order up to level is the same in both cases. Hence, the firm keeps a slightly higher stock in case E. This suggests that the firm is better able to plan inventory purchases during disruption in case E (where the hazard rate is increasing and the expected length of phases is decreasing) compared to case D (where the hazard rate is decreasing and the expected length is decreasing).

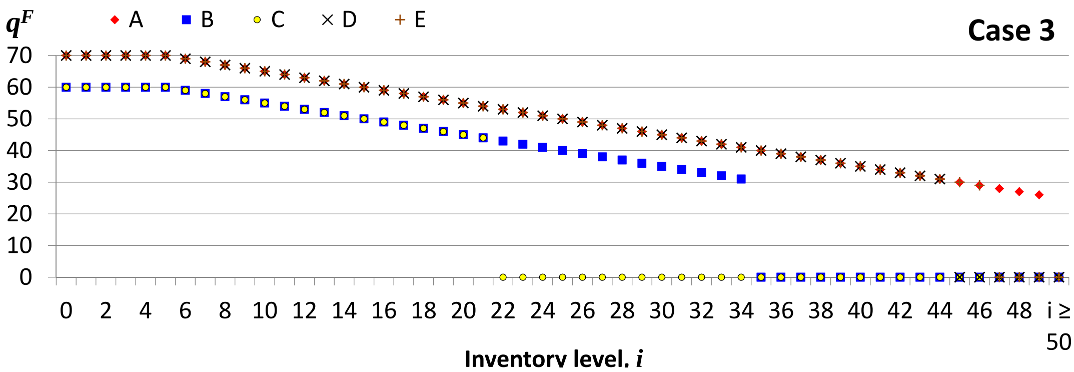


Figure 6.2. M5*InfCons*, state u: The optimal order from supplier F.

Table 6.2. M5*InfCons*, state u: Optimal order from supplier F

α	Scenario				
	A	**B**	**C**	**D**	**E**
1	(19,70)	(11,65)	(9,50)	(15,70)	(18,70)
2	(43,70)	(32,60)	(17,60)	(39,70)	(41,70)
3	(49,70)	(34,60)	(21,60)	(44,70)	(46,70)
4	(52,70)	(37,55)	(23,55)	(46,70)	(49,70)
5	(54,70)	(38,55)	(24,55)	(48,70)	(49,70)

From figure 6.3, in cases A, D and E, order up to level, S are stagnant with an increase in the expected up time, π_u. However, we can see a positive correlation between S and π_u in cases B and C, the value of S increases when π_u increases. Under this condition, if the expected length of recovery phases is shorter (or the hazard rate in each recovery phase is relatively higher), it is optimal for the firm to carry more inventory. However, if the expected length of recovery phases is longer or, decreasing or increasing as the recovery plan progresses (the hazard rate in each recovery phase is lower or, increases or decreases), the firm will keep the same inventory level across the up time of the offshore supplier.

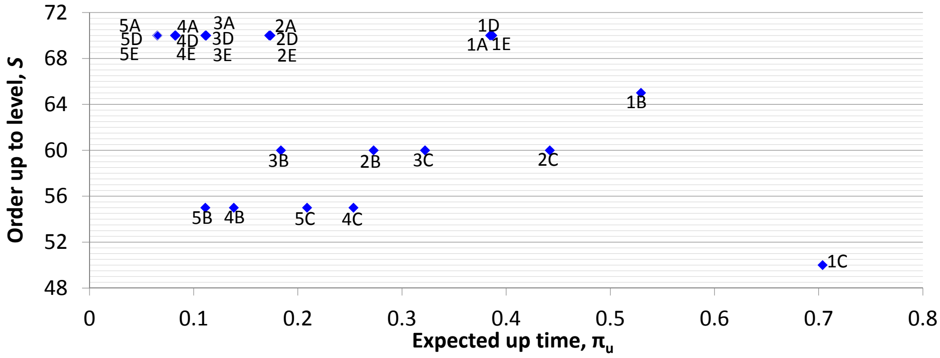


Figure 6.3. M5*InfCons*: The relationship between the order up to level and the expected up time.

If the offshore supplier is expected to be in the down state, we can see a positive correlation in each recovery phase between reorder point, s and the expected down time, π_w. This is illustrated in figures 6.4, 6.5, 6.6 and 6.7. From figure 6.4, when the recovery process is in phase 1, reorder point, s increases with an increase in the expected down time, π_w. The same pattern also occurs in other recovery phases, as illustrated in figures 6.5, 6.6 and 6.7. The inventory level at which the firm starts to order from the offshore supplier increases if the expected length of each recovery phase increases.

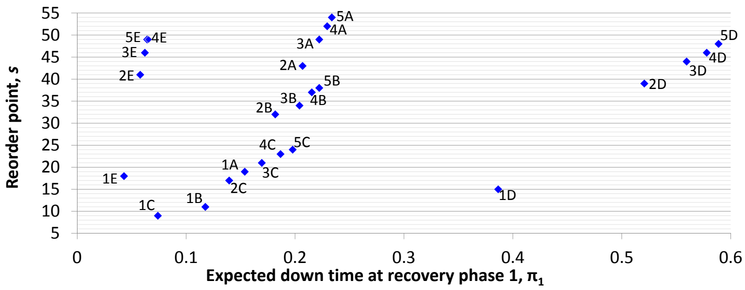


Figure 6.4. M5*InfCons*: The relationship between the reorder point and the expected down time in recovery phase 1.

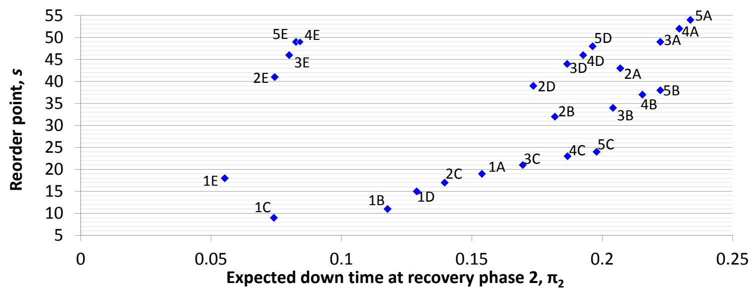


Figure 6.5. M5*InfCons*: The relationship between the reorder point and the expected down time in recovery phase 2.

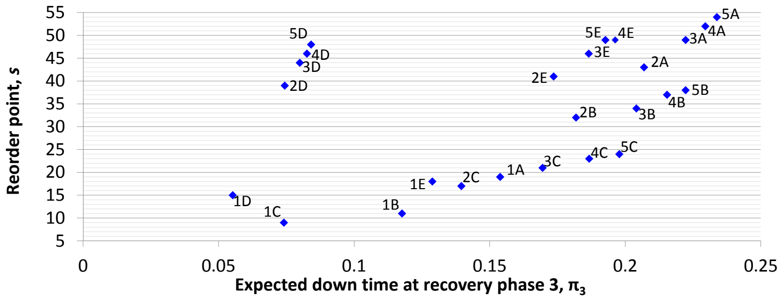


Figure 6.6. M5*InfCons*: The relationship between the reorder point and the expected down time in recovery phase 3.

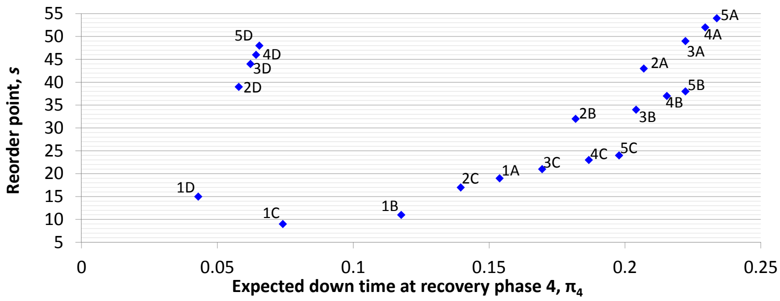


Figure 6.7. M5*InfCons*: The relationship between the reorder point and the expected down time in recovery phase 4.

If supplier F is in the down state (or state w), the order quantity from the onshore supplier is not influenced much by the supply disruption probability (α). Figure 6.8 shows how the order quantity depends on the phase of the recovery plan for case A and one case of α. The quantity ordered from supplier N decreases as the phase of the recovery plan increases. Hence, the information about the phase of the recovery plan is useful to the firm as it can reduce the quantity ordered from the onshore supplier as the recovery approaches its completion. As we expected, the firm will only place order if there is an immediate shortage (i.e., $i < D$).

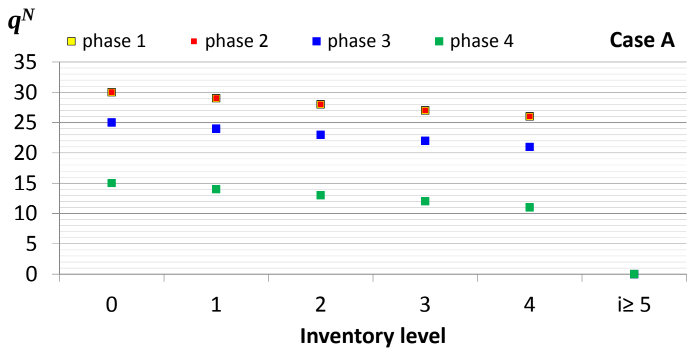


Figure 6.8. M5*InfCons*: The optimal order from supplier N during the recovery phase process for each case.

The optimal ordering policy from supplier N (i.e., an (s, S) policy) for all cases of supply disruption probability, α, and hazard rate, p_j, in each recovery phase, j, are tabulated in table 6.3. This table shows that the supply disruption probability has a minor influence on the order policy. For example, consider the order quantity placed with supplier N when recovery is in phase 4. If recovery process is in phase 4, the order quantity from supplier N decreases with an increase in α values. In this situation, we can conclude that as the risk of disruption at the offshore supplier increases, the firm will keep less stock if the offshore supplier is close to recovering from the disruption. This is possibly to ensure that the firm is in a position to order from the offshore supplier when the disruption ends, and so take advantage of the cheaper unit cost, before this supply is disrupted once more.

The ordering policy of M5InfSto

If supplier F is state u, the properties of optimal ordering policy from supplier N and supplier F are similar to the constant demand model. The firm only places an order with supplier N if there is an immediate shortage at every decision epoch in all cases. In this situation, the quantity ordered from supplier N is just enough to meet the anticipated shortage. However, the optimal order placed with supplier F varies from case to case, depending on the supply disruption probability and the expected length of each phase of the recovery plan.

Table 6.3. M5*InfCons*, state u: Optimal order from supplier N

α	recovery phase, j	Scenario				
		A	**B**	**C**	**D**	**E**
1	1	(4,30)	(4,30)	(4,25)	(4,30)	(4,30)
	2	(4,30)	(4,25)	(4,20)	(4,20)	(4,30)
	3	(4,25)	(4,20)	(4,15)	(4,15)	(4,25)
	4	(4,25)	(4,10)	(4,10)	(4,10)	(4,20)
2	1	(4,30)	(4,30)	(4,25)	(4,30)	(4,30)
	2	(4,30)	(4,25)	(4,20)	(4,20)	(4,30)
	3	(4,25)	(4,20)	(4,15)	(4,15)	(4,25)
	4	(4,25)	(4,10)	(4,10)	(4,10)	(4,20)
3	1	(4,30)	(4,30)	(4,25)	(4,30)	(4,30)
	2	(4,30)	(4,25)	(4,20)	(4,20)	(4,30)
	3	(4,15)	(4,10)	(4,10)	(4,10)	(4,20)
	4	(4,15)	(4,10)	(4,10)	(4,10)	(4,20)
4	1	(4,30)	(4,30)	(4,25)	(4,30)	(4,30)
	2	(4,30)	(4,25)	(4,20)	(4,20)	(4,30)
	3	(4,25)	(4,20)	(4,15)	(4,15)	(4,25)
	4	(4,15)	(4,10)	(4,10)	(4,10)	(4,20)
5	1	(4,30)	(4,25)	(4,25)	(4,30)	(4,30)
	2	(4,30)	(4,25)	(4,25)	(4,30)	(4,30)
	3	(4,25)	(4,15)	(4,15)	(4,15)	(4,25)
	4	(4,15)	(4,10)	(4,10)	(4,10)	(4,20)

From figure 6.9, in case 3 ($\alpha = 0.3$), in cases A, B and C (i.e., the hazard rate in each phase is equal), the quantity ordered from the offshore supplier and the inventory level at which the firm start places order decrease with an increase in the hazard rate in each recovery phase. In cases D and E (i.e., the hazard rate in each phase is increasing or decreasing), the order quantity is similar as in case A, but the firm will waits until inventory level is lower before ordering. In general, the point at which the firm starts to order decreases as the expected length of disruption decreases. The property of optimal policy in each case is similar as in the constant demand model.

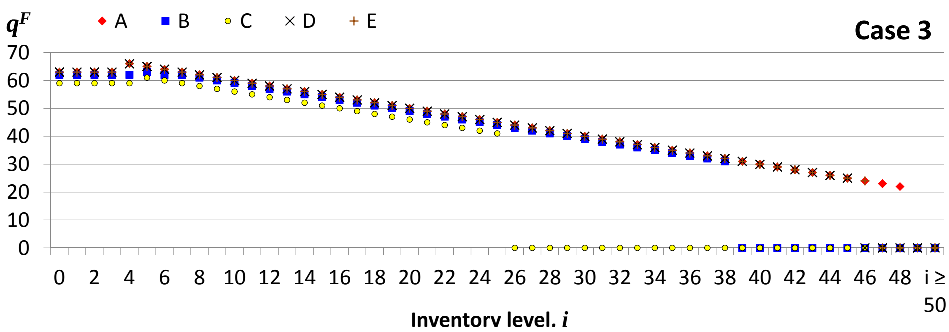


Figure 6.9. M5*InfSto*, state u: The optimal order from supplier F.

Table 6.4 shows an (s, S) policy in each case and how the parameters vary in these 25 cases. From this table, we see the incentive to order from supplier F decreases as α increases. In case of equal hazard rates in each phase, the incentive to order decreases too as the hazard rate increases and so the expected length of each phase decreases. As we expected, the firm keeps higher stock of cheap items from the offshore supplier when the chance of this supplier remaining up is getting slim and when the expected length of disruption is increasing. Different situation occurs in case C. If the hazard rate in each phase is higher, the incentive to order increases if α is relatively lower (i.e., $\alpha < 0.5$), but the incentive to order decreases if α is relatively higher (i.e., $\alpha \geq 0.5$). In this situation, it is preferable for the firm to keep less stock if the chance of the offshore supplier remaining up is lower or higher. It is logical since the length of phases is the shortest. If the expected up time for the offshore supplier is lower, the firm can always place an order with the onshore supplier for immediate shortage and, if the expected up time for the offshore supplier is higher, it is optimal to keep less stock in the inventory since the firm can always place order with this supplier in next ordering period. The reorder point is slightly higher in case E compared to case D while the order up to level is the same in both cases. Hence, the firm keeps a slightly higher stock in case E. This suggests that the firm is better able to plan inventory purchases during disruption in case E (where the hazard rate is increasing and the expected length of phases is decreasing) compared to case D (where the hazard rate is decreasing and the expected length is decreasing).

Table 6.4. M5*InfSto*, state u: Optimal order from supplier F

α	Scenario				
	A	**B**	**C**	**D**	**E**
1	(20,70)	(17,70)	(14,60)	(18,70)	(19,70)
2	(43,70)	(35,70)	(22,65)	(40,70)	(41,70)
3	(48,70)	(38,68)	(25,66)	(45,70)	(46,70)
4	(51,70)	(40,66)	(26,65)	(47,70)	(49,70)
5	(52,70)	(41,65)	(27,64)	(49,70)	(50,70)

If the up time for the offshore supplier is expected to be longer, it is optimal for the firm to carry more cheaper items from the offshore supplier as the up time increases or keeps the same inventory level across the up time, as illustrated in figure 6.10. From this figure, in cases A, D and E, order up-to level, S are stagnant with an increase in the expected up time, π_u. However, we can see a positive correlation between S and π_u in cases B and C, the value of S increases when π_u increases. The pattern of the effect of up time towards order up-to level in this condition is similar to the constant demand model.

If the offshore supplier is expected to be in the down state, we can see a positive correlation in each recovery phase between reorder point, s and the expected down time, π_w. This is illustrated in figures 6.11, 6.12, 6.13 and 6.14. From figure 6.11, when the recovery process is in phase 1, in cases A, B and C (i.e., the hazard rate in each phase is equal), reorder point, s increases with an increase in the expected down time, π_w. The same pattern occurs in cases D and E (i.e., the hazard rate in recovery phases are increasing or decreasing). The same pattern also occurs in other recovery phases, as illustrated in figures 6.12, 6.13 and 6.14. Reorder point, s increases as the expected down time, π_w increases. During phased recovery, the inventory level at which the firm starts to order from the offshore supplier increases if the length of each phase is expected to be longer.

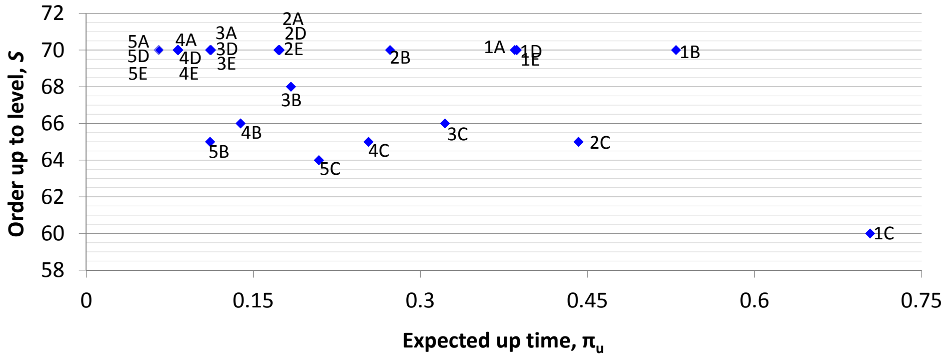


Figure 6.10. M5*InfSto*: The relationship between the order up to level and the expected up time.

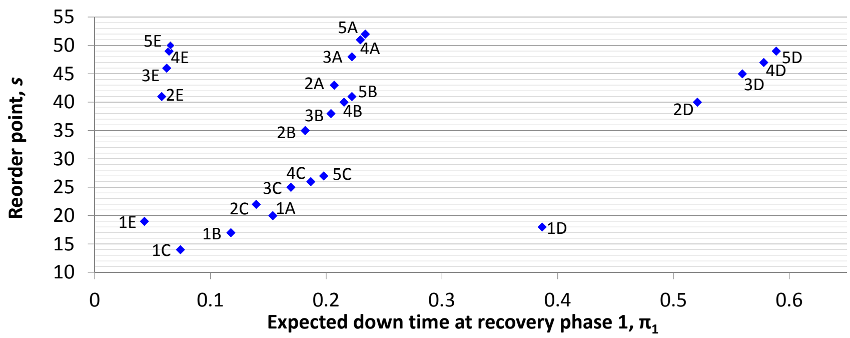


Figure 6.11. M5*InfSto*: The relationship between the reorder point and the expected down time in recovery phase 1.

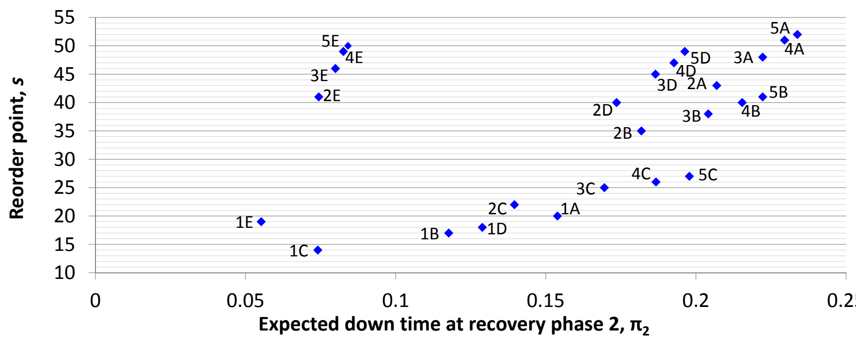


Figure 6.12. M5*InfSto*: The relationship between the reorder point and the expected down time in recovery phase 2.

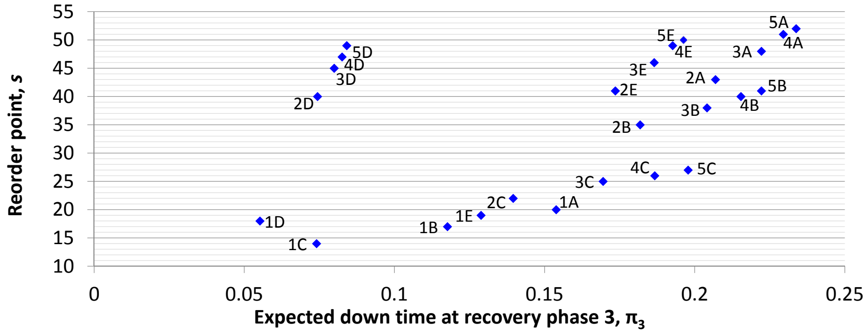


Figure 6.13. M5*InfSto*: The relationship between the reorder point and the expected down time in recovery phase 3.

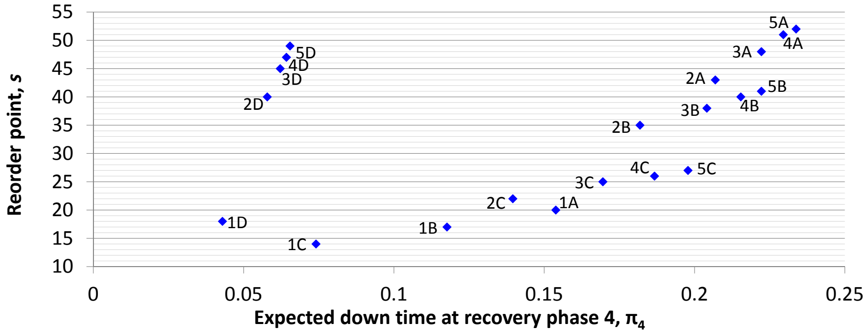


Figure 6.14. M5*InfSto*: The relationship between the reorder point and the expected down time in recovery phase 4.

If supplier F is in state w, similar to the constant demand model, the order quantity from the onshore supplier is not influenced much by the supply disruption probability (α), as illustrated in figure 6.15. This figure shows how the order depends on the phase of the recovery plan for case A and one case of α. The quantity ordered from supplier N decreases as the phase of the recovery plan increases. We can conclude that it is important for the firm to has information about the phase of the recovery plan as it can reduce the quantity ordered from the onshore supplier when the recovery approaches its completion. As we expected, the firm will only place order if there is an immediate shortage.

Table 6.5 shows how the hazard rate at each recovery phase has more influence on the order policy, which the optimal ordering for each case is considered as an (s, S) policy. For example, if recovery is assumed to be at phase 4, the order up to level decreases with an increase in the hazard rate at each phase (i.e., cases A, B and C). In this situation, we can conclude that as the length of disruption at the offshore supplier decreases, the firm will keep less stock if the offshore supplier is close to recovering from the disruption. This is possibly to ensure that the firm is in a position to order from the offshore supplier when the disruption ends, and so take advantage of the cheaper unit cost, before this supply is disrupted once more. However, the firm needs to carry more stock if the length of disruption is expected to be longer, as a preparation during a longer disruption and so avoiding the expensive unit cost ordered from the onshore supplier.

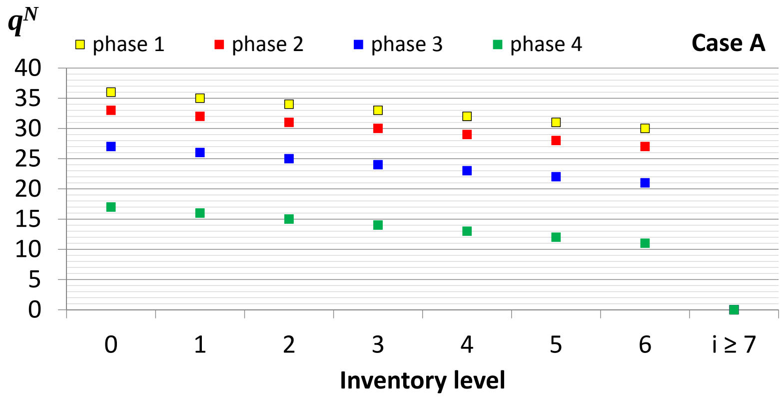


Figure 6.15. M5*InfSto*, state w: The optimal order from supplier N in Case A.

Table 6.5. M5*InfSto*, state w: Optimal order from supplier N

recovery phase, j	Scenario				
	A	**B**	**C**	**D**	**E**
1	(6,36)	(6,33)	(7,27)	(6,34)	(6,36)
2	(6,33)	(6,28)	(7,22)	(6,26)	(6,33)
3	(6,27)	(6,22)	(7,18)	(6,18)	(7,30)
4	(6,17)	(6,15)	(6,13)	(6,13)	(6,22)

Discussion

During the phased recovery process, we discover that the expected length of each recovery phase has more influence on the ordering policy than the supply disruption probability. However, the supply disruption probability has minor influence on the ordering policy in the constant demand model, but not in the stochastic demand model. In the constant demand model, it is optimal to order less from the onshore supplier when the risk of disruption at the offshore supplier increases. In addition, the firm will keep less stock if the offshore supplier is close to recovering from the disruption. In the stochastic demand model, as the expected length of each phase decreases, the firm will keep less stock especially if this supplier is close to recovering from the disruption. However, if the expected length of each phase increases or the expected of phases increasing as the recovery plan progresses, the firm will carry more stock as a preparation during a longer recovery period and so to avoid placing order with expensive onshore supplier. In general, we can conclude that, information about the phase of the recovery plan is useful to the firm in managing supply disruption as the findings have showed that it can reduce the quantity ordered from the onshore supplier when the recovery approaches its completion.

If the offshore supplier is in normal operation, in both the constant and stochastic demand models, the properties of order policy for the onshore supplier are the same. The firm will only place order with this supplier if the inventory level is so low that there is a high risk of immediate shortages. The findings show that the order quantity from the onshore supplier is only needed as backup if the firm does not have enough stock in inventory, which is similar to the properties of the ordering policy from the onshore supplier in models 2, 3 and 4. Under this situation, we also discover that the risk of supply disruption and the hazard rate in each recovery phase have impact on the optimal order placed with the offshore supplier. In both constant and stochastic demand models, the firm will order a smaller quantity and waits until inventory level is lower before placing order with the offshore supplier as the risk of disruption increases and the expected length of each recovery phase decreases.

6.2.6 The Impact of the Transition Probabilities on the Long-run Average Costs

In this section, we discuss the result on how the values of transition probability can affect the properties of the optimal policies costs under the infinite-horizon Model 5 which covered the experiments with the constant and stochastic demand settings.

The optimal policies costs in the infinite-horizon Model 5

From figure 6.16, we can see that the pattern of the long-run average cost, g across the cases is similar as for both constant and stochastic demand. In addition, the pattern of g is also the same for each α cases. In each case of equal hazard rate in each recovery phase, p_j (cases A, B and C), g decreases as the hazard rate in each recovery phase increases in all α cases. The value of g in case D (i.e., the hazard rate getting bigger in the recovery process) is always higher than in case E (i.e., the hazard rate getting smaller in the recovery process). As we expected, the firm will face higher cost if the recovery process will take longer period than shorter period of recovery process. In addition, we can see that g in each case is relatively the same (except in case C) when the risk of the offshore supplier facing a disruption is high ($\alpha > 0.5$). From the findings, we can conclude that the hazard rate in each recovery phase has more impact on g than the supply disruption probability.

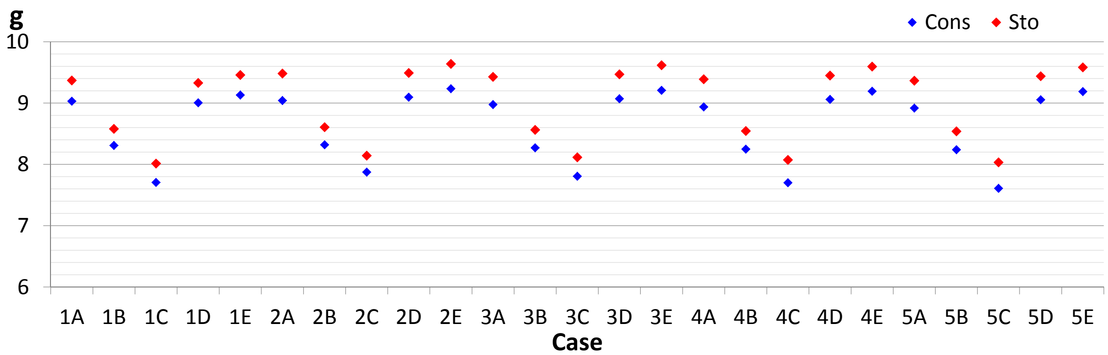
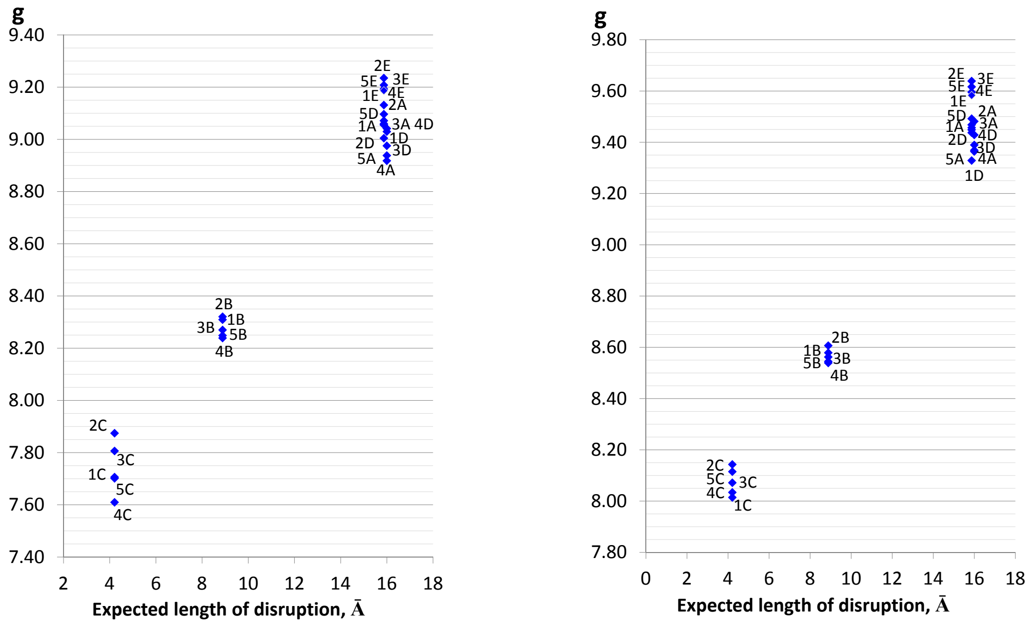


Figure 6.16. M5*Inf*: Optimal long-run average cost in different cases for constant and stochastic demand models

If we look from the aspect of the expected length of disruption, $\bar{A}$, from figure 6.17, in both the constant and stochastic demand models (see figures 6.17a and 6.17b), we can see that there are positive linear relationships between the long-run average cost, g and $\bar{A}$. As we expected, the firm will face higher cost if the length of disruption at the offshore supplier is expected to be longer in both constant and stochastic demand.



(a) M5*InfCons*: g *vs.* $\bar{A}$ (b) M5*InfSto*: g *vs.* $\bar{A}$

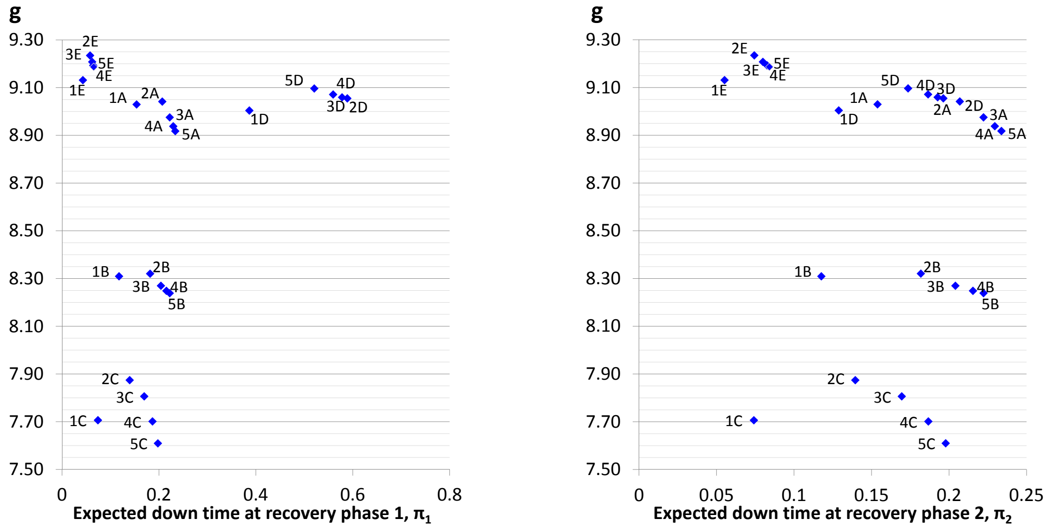
Figure 6.17. M5*Inf*: The relationship between the long-run average cost and the expected length of disruption

From the aspect of the expected down time at each recovery phase, π_j, in the constant and stochastic demand models, the long-run average cost, g also have relationships with π_j. This is illustrated in figures 6.18, 6.19, 6.20 and 6.21. From figures 6.18a and 6.20a, in recovery phase 1 in both constant and demand models, we can see a negative non-linear relationship between g and π_1, which g decrease when π_1 increases in cases B and C. In cases A, D and E, there is almost quadratic relationship between g and π_1. The values of g decreases when $0 \leq \pi_1 \leq 0.4$ and increases when $\pi_1 > 0.4$. The findings show that, the firm will face lower g if the expected down time in recovery phase 1 is lower, but g will increase when the expected down time in recovery phase 1 increases. The pattern of relationship between the long-run average cost and the expected down time for other recovery phases in both constant and stochastic demand models are contradict as to the expected down time in recovery phase 1, except in case B. This is illustrated in figures 6.18b, 6.20b, 6.19a, 6.21a, 6.19b and 6.21b. From these figures, in case A, D and E, g decreases when π_j ($j > 1$) in recovery phases 2, 3 and 4 increase. However, in case C, we can see a positive quadratic relationship between g and π_j. In case C, the firm faces with lower g and g increases when $0 \leq \pi_j \leq 0.1$. However, g decrease as π_j increases if $\pi_j > 0.1$. From the findings, we can conclude that in the beginning of recovery process, the firm should be more careful if the down time at the first phase of recovery is expected to be longer if the hazard rate in each recovery phase is lower or getting smaller or getting bigger as it can increases the long-run average cost. In addition, the firm will also facing higher cost in the middle of recovery process in each phase even though the hazard rate in each recovery phase is higher. Perhaps, after a completion of the first phase of recovery process, there will be additional cost that the firm has to face in the transition of recovery phase from the current phase to the next phase of recovery.

Discussion

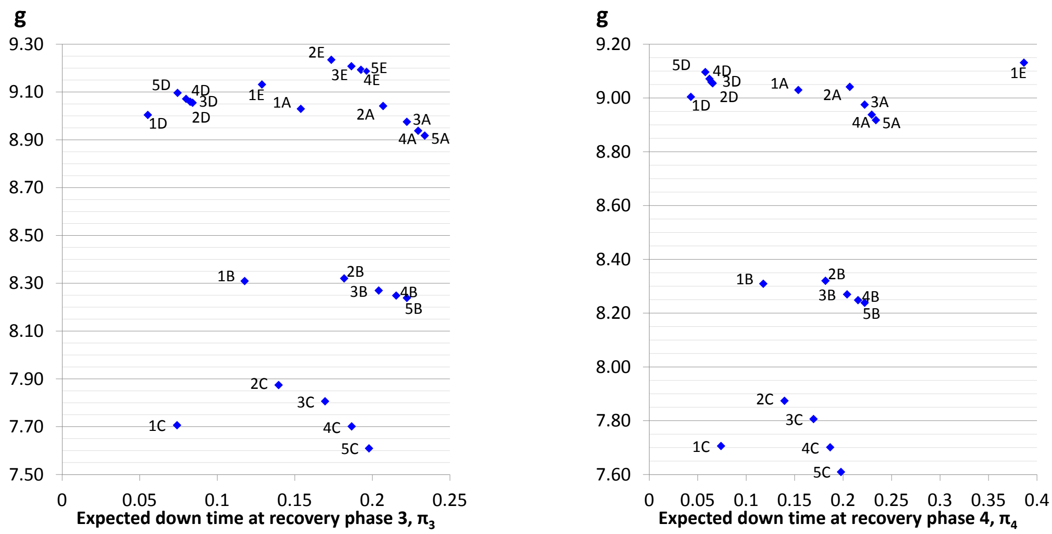
Under the finite-horizon planning model under both constant and stochastic demand settings, based on the relative value, we show that the recovery process that progresses phase by phase can decrease the minimum cost in each phase. However, the minimum cost will increase if the phases of recovery increases. This is explained from the findings in Model 5. For example, the minimum cost for the firm to recover from the disruption are higher when the recovery process at the offshore supplier are in phases 1 and 3, and lower when the recovery process are in phases 2 and 4. Based on the findings, we can conclude that the firm will face higher cost upon the occurrence of the disruption and the recovery process is just started. In addition, the minimum cost most probably will increase after the offshore supplier has just completed one phase of recovery process. However, the firm will have lower minimum costs upon the completion of recovery process for the offshore supplier. Hence, the firm will incur lower minimum costs if the offshore supplier is able to complete the recovery process and back to normal operation at the next period.

Under the infinite-horizon planning model under both constant and stochastic demand settings, we discover that the expected length of each recovery phase (or the hazard rate) has more impact on the long-run average cost than the supply disruption probability. The long-run average cost is lower when the hazard rate is higher. In addition, the firm will also having lower average cost when the expected length of phases is increasing as the recovery plan progresses (e.g., case E).



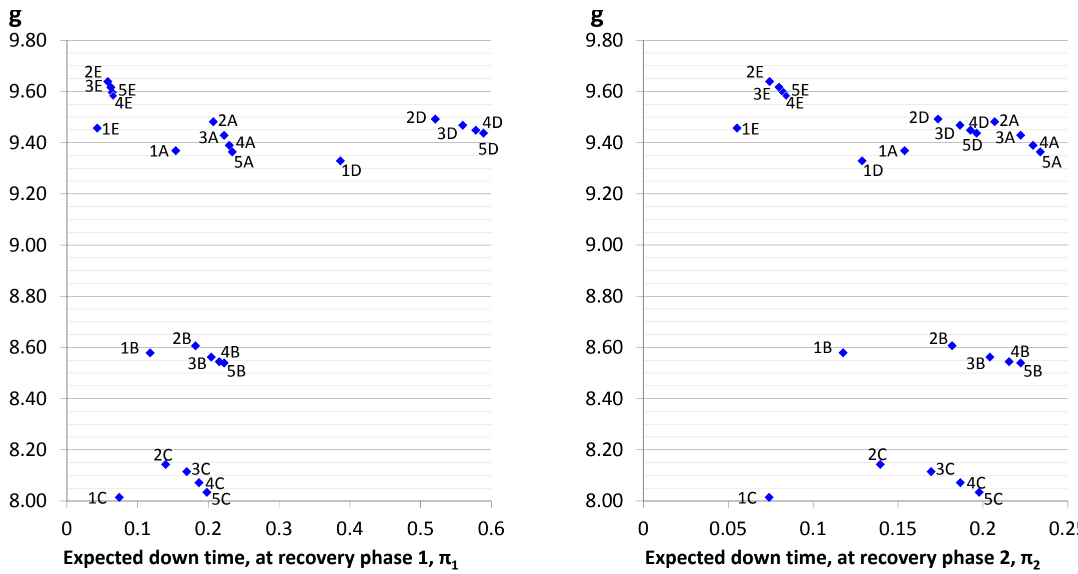
(a) g *vs.* π_1 (b) g *vs.* π_2

Figure 6.18. M5*InfCons*: The relationships between the long-run average cost and the expected down time in at recovery phases 1 and 2



(a) g *vs.* π_3 (b) g *vs.* π_4

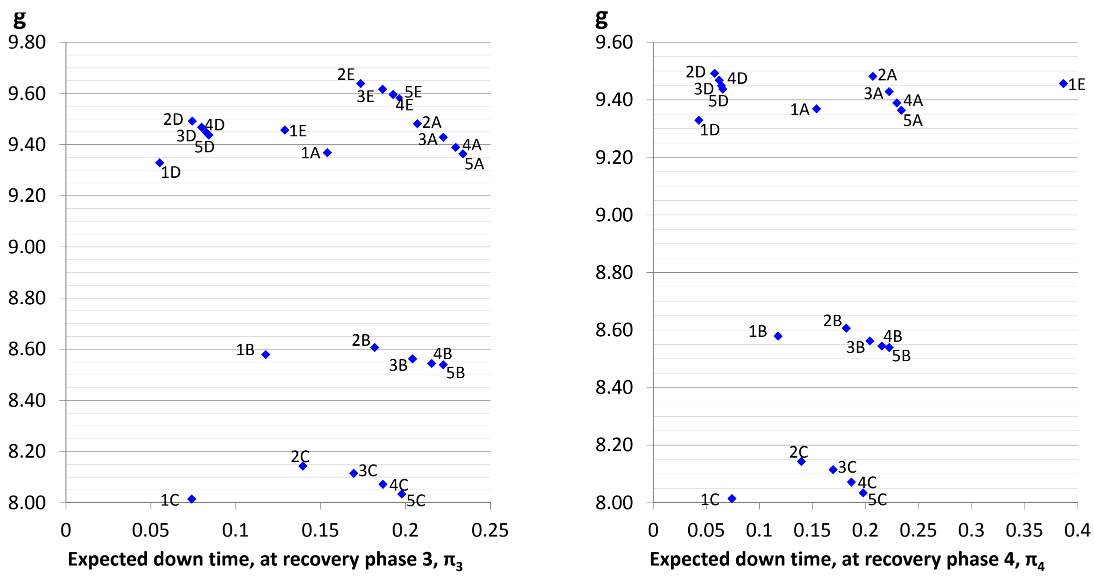
Figure 6.19. M5*InfCons*: The relationships between the long-run average cost and the expected down time in at recovery phases 3 and 4



(a) g *vs.* π_1

(b) g *vs.* π_2

Figure 6.20. M5*InfSto*: The relationships between the long-run average cost and the expected down time in at recovery phases 1 and 2



(a) g *vs.* π_3

(b) g *vs.* π_4

Figure 6.21. M5*InfSto*: The relationships between the long-run average cost and the expected down time in at recovery phases 3 and 4

6.2.7 The Impact of the Transition Probability Values on the Performance of the Policies

In this section, we discuss the performance of the fill rate in section 6.2.7a and the average inventory level in section 6.2.7b under the ordering policy of M5*InfSto*.

Fill rate

From figure 6.22, the percentage of demand satisfied from stock in hand in all cases are estimated to lie between 99.58% and 99.795% with 95% confidence interval. From this figure, we can see that the pattern of fill rate for each case of α is the same. The fill rate in each hazard rate case decreases with an increase in α values, except in cases C and D. As we expected, the fill rate improves as the risk of disruption decreases. In case of equal hazard rate (cases A, B and C), the fill rate increases when the hazard rate increases in each recovery phase. The fill rate in case D is relatively similar to the fill rate in case A and smaller than in case E. From the findings, we can conclude that supply disruption probability and the hazard rate in each recovery phase can affect the fill rate. Even though in some cases the fill rate seems quite low, the variation in the fill rates for other values of the disruption risk is within the range of the confidence intervals of the simulation results.

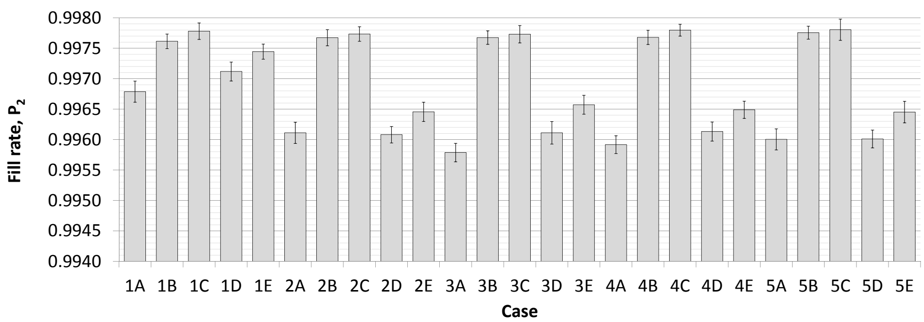
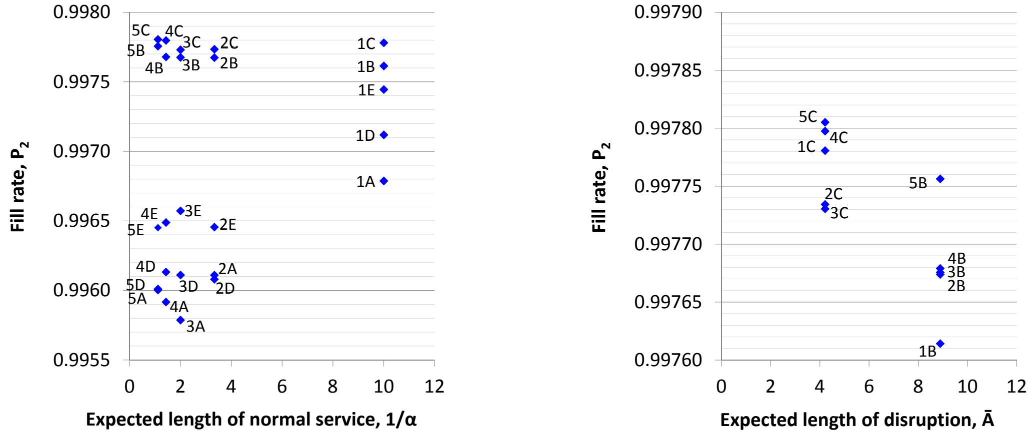


Figure 6.22. M5: Fill rate in each case

From the aspect of the expected normal service, $1/\alpha$, and the expected disruption length, $\bar{A}$, the fill rate, P_2 has no relationship with $1/\alpha$ and $\bar{A}$, which illustrated in figure 6.23. In addition, the fill rate, P_2 also has a relationship with the expected up time, π_u. From figure 6.24, there is a non-linear relationship between P_2 and π_u, which we can see a positive weak correlation. P_2 increases as π_u increases. From findings, we could say the capability of the firm to satisfy demand will increase if the offshore supplier is expected to operate longer.



(a) P_2 $vs.$ $1/\alpha$ (b) P_2 $vs.$ $\bar{A}$

Figure 6.23. M5: The relationships between fill rate and the expected normal service and the down time.

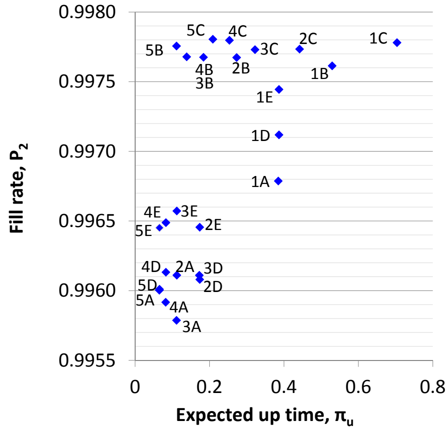


Figure 6.24. M5: The relationship between fill rate and the expected up time

Average inventory level

From figure 6.25, the average inventory level, I_A, in all cases are estimated to lie between 29.06 and 37.35 with 95% confidence interval, which lie in almost half of the maximum inventory level. It is quite noticeable that I_A decreases as α and the hazard rate in each recovery phase values increases. Due to low risk of disruption, it is optimal to carry less inventory to avoid high holding cost in the inventory. In equal hazard rate in each recovery phase case, I_A increases as the hazard rate in each phase increases. However, I_A decreases when the hazard rate is getting bigger (or the recovery process is getting longer). As we expected, the firm will carry more stock if the disruption length is lower.

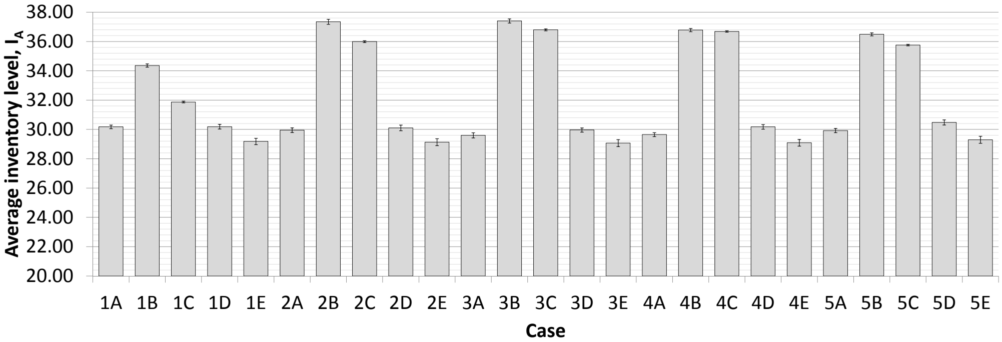
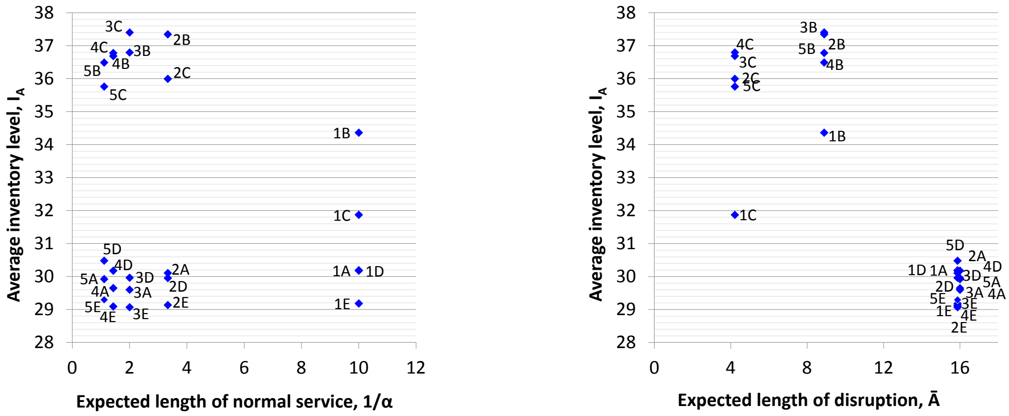


Figure 6.25. M5: Average inventory level in each case α.

From the aspect of the expected normal service, $1/\alpha$, and the expected disruption length, $\bar{A}$, the average inventory level, I_A, has no relationship with $1/\alpha$ and $\bar{A}$, as illustrated in figure 6.26.



(a) I_A *vs.* $1/\alpha$

(b) I_A *vs.* $\bar{A}$

Figure 6.26. M5: The relationships between average inventory level and the expected normal service and the expected disruption length

From figure 6.27, in cases A, D and E, I_A is almost stagnant with an increase in π_u. However, in cases B and C, I_A increases as π_u increase. From findings, it is optimal for the firm to carry more stock when the offshore supplier is expected to operate in a longer period.

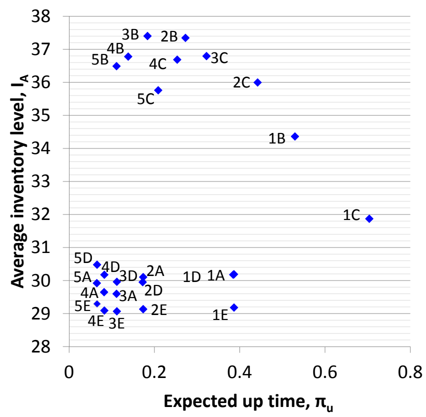


Figure 6.27. M5: The relationship between average inventory level and the expected up time

Discussion

In Model 5 analyses, similar with the analyses of Models 2, 3 and 4, the performance of the firm's ordering policy has been measured with the values of fill rate and average inventory level. Based on the fill rate, the firm still has high capability to satisfy demand from the customer when it chooses to use the optimal ordering policies. The percentage of demand satisfied from stock in hand is still high even though there are high supply disruption probability. In most situations, the firm will keep stock which is half of the maximum inventory level. However, we discover that the fill rate and the average inventory level have no relationship with the expected length of normal service and the expected length of disruption at the offshore supplier.

6.2.8 Conclusion

Model 5 has been developed to demonstrate the optimal ordering policy for the firm for its suppliers especially from the onshore supplier during the recovery process during a disruption. In Model 5, we addressed how the firm's optimal ordering policies can be affected by the risk of supply disruption and the expected length of each recovery phase at the offshore supplier. From the findings, during the recovery process, we discover that the expected length of each recovery phase has more impact on the optimal order quantity from the onshore supplier, the minimum cost and the long-run average cost. The firm will order more from the onshore supplier if the expected length of each recovery phase is lower or the expected length of phases increases as the recovery plan progresses. However, the firm will reduce the order quantity placed with this supplier if the expected length of each recovery phase is higher or the expected length of phases decreases as the recovery plan progresses. The results from Model 5 have provided us with a basic understanding on how the firm can manage the inventory during the recovery process at the offshore supplier during the disruption. Information about the phase of the recovery process can be used by the firm to help plan inventory purchases and it has been analytically tested in Model 5. This model 5 will be considered as a benchmark model for next model where we will consider the element of the value of disruption information during the recovery. In conclusion, a benchmark of the firm's optimal policy is established for further analyses on the firm's ordering process during the recovery process by considering additional information on supply disruption at the offshore supplier. For this reason, therefore, in the next section, we introduce the phased recovery model with additional disruption information or Model 6.

6.3 Phased Recovery Model with Additional Disruption Information

Model 6 is an extension of Model 5, in which advance information of the length of a phase of the recovery is available at the beginning of each phase. Hence, Model 6 extends Model 5 in the same way as Model 3 extends Model 2. The analysis of Model 6 focuses on the influence of advance disruption information on the firm's ordering policy, as the firm can has better supply disruption mitigation plan for recovery during the disruption. We could say the acquisition of advance information about the disruption under Model 6 is similar to Model 3, but these two models have a difference on information about the length of recovery phases (or the length of disruption). In Model 3, the firm just knows the distribution of the overall length of disruption, while in Model 6, the firm knows the distribution of the length of individual recovery phases. Nonetheless, we expect Model 3 to be better than Model 6. In Model 3, the firm has information about the length of disruption as soon as the disruption occurs, but not in

Model 6 where the firm only knows about the length of the current phase. Information on the length of recovery phases maybe practical for the firm to update its recovery plan. However, advance information on the disruption length before the occurrence of disruptions maybe more useful for a better mitigation plan. The model with better optimal ordering policies can be identified as the one having lower costs, fewer items ordered from expensive onshore supplier and perhaps higher fill rates and lower inventory levels.

The structure of this section is as follows. We describe Model 6 and its assumptions in sections 6.3.1 and 6.3.2, followed by the formulation of the ordering decision problem under supply disruption via the DMDP in section 6.3.3. Then, in section 6.3.4, we present the transition probability values used when we were carrying out the numerical experiment. The results and findings are reported in sections 6.3.5, 6.3.6 and 6.3.7. Then, a comparison between Model 6 and Model 3 is discussed in section 6.3.8. Finally, the conclusion for Model 6 is presented in section 6.3.9.

6.3.1 Model Description

The firm seeks to split the order between the onshore supplier (or supplier N) and the offshore supplier (or supplier F), this time under the assumption that the firm has additional information about the phased recovery process at supplier F. As in Model 5, supplier N is always reliable and supplier F may face disruption. During normal operations, the firm can order from both suppliers. However, during the recovery process after disruption to supplier F, the firm can only order from supplier N. The length of disruption is unknown when a disruption occurs, but the firm has knowledge of a disruption from the outset of the occurrence of the disruption.

The Markov model of the recovery process at supplier F is as follows. The recovery process is assumed to consist of R phases. The phase of the recovery process is denoted by j where $j = 0, 1, \ldots, R$ where phase 0 represents normal operations. The duration of the phases are assumed to follow known probability distributions with finite support. Let W

denote the maximum duration of any phase of the recovery plan. Following the occurrence of a disruption, supplier F enters phase 1 of the recovery plan and the duration of the phase is determined. In other words, on entering phase 1 of recovery, the firm is aware how long the phase will last. On completion of phase 1, supplier F enters phase 2 of the recovery plan and the duration of this phase is determined. The recovery process continues in this way until, on completion of phase R, normal operations are restored at supplier F. As before it is assumed that the length of a period of normal operation of supplier F follows a geometric distribution. During disruption, the state of supplier F is represented by two integer variables: j the current phase of the recovery plan and k the number of periods remaining in the current phase of the recovery plan. For consistency with this representation, the state of supplier F during normal operations is represented by two zeros (i.e. $(0, 0)$). For a better understanding, the transitions between normal operation and phases of recovery for supplier F are illustrated in figure 6.28.

In figure 6.28, α represents the probability of supplier F failing due to disruption and $p_j(k)$ represents the probability that the duration of phase j of the recovery plan is k periods. Whenever supplier F is in state $(0, 0)$, the process either remains in state $(0, 0)$, with probability $1 - \alpha$, or moves to state $(1, k)$, with probability $\alpha p_1(k)$. From state j, k with $k > 1$, the process moves to state $(j, k - 1)$ with probability 1. When supplier F is in state $(j, 1)$, it is known that phase j of the recovery plan will end in the next period. If $j < F$, then the recovery process proceeds to the next phase and the state of the supplier moves to state $(j + 1, k)$ with probability $p_{j+1}(k)$. While if $j = 0$, the recovery is completed in the next period and the state of supplier F moves to $(0, 0)$. The formulation of the ordering decision problem is presented in the next section.

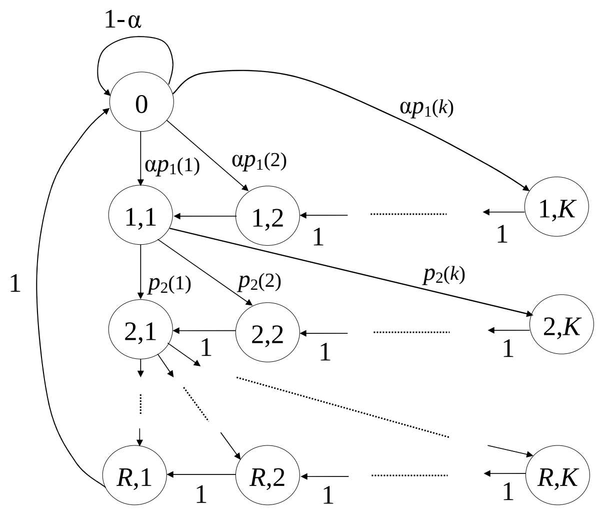


Figure 6.28. Markov chain model of the recovery process in Model 6

6.3.2 Model Assumptions

The assumptions of Model 6 are as follows:

a. The option of sourcing from supplier F is at risk of disruptions. The order from supplier F arrives either in full or not at all. However, the status of supplier N is perfectly reliable.

b. The length of disruption is unknown when a disruption occurs, but the firm has knowledge of a disruption from the outset of the occurrence of the disruption.

c. The recovery process is assumed to consist of R phases, but the firm has no information about the duration of the recovery phases.Therefore, the duration of the phases are assumed to follow known probability distributions with finite support

d. The length of a period of normal operation of supplier F follows a geometric distribution.

e. The firm's inventory planning horizon is discrete.

f. Demand is either deterministic or stochastic. The stochastic demand distribution follows the truncated Poisson distribution, $P(d_t) \sim Pois(\lambda, K)$.

g. Customers do not accept backorders, thus the firm encounters lost sales. The firm is charged with a penalty cost for each unit of demand that cannot be satisfied in a period, $PNLTY$.

h. The firm incurs a holding cost for inventory held during period t, $HOLD$.

6.3.3 Model Formulation

In this section, we explain the formulation of the problem analysed in Model 6 as a DMDP model and present the optimality equation.

Components of the DMDP for Model 6

The components of DMDP for Model 6 are as follows:

Decision Epochs

A decision is taken at the beginning of each period. Let t denote the decision epoch when there are t periods remaining in the planning horizon, $t = 1, 2, \ldots, T$.

States

At each decision epoch, the firm reviews the inventory level, i, and state of supplier F, (j, k) with $j \in \{0, 1, \ldots, F\}$ and $k \in \{0, 1, \ldots, W\}$. The parameters, j and k comprise the state of the process y, such that $y = (i, j, k)$. The state space, Y, of Model 6 is given by:

$$Y = \{(i, j, k) : \; i \in \{0, 1, \cdots, I\}, \; j \in \{0, 1, \cdots, F\}, \; k \in \{0, 1, \cdots, W\}, \; j = 0 \Leftrightarrow k = 0\}$$

Actions

Based on the current state, the firm then decides on the quantity to be ordered from supplier N and supplier F. The action is denoted by $b = (q^N, q^F)$ and the set of admissible actions,

$B(y)$ is given by:

$$B(i,0,0) = \left\{(q^F,q^N): \quad q^F,q^N \geq 0 \quad \& \quad q^F+q^N \leq I-i\right\} \quad \text{for} \quad 0 \leq i \leq I.$$

$$B(i,j,k) = \left\{(0,q^N): \quad q^N \in \{0,......,I-i\}\right\} \text{ for } 0 \leq i \leq I, \;\; 0 < j \leq F \text{ and } 0 < k \leq W.$$

Under the admissible action set of $B(i,0,0)$, the firm can choose to order up to $I-i$ items from supplier N only or from supplier F only or from both the suppliers. Whilst during recovery of supplier F, under the admissible action set of $B(i,j,k)$, the decision is to place an order for up to $I-i$ items with supplier N only.

Transition probabilities

We model changes in the inventory level and changes in the states of supplier F, separately. The transition matrix describing changes in the inventory level depends on the order quantities and is the same as in previous models. See section 4.2.3a for a full description. The transition matrix describing changes in the state of supplier F follows from figure 6.28 above. The transition matrix is denoted by X and is formally presented below.

$$X = \begin{array}{c} \\ (0,0) \\ (1,1) \\ (1,2) \\ \vdots \\ (F,1) \\ (F,2) \\ \vdots \\ (F,W) \end{array} \begin{array}{c} \begin{array}{cccccccccc} (0,0) & (1,1) & (1,2) & \cdots & (2,1) & (2,2) & \cdots & (F,1) & (F,2) & \cdots & F,W \end{array} \\ \begin{pmatrix} 1-\alpha & \alpha p_1(1) & \alpha p_1(2) & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & p_2(1) & p_2(2) & \cdots & 0 & 0 & \cdots & 0 \\ 0 & 1 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 1 & \cdots & 0 \\ 0 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \\ \vdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots & \vdots & \cdots & \vdots \\ 1 & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 & 0 & \cdots & 0 \end{pmatrix} \end{array}$$

One-step costs:

The one-step cost as a result of action b in state y consists of the ordering cost, $ORDER$, the holding cost, $HOLD$ and the penalty cost, $PNLTY$. In the one-step cost with stochastic demand, the values of $HOLD$ and $PNLTY$ depend on the random demand during the period. The one-step costs for Model 6 with the constant and stochastic demand settings are based on the same assumptions as the one-step costs for the models in previous chapters. See section 3.2.1 for a detailed explanation.

The one-step cost is denoted by $C_t^y(b)$ and this cost under a constant demand setting is given by:

$$\begin{aligned} C_t^y(b) &= ORDER + HOLD + PNLTY \\ &= \sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \; h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\ &\quad + \; m\Big(\max(D_t - i - q^N, 0)\Big) \end{aligned}$$

and the one-step cost under a stochastic demand setting is given by:

$$\begin{aligned} C_t^y(b) &= ORDER + \Big(E_{D_t}\Big(HOLD + PNLTY\Big)\Big) \\ &= \sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t)\Big\{h\Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) \\ &\quad + \; m\Big(\max(d_t - i - q^N, 0)\Big)\Big\} \end{aligned}$$

Optimality equation

Let $V_t(y)$ denote the minimum cost over the remainder of the planning horizon when the process is in state y at decision epoch t. The optimality equation for Model 6 with constant demand is given by:

$$
\begin{aligned}
V_t(i, j, k) = \min_{b \in B(y)} \Bigg\{ & \sum_{sp \in \{N,F\}} (\delta(q^{sp}) c^{sp} + q^{sp} v^{sp}) + m\Big(\max(D_t - i - q^N, 0)\Big) \\
& + h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\
& + X_{(j,k),(0,0)} V_{t-1}\Big(\max(i + q^N - D_t, 0) + q^F, 0, 0\Big) \\
& + \sum_{z=1}^{F} \sum_{\ell=1}^{W} X_{(j,k),(z,\ell)} V_{t-1}\Big(\max(i + q^N - D_t, 0) + q^F, z, \ell\Big) \Bigg\}
\end{aligned}
$$

Similarly, the optimality equation for Model 6 with stochastic demand is given by:

$$
\begin{aligned}
V_t(i, j, k) = \min_{b \in B(y)} \Bigg\{ & \sum_{sp \in \{N,F\}} (\delta(q^{sp}) c^{sp} + q^{sp} v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t) \Big\{ m\Big(\max(d_t - i - q^N, 0)\Big) \\
& + \Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) \\
& + X_{(j,k),(0,0)} V_{t-1}\Big(\max(i + q^N - d_t, 0) + q^F, 0, 0\Big) \\
& + \sum_{z=1}^{F} \sum_{\ell=1}^{W} X_{(j,k),(z,\ell)} V_{t-1}\Big(\max(i + q^N - d_t, 0) + q^F, z, \ell\Big) \Big\} \Bigg\}
\end{aligned}
$$

Using these equations, we seek to minimise $V_t(y)$ and to find the optimal quantities to be ordered from supplier N and supplier F, depending on the recovery phases and the rates of transition between recovery phases in the transition matrix, X. We are particularly interested in carrying out numerical experiments to investigate how the values in this transition matrix affect the firm's ordering policy..

6.3.4 Choice of Parameters Values

In this section, we present various transition probability values used for the numerical analysis. Our objectives are to analyse how the optimal policy changes with different transition probabilities and to identify better optimal ordering policies for Models 6 and 3. In this numerical study, the offshore supplier is assumed to have one up state, three phases in the recovery plan, thus $j = 3$ and each individual recovery phase can last for up to four periods. For the experiment, we consider four scenarios of the length of recovery phases as follows. For a better understanding, the number of periods in each phase and $p_j(k)$ for all scenarios are presented in table 6.6.

In the first scenario, we assume that each phase is up to two periods long, thus $W = 2$ in each phase. In the second and third scenarios, the periods increase and decrease, respectively, with increases in phase. Hence, the maximum number of periods in these cases vary in each phase. The idea of the second and third scenarios are conceptually linked with the idea of the disruption information availability for the firm.

In the second scenario, the firm has very limited access to disruption information, the firm is uncertainty about the length of later phases. The progress to learn about the disruption during the recovery process is slower as the offshore supplier has little idea of the cause of disruption and of what needs to be done to recover. Hence, the number of periods increases in later phases.

In the third scenario, there is an assumption that the firm has access to disruption information and, as such there should be more certainty about the length of later phases. The progress to learn about the disruption during the recovery process is faster as the offshore supplier has a better idea of the cause of disruption and of what needs to be done to complete recovery. Hence, the number of periods decreases in later phases.

In the final scenario, the idea is linked to the idea of a structured recovery planning process. In this scenario, the firm takes the first phase as an initial state to assess the impact of the disruption on the optimal policy. The assumption in this case is that the firm has detail information on the disruption. Hence, the first phase is typically short. Then, the corrective actions on the impact of the disruption are implemented in the second phase which takes longer a longer time. In this phase, these corrective actions may involve errors during the actions implementation, difficulties to be solved and requirement for the firm to learn any new discovery about the disruption. Hence, there can be more periods in this phase. Finally, in the final phase, the firm will be initialising normal operation upon the completion of the recovery plan, with an assumption that this action it already stated in the firm's SCRM procedure. Hence, there will be few periods in this final phase. Overall, in the final scenario, the maximum number of periods is lower in the first and last phases but higher in the second phase. We define the probability that the offshore supplier requires k periods to be $p_j(k)$.

To do the experiment, we combine various probabilities of supplier F failing due to disruption, α, and the scenarios of the length of recovery phases, and produce a set of 20 cases. We number and numerate the cases according to α values and the scenarios of the recovery length, respectively, as illustrated in table 6.8. For example for case 1A, number **1** is used to represent the corresponding values of α, which are tabulated in table 6.7. For a comparison of Model 6 and Model 3, in this Model 6, we use the same α values so as to be consistent with the α cases considered in the analysis of Model 3. Letter **A** is used to represent the set of recovery length scenarios, which are tabulated in table 6.6.

Similar to the analysis in Model 5, we are also interested in examining the impacts of the values of the expected length of an interval of normal service, $1/\alpha$, the expected length of recovery phases (or the expected length of disruption), $\bar{A}$, the proportion of time for which the offshore supplier is up, π_0, respectively, on the optimal policy. These values are calculated by using the same formulae' of $\bar{A}$, π_0, as shown in section 6.2.4 and tabulated in table 6.8.

In this analysis, we also make a comparison between Model 6 and Model 3. Several new distributions of the overall length of disruption are constructed for additional experiment in Model 3 and these distributions correspond to the scenarios of the length of recovery phases in the analysis of Model 6. To produce these probability distributions, we consider all possible combinations of lengths of phases in the recovery plan to produce a series of possible disruption scenarios, $A_1, A_2, \ldots, A_J$.. Let $L(A_i)$ be the length of scenario A_i and $P(A_i)$ be the probability of scenario A_i. These values can be calculated as $L(A_i) = k_1 + k_2 + ... + k_R$ and $P(A_i) = p_1(k_1)p_2(k_2)...p_R(k_R)$ for some $k_1, k_2, ..., k_R$. The calculation is illustrated for scenarios A to D in tables 6.10 to 6.13. From these values, it is possible to calculate the probability that the disruption length is j periods as $\sum_{i:L(A_i)=j} P(A_i)$ and construct the distribution of disruption length for new scenario of the disruption length in Model 3. These distributions are tabulated in table 6.9 and to avoid confusion in interpreting the outcome of this additional experiment in Model 3, we use small letters in this table to represent the scenarios of the disruption lengths.

The additional experiment for Model 3 is conducted with various combination of values of α cases and new constructed scenarios on the length of disruption and produces a set of 20 cases, which similar to the set of case in Model 6. The cases of α are considered from table 6.7 and disruption length scenarios from table 6.9.

In what follows, we first present results on the effects of the cases on the properties of the ordering decisions, then results relating to the effects on the properties of the costs of policies, and finally the results on the effects of the fill rate and the average inventory under the stochastic demand model analysis. We also make a comparison in these three areas of analyses between Model 6 and Model 3. Note that, the analyses in Model 6 focus on the infinite horizon plan model for both constant and stochastic demand settings.

Table 6.6. The values of the probability of recovery length in each phase.

Case	**Phases, j**									
	1			2				3		
	length, k			**length, k**				**length, k**		
	1	2	3	1	2	3	4	1	2	3
A	0.3	0.7	-	0.5	0.5	-	-	0.6	0.4	-
B	1	-	-	0.4	0.6	-	-	0.5	0.3	0.2
C	0.5	0.3	0.2	0.4	0.6	-	-	1	-	-
D	1	-	-	0.4	0.3	0.2	0.1	1	-	-

Table 6.7. The input α values

Case	1	2	3	4	5
α	0.1	0.3	0.5	0.7	0.9

Table 6.8. The values of $1/\alpha$, $\bar{A}$ and π_u in each case.

Case	$1/\alpha$	$\bar{A}$	π_u
1A	10.00	12.93	0.40
2A	3.33	12.93	0.20
3A	2.00	12.93	0.10
4A	1.43	12.93	0.10
5A	1.67	12.93	0.10
1B	10.00	15.50	0.40
2B	3.33	15.50	0.20
3B	2.00	15.50	0.10
4B	1.43	15.50	0.10
5B	1.67	15.50	0.10
1C	10.00	15.50	0.40
2C	3.33	15.50	0.20
3C	2.00	15.50	0.10
4C	1.43	15.50	0.10
5C	1.67	15.50	0.10
1D	10.00	22.83	0.30
2D	3.33	22.83	0.10
3D	2.00	22.83	0.10
4D	1.43	22.83	0.10
5D	1.67	22.83	0.10

Table 6.9. The values of transition probability for new scenarios in Model 3.

Case	Length of disruption, j					
	1	2	3	4	5	6
a	0	0	0.09	0.36	0.41	0.14
b	0	0	0.20	0.42	0.26	0.12
c	0	0	0.20	0.42	0.26	0.12
d	0	0	0.40	0.30	0.20	0.10

Table 6.10. The values of probability of possible recovery length in scenario a.

Possible scenario	1	2	3	4	5	6	7	8
Phase 1	1	1	1	1	2	2	2	2
Phase 2	1	1	2	2	1	1	2	2
Phase 3	1	2	1	2	1	2	1	2
length of disruption, A_i	3	4	4	5	4	5	5	6
Probability, $P(A_i)$	0.09	0.06	0.09	0.06	0.21	0.14	0.21	0.14

Table 6.11. The values of probability of possible recovery length in scenario b.

Possible scenario	1	2	3	4	5	6
Phase 1	1	1	1	1	1	1
Phase 2	1	1	1	2	2	2
Phase 3	1	2	3	2	2	3
Length of disruption, A_i	3	4	5	4	5	6
Probability, $P(A_i)$	0.2	0.12	0.08	0.3	0.18	0.12

Table 6.12. The values of probability of possible recovery length in scenario c.

Possible scenario	1	2	3	4	5	6
Phase 1	1	1	2	2	3	3
Phase 2	1	2	1	2	1	2
Phase 3	1	1	1	1	1	1
Length of disruption, A_i	3	4	4	5	5	6
Probability, $P(A_i)$	0.2	0.3	0.12	0.18	0.08	0.12

Table 6.13. The values of probability of possible recovery length in scenario d.

Possible scenario	1	2	3	4
Phase 1	1	1	1	1
Phase 2	1	2	3	4
Phase 3	1	1	1	1
Length of disruption, A_i	3	4	5	5
Probability, $P(A_i)$	0.4	0.3	0.2	0.1

6.3.5 The Impact of Different Transition Probabilities on the Ordering Decision

In this section, we explain how various transition probabilities values in each case can affect the properties of firm's ordering decision. The discussion covers the infinite-horizon model under the constant and stochastic demand settings (later known as M6*InfCons* and M6*InfSto*, respectively). The analyses of the constant and stochastic demand models are reported in sections 6.3.5a and 6.3.5b respectively.

The optimal ordering policy of M6*InfCons*

If supplier F is in state u.

An optimal order placed with supplier N in this model is similar to Model 5 and models in the previous chapters. The firm will only place the order with this supplier if there is an immediate shortage (i.e., $i < 5$). In this situation, the quantity ordered from supplier N is just enough to meet the immediate shortage (i.e., $5 - i$). This is illustrated in figure 6.29. However, the optimal order placed with the offshore supplier depends on the supply disruption probability and the length of recovery phases scenarios. From figure 6.30, in case 2 ($\alpha = 0.3$), scenario A (i.e., equal length in each recovery phases) is different from scenarios B, C and D. In scenario A, the firm orders a bigger quantity order. Scenarios B, C and D are very similar with only minor differences on the inventory level at which the firm starts to order. In general, the point at which the firm starts to order depends on the pattern of the length of recovery for each phases.

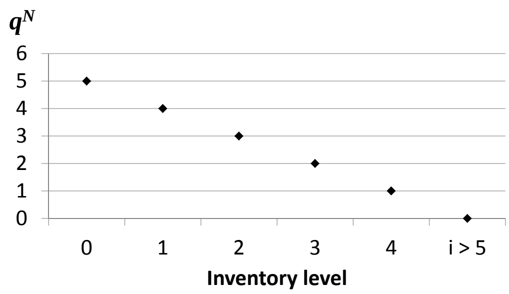


Figure 6.29. M6*InfCons*, state u: The optimal order from supplier N.

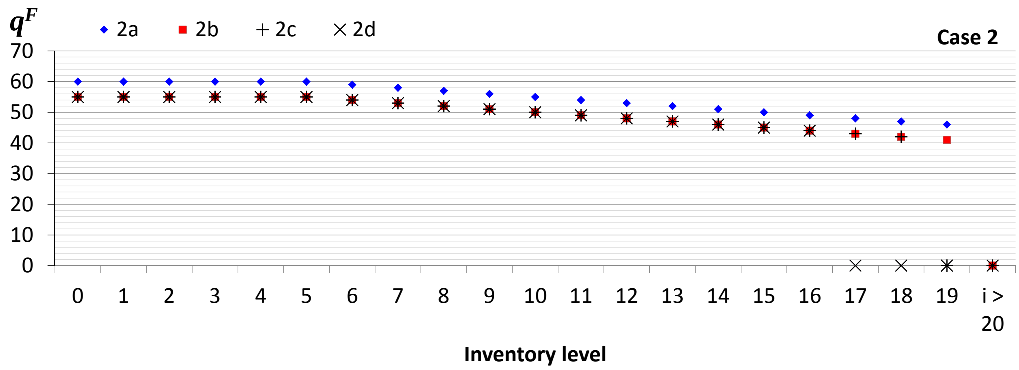


Figure 6.30. M6*InfCons*, state u: The optimal order from supplier F.

Table 6.14 shows the optimal ordering policy from supplier F (i.e., (s, S) policy) in the 20 cases considered. From this table, we see that the incentive to order from supplier F decreases as α increases. As we expected, the firm keeps a higher stock of cheap items from the offshore supplier when the chance of the offshore supplier remaining up is getting slim and when the expected length of disruption is increasing. The policy in scenario of length in recovery phases increasing (scenario B) is similar as to in scenario of length in recovery phases decreasing (scenario C). The reorder point, s, in these scenarios are slightly lower than the reorder point in scenario A (i.e., the length in each recovery phase is equal). When $\alpha \geq 0.5$, the reorder point, s, and the order up-to level, S, in scenario D are slightly lower than in other scenarios. Overall, the firm keeps higher stock in case A. This suggests that the firm is better able to plan inventory purchases during disruption in scenario A (where the recovery length in each phase is equal) compare to other scenario, especially as in scenario D (a policy with lower reorder point and order up-to level when the risk of disruption increases).

Table 6.14. M6*InfCons*: Optimal order from supplier F

Scenario	α				
	1	**2**	**3**	**4**	**5**
A	(9,55)	(19,65)	(22,65)	(24,65)	(24,65)
B	(9,55)	(19,60)	(21,65)	(23,65)	(24,60)
C	(9,55)	(18,60)	(21,65)	(23,65)	(24,60)
D	(9,55)	(16,60)	(20,60)	(22,60)	(23,60)

If the normal operation at the offshore supplier is expected to be longer, it is optimal for the firm to carry less cheap items from this supplier, as illustrated in figure 6.31. From this figure, we can see a non-linear relationship between the order up-to level, S, and the expected normal, π_u, where S decreases as π_u increases.

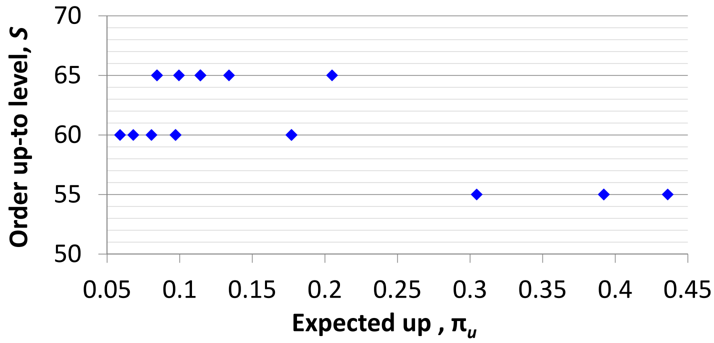


Figure 6.31. M6*InfCons*: The relationship between the order up to level and the expected up time

If supplier F is in state w.

The order quantity from supplier N is not influenced much by the supply disruption probability (α). However, figure 6.32 shows how the quantity ordered from this supplier depends on the phase of the recovery plan in scenario C and one case of α. The quantity ordered from supplier N decreases as the length of the recovery phases increases. As we expected, the firm will only place an order with this supplier if there is an immediate shortage (i.e., $i < 5$). These findings suggest that the firms would benefit from having information on the length of the recovery phases. For example, in this case, it tells the firm to reduce the quantity ordered from the onshore supplier upon the completion of the recovery phases.

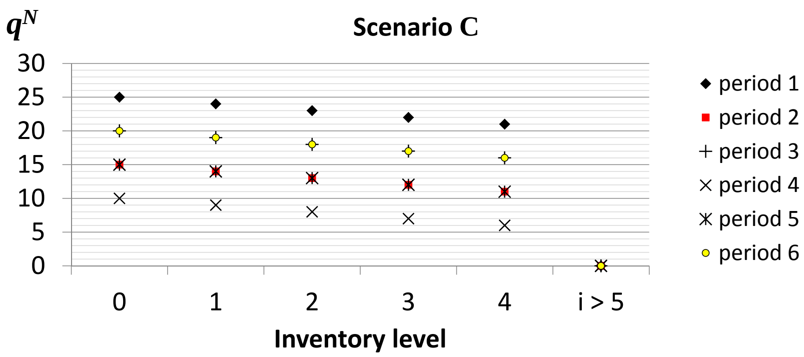


Figure 6.32. M6*InfCons*: The optimal order from supplier N during the recovery phase process for each case.

Let us now consider optimal ordering from supplier N as an (s, S) policy. The order policy from supplier N for all cases of supply disruption probability, α, and all scenarios of the length of recovery phases are tabulated in table 6.15. This table shows that the supply disruption probability has a minor influence on the order policy. For example, consider the order quantity placed with supplier N when recovery is in period 2. If recovery process is in period 2 and scenario C, the order quantity from supplier N decreases with an increase in α values and, in each α case, the quantity ordered from supplier N varies, depending on the scenario of the length of recovery phases. Overall, lower order quantity is in scenario D for all α cases.

Table 6.15. M6*InfCons*: Optimal order from supplier N

recovery period, j	scenario	α				
		1	**2**	**3**	**4**	**5**
1	A	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
	B	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
	C	(4,25)	(4,20)	(4,20)	(4,20)	(4,20)
	D	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
2	A	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
	B	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	C	(4,30)	(4,25)	(4,25)	(4,25)	(4,25)
	D	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
3	A	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
	B	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
	C	(4,30)	(4,30)	(4,30)	(4,30)	(4,30)
	D	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
4	A	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
	B	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)
	C	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	D	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
5	A	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)
	B	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	C	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
	D	(4,30)	(4,30)	(4,30)	(4,30)	(4,30)
6	A	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	B	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
	C	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)
	D	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)

The optimal ordering policy of M6*InfSto*

If supplier F is in state u.

An optimal order placed with the onshore supplier, supplier N, in this model is similar to to the constant demand model. The firm will only place the order with supplier N if the inventory level is very low. In this situation, the quantity ordered from supplier N is just enough to meet the immediate shortage (i.e., $5 - i$). This is illustrated in figure 6.33. However, the optimal order placed with the offshore supplier, supplier F, varies from case to case, depending on the supply disruption probability and the length of recovery phases scenario. From figure 6.34, in case 4 ($\alpha = 0.7$), scenario A (i.e., equal length in each recovery phases) is different from scenarios B, C and D. In scenario A, the firm orders a bigger quantity order. scenarios B, C and D are very similar with only minor differences on the inventory level at which the firm starts to order. In general, the point at which the firm starts to order depends on the pattern of the length of recovery for each phases.

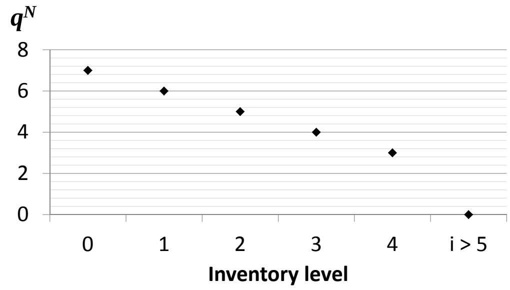


Figure 6.33. M6*InfSto*, stste u: The optimal order from supplier N.

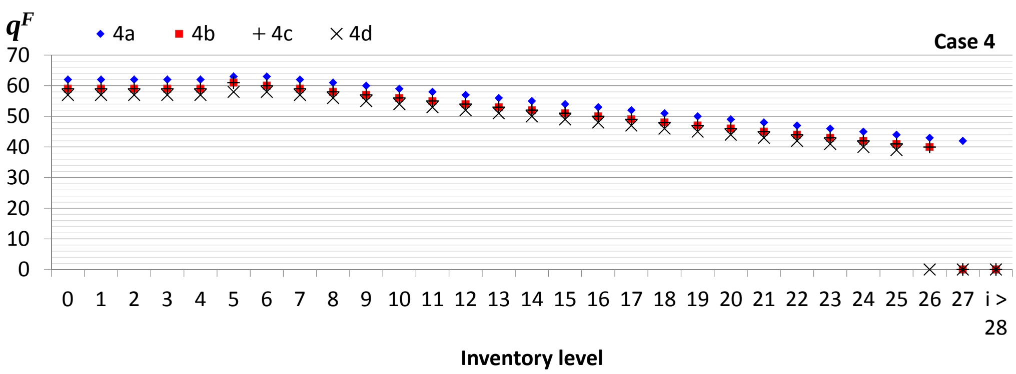


Figure 6.34. M6*InfSto*, state u: The optimal order from supplier F.

The results suggest that the optimal ordering policy from the offshore supplier is the (s, S) policy. Table 6.16 shows how the parameters vary in the 20 cases considered. From this table, we see that the incentive to order from supplier F increases as α increases (except when $\alpha > 0.7$). As we expected, the firm keeps a higher stock of cheap items from the offshore supplier when the chance of the offshore supplier remaining up is getting slim and when the expected length of disruption is increasing. The policy in scenario of length in recovery phases increasing (scenario B) is similar to as in scenario of length in recovery phases decreasing (scenario C). The reorder point in these scenarios are slightly lower than the reorder point in scenario A (i.e., the length in each recovery phase is equal). When $\alpha \geq 0.5$, the reorder point and order up-to level in scenario D are slightly lower than in scenario A. Overall, the firm keeps a slightly lower stock in case D. This suggests that the firm is better able to plan inventory purchases during disruption in scenario A (where the recovery length in each phase is equal) compared to other scenario, especially as in scenario D (a policy with lower reorder point and order up-to level when the risk of disruption increases). The pattern of optimal ordering from the offshore supplier during normal operation in this model is the same as in the constant demand model.

Table 6.16. M6*InfCons*: Optimal order from supplier F

α	Scenario				
	1	**2**	**3**	**4**	**5**
A	(9,50)	(19,60)	(22,60)	(24,60)	(24,60)
B	(9,50)	(19,55)	(21,60)	(23,60)	(24,55)
C	(9,50)	(18,55)	(21,60)	(23,60)	(24,55)
D	(9,50)	(16,55)	(20,55)	(22,55)	(23,55)

From figure 6.35, the value of S decreases when π_u increases. From the findings, we can see that it is optimal for the firm to carry less cheap items from this supplier if the expected normal operation at the offshore supplier is longer. The relationship between S and π_u is a non-linear. The pattern of this relationship is similar as in the constant demand model.

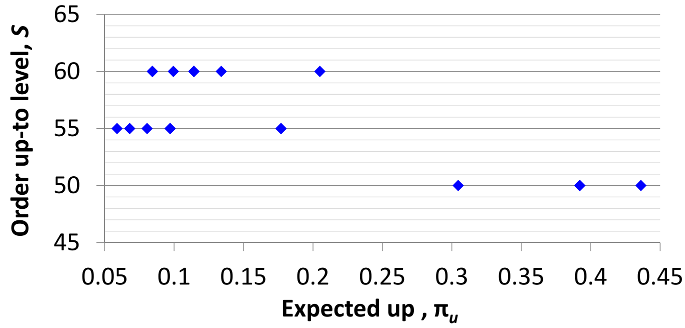


Figure 6.35. M6*InfSto*: The relationship between the order up to level and the expected up time.

If supplier F is in state w.

The order quantity from the onshore supplier depends on the scenario of the length in recovery phases. However, it is not influenced much by the supply disruption probability (α). This is illustrated in figure 6.36. In scenario A, from this figure, the quantity ordered from supplier N decreases as the length of the recovery phases increases. As we expected, the firm will only place order if there is an immediate shortage (i.e., $i < 8$). Similar to the constant demand model, we can see that information on the length of the recovery phases is useful for the firm as it can reduce the quantity ordered from the onshore supplier upon the completion of recovery phases.

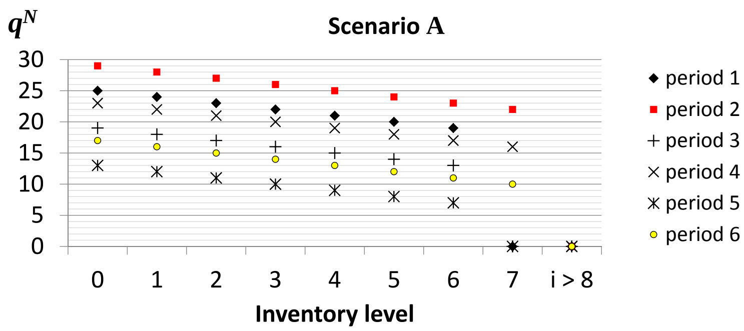


Figure 6.36. M6*InfSto*: The optimal order from supplier N during the recovery phase process for each case.

Let us now consider the case where the firm has an (s, S) policy for ordering from supplier N the order policy from supplier N for all cases of supply disruption probability, α, and all scenarios of the length of recovery phases are tabulated in table 6.17. This table shows that the supply disruption probabilities do not influence the order policy. However, the order quantity from supplier N does depend on the scenario of the length of recovery phase. In each recovery period, the policy varies, from scenario to scenario. For example, in recovery period 3, the policy in scenario A is slightly better than the policies in other scenarios, where the firm carry less items from expensive onshore supplier.

Discussion

During normal operation, in both constant and stochastic demand models, the properties of optimal ordering policy from the onshore and offshore supplier are not much different from other optimal policies in Model 5. In on hand, it is optimal for the firm to place order with the onshore supplier only if the inventory is very low and the quantity order is used to fill up the immediate shortage. In the other hand, the order policy from the offshore supplier depends on the risk of supply disruption and the scenario of the length of recovery phases. It is optimal to order more from the offshore supplier if the risk of supply disruption is increased. We also discover that ordering policy from the offshore supplier in scenario D is better than policies in other scenarios. With a structured and detail action plan in the recovery process as in scenario D, this plan can help the firm to have better optimal policy in managing its inventory during the disruption.

During crisis operation, the firm just can rely on the order from the onshore supplier. The quantity ordered from this supplier varies on the recovery period for all the length of recovery phases scenarios. The firm will carry more items from the supplier when the recovery period increases. However, upon the completion of the recovery process (i.e., recovery process is at the maximum recovery period), the firm will decrease the order. As we expected, the firm will carry less items from the onshore supplier at the end of the recovery process since it has a chance to place an order with cheaper offshore supplier in the next purchase.

Table 6.17. M6*InfSto*: Optimal order from supplier N

recovery period, j	scenario	α				
		1	**2**	**3**	**4**	**5**
	A	(7,25)	(7,25)	(7,25)	(7,25)	(7,25)
	B	(7,26)	(7,26)	(7,26)	(7,26)	(7,25)
	C	(7,24)	(7,24)	(7,24)	(7,24)	(7,24)
1	D	(7,24)	(7,24)	(7,24)	(7,24)	(7,24)
	A	(7,29)	(7,29)	(7,29)	(7,29)	(7,29)
	B	(7,19)	(7,19)	(7,19)	(7,19)	(7,19)
	C	(7,28)	(7,28)	(7,28)	(7,28)	(7,28)
2	D	(7,17)	(7,17)	(7,17)	(7,17)	(7,17)
	A	(7,19)	(7,19)	(7,19)	(7,19)	(7,19)
	B	(7,23)	(7,23)	(7,23)	(7,23)	(7,23)
	C	(7,32)	(7,32)	(7,32)	(7,32)	(7,32)
3	D	(7,22)	(7,22)	(7,22)	(7,22)	(7,22)
	A	(7,23)	(7,23)	(7,23)	(7,23)	(7,23)
	B	(7,13)	(7,13)	(7,13)	(7,13)	(7,13)
	C	(7,17)	(7,17)	(7,17)	(7,17)	(7,17)
4	D	(7,26)	(7,26)	(7,26)	(7,26)	(7,25)
	A	(7,13)	(7,13)	(7,13)	(7,13)	(7,13)
	B	(7,17)	(7,17)	(7,17)	(7,17)	(7,17)
	C	(7,22)	(7,22)	(7,22)	(7,22)	(7,22)
5	D	(7,31)	(7,31)	(7,31)	(7,31)	(7,31)
	A	(7,17)	(7,17)	(7,17)	(7,17)	(7,17)
	B	(7,22)	(7,22)	(7,22)	(7,22)	(7,22)
	C	(7,13)	(7,13)	(7,13)	(7,13)	(7,13)
6	D	(7,13)	(7,13)	(7,13)	(7,13)	(7,13)

6.3.6 The Impact of Different Transition Probabilities on the Long-run Average Costs

In this section, we explain how various transition probabilities values in each case can affect the properties of the long-run average costs under the infinite-horizon Model 6, which covered the experiment with the constant and stochastic demand settings.

From figure 6.37, the pattern of long-run average costs, g, in each α case is the same across the length of recovery phases scenarios in both constant and stochastic demand models. The highest g occurs in scenario A and the lowest g occurs in scenario D. The values of g in scenarios B and C are the same. From the findings, we can see that varied values of g are more influenced by different scenarios in the length of recovery in each phase, but not the supply disruption probability. To reduce the cost, the firm can suggests the offshore supplier to has better structured recovery plan, as in scenario D, rather than start making plan on the recovery based on the observation on the current disruption event, as in scenarios B and C.

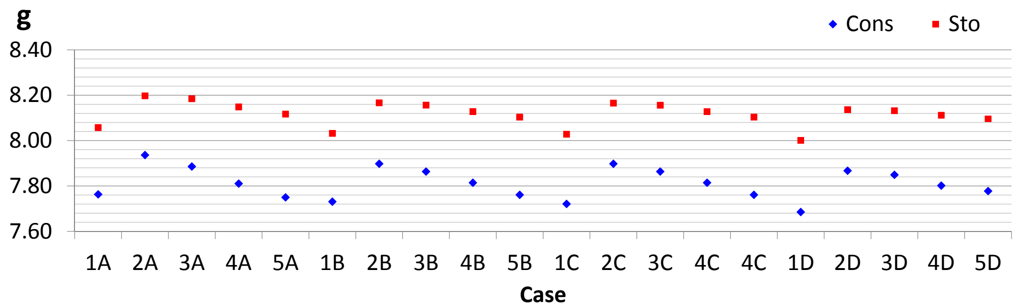
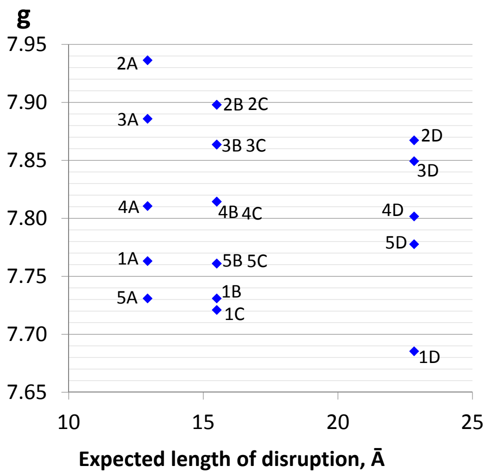
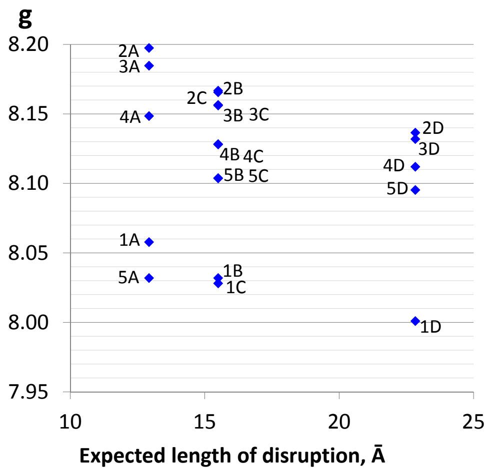


Figure 6.37. M6*Inf*: Optimal long-run average cost in different cases for constant and stochastic demand models

Figure 6.38 illustrates the relationships between the long-run average cost, g, and the expected length of disruption, $\bar{A}$. In the constant demand model, from figure 6.38a, in general there is no relationship between g and $\bar{A}$. However, we can see an approximately linear relationship between g and $\bar{A}$ if we consider the relationship by α cases. In each α case, the values of g decreases with an increase in $\bar{A}$, except in case 5 ($\alpha = 0.9$). From the findings, we can see that the long-run average cost decreases when the expected length of disruption increases (except in lower supply disruption probability cases, $\alpha < 0.3$). In the stochastic demand model, figure 6.38b, we can see a similar relationship between the long-run average cost, g, and the expected length of disruption, $\bar{A}$, as in the constant demand model. In each α case, g decreases with an increase in $\bar{A}$. In both constant and stochastic demand models, as we expected, the firm will face lower cost when the expected disruption length at the offshore supplier is shorter.



(a) M6*InfCons*: g *vs.* $\bar{A}$



(b) M6*InfSto*: g *vs.* $\bar{A}$

Figure 6.38. M6*Inf*: The relationship between the long-run average cost and the expected length of disruption

6.3.7 The Impact of Different Transition Probabilities on the Performance of the Policies

In this section, we discuss the performance of the ordering policy under the infinite horizon plan and stochastic demand, focussing on the performance of the fill rate (section 6.3.7a) and the average inventory level (section 6.3.7b).

Fill rate

From figure 6.39, the rate at which the firm can fill up customer's demand from the existing inventory in all cases are estimated to lie between 99.76% and 99.78% with 95% of confidence interval. From this figure, we can see that higher fill rates occur mostly in case 1 (i.e., α = 0.1) and lower fill rates occur mostly in case 5 (i.e., $\alpha = 0.9$). This is logical since the increase in the risk of supply disruption can decrease the fill rate values. Nonetheless, the variation in fill rate values from the simulation run in each case is still within the range of good performance for the firm, with the fill rate values in all cases being more that 99.70%. In scenario A (i.e., equal length in each recovery phase), the fill rate increases if $\alpha < 0.5$ but decreases when $\alpha \geq 0.5$. The same pattern of fill rate values occur in scenarios B and C (i.e., the length of recovery phases are increasing or decreasing) across α values. However, in scenario D, the fill rate decreases as the supply disruption probability increases. From the findings, we can conclude that supply disruption probability and various pattern of length of recovery phases can affect the fill rate.

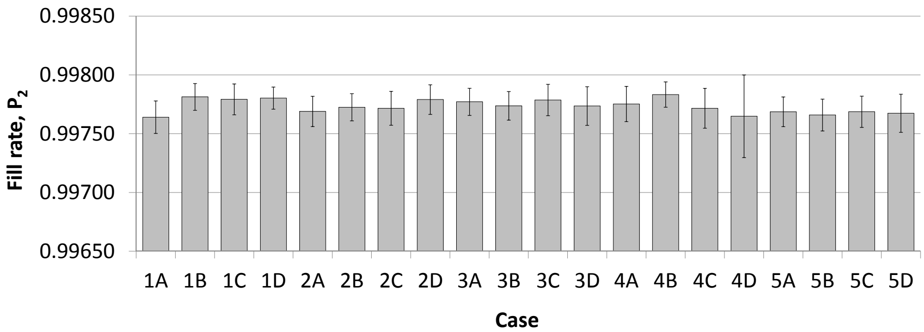
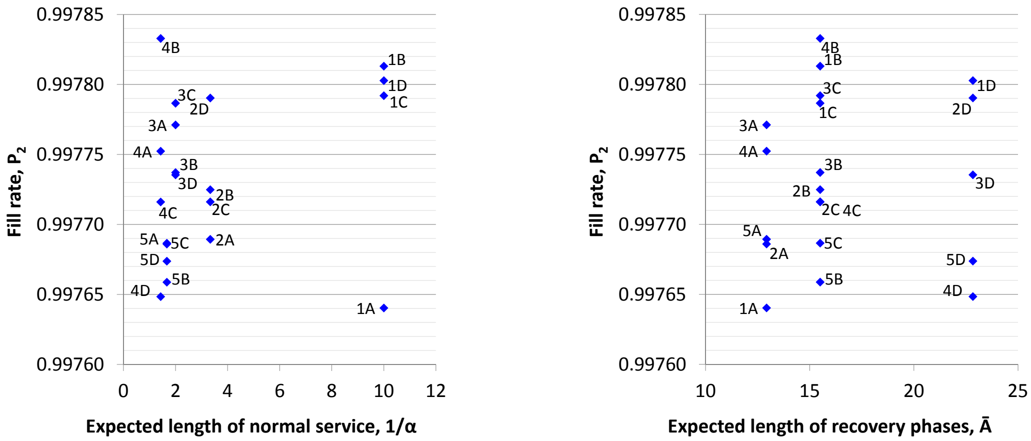


Figure 6.39. M6: Fill rate in each case

If we examine the association between the fill rate and the expected length of normal service and recovery phases at the offshore supplier, the fill rate values, P_2, have approximately linear relationships with the expected length of normal service, $1/\alpha$, and the expected length of recovery phases, $\bar{A}$. This is illustrated in figure 6.40. From figure 6.40a, if we exclude case 1 (i.e.,$\alpha = 0.1$) from this plot , P_2 increases as $1/\alpha$ increases. From figure 6.40b, if we exclude scenario D, P_2 also increases as $\bar{A}$ increases.

There is also no relationship between the fill rate values and the expected up time, π_u, as illustrated in figure 6.41.



(a) P_2 $vs.$ $1/\alpha$ (b) P_2 $vs.$ $\bar{A}$

Figure 6.40. M6: The relationship between fill rate and the expected lengths of normal service and the recovery phases.

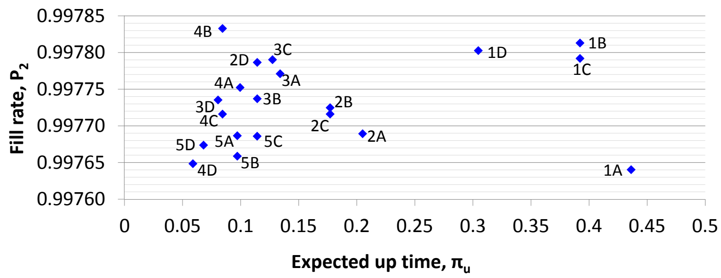


Figure 6.41. M6: The relationship between fill rate and the expected up time

Average inventory level

From figure 6.42, the average inventory level, I_A, in all scenarios are estimated to lie between 31.77 and 38.20 with 95% confidence interval which lie at almost half of the maximum inventory level. In each scenario, I_A increases as α increases when $\alpha < 0.5$ and vice versa when $\alpha \geq 0.5$. As we expected, if the risk of supply disruption increases, the firm will carry more stock in the inventory. The average inventory level, I_A, has negative relationships with the expected length of normal service, $1/\alpha$, and the expected length of recovery phases, $\bar{A}$. The decreases of I_A with the increases of $1/\alpha$ and $\bar{A}$ are non-linear. This is illustrated in figure 6.43. From figure 6.43a, I_A decreases as $1/\alpha$ increases. From figure 6.43b, if we exclude case 1 (i.e., $\alpha = 0.1$) from this plot, I_A also increases as $\bar{A}$ increases. The average inventory, I_A, also has a negative relationship with the expected up time, π_u. I_A $vs.$ decreases as π_u increases (see figure 6.44).

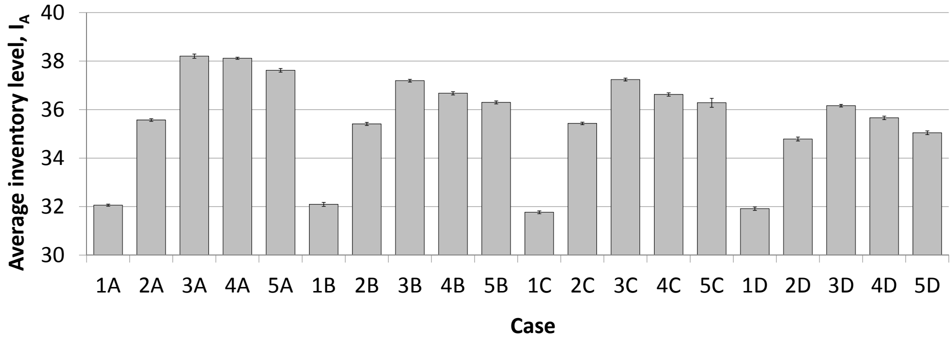
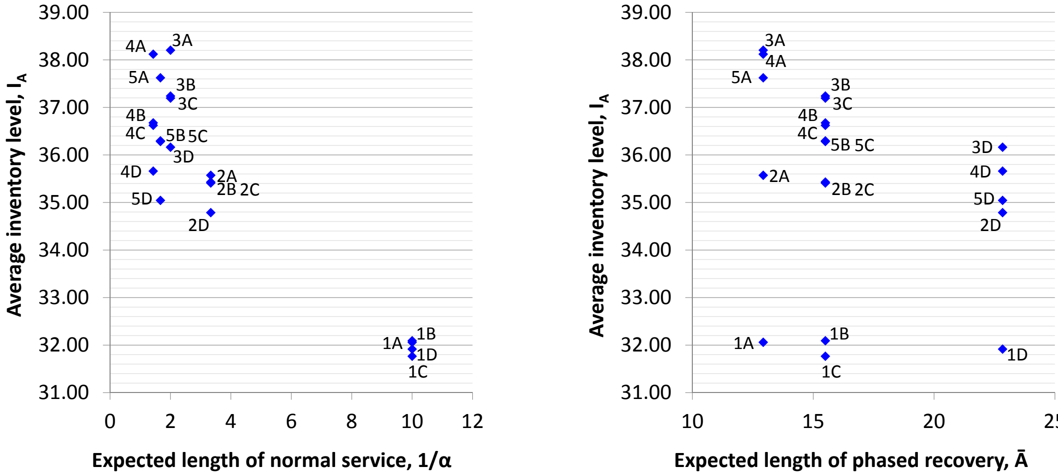


Figure 6.42. M6: Average inventory level in each recovery length scenario for all cases.



(a) I_A *vs.* $1/\alpha$ (b) I_A *vs.* $\bar{A}$

Figure 6.43. M6: The relationship between average inventory level and the expected normal service and the expected length of recovery phases

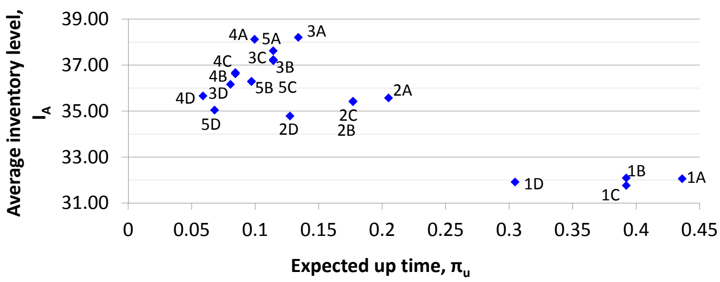


Figure 6.44. M6: The relationship between average inventory level and the expected up time

Discussion

In Model 6 analyses, similar with the analyses of Model 5, the performance of the firm's ordering policy has been measured with the values of fill rate and average inventory level. Based on the fill rate, the firm still has high capability to satisfy demand from the customer when it chooses to use the optimal ordering policies. The percentage of demand satisfied from stock in hand is still high even though there are high supply disruption probability. In most situations, the firm will keep stock which is half of the maximum inventory level. We also discover that the fill rate and the average inventory level have no relationship with the expected length of normal service and the expected length of disruption at the offshore supplier.

6.3.8 On the Value of Additional Information on the Length of Recovery Phases

In the previous section, we mention that Model 6 has been developed in a similar way of Model 3. The only difference between these two model is about information on the length of disruption (in Model 3) and the length of recovery phases (in Model 6). Nevertheless, we expect Model 3 to be better than Model 6. Model 3 has information on the length of disruption as soon as the disruption occurs, while in Model 6, it only knows about the length of the current phase. It maybe useful for the firm to have information about the length of recovery phases, however, we think that having information before the occurrence of disruptions is more useful for the firm for a better supply disruption mitigation plan. Therefore, in this section, we conduct a comparison of Model 3 and Model 6 to identify a model with better optimal ordering policies. The better model is selected based on several conditions, which are fewer items ordered from expensive onshore supplier, lower long-run average costs, and perhaps higher fill rates and lower inventory levels. These findings are presented in sections 6.3.8a, 6.3.8b and 6.3.8c.

The comparison on the optimal ordering decision between Model 6 and Model 3

This section focuses on the order quantity placed with the onshore supplier, q^N, when the offshore supplier is down. The difference of q^N between Model 6 and Model 3 is examined to compare which model has better optimal ordering decision. The (s, S) policy is use to find the difference q^N between these two models and we refer to the parameter of order up-to level, S, as a reference to identified which model will carry less items from the onshore supplier.

From figures 6.45 and 6.46, in the constant and stochastic demand models, a positive value of difference of order up-to level between Model 6 and Model 3 (i.e., diffq^NM6-q^NM3), shows that optimal ordering policy in Model 3 is better than Model 6 and vice versa if the difference is a negative value. From these figures, both Model 6 and Model 3 carry fewer items from the onshore supplier, depending on the length of recovery phases scenarios. For example, if the length is in the first half of the recovery process ($k < 3$), the ordering decision in Model 3 is better than in Model 6. However, the ordering decision in Model 6 is better than in Model 3 in the second half of the recovery process ($k \geq 3$). From the findings, we can say that information on the disruption length before the occurrence of the disruption, as in Model 3 maybe useful for the firm in the beginning of recovery process at the offshore supplier. However, information on the length of recovery phases, as in Model 6 is more useful after some times in the recovery process in order to complete the recovery plan. Therefore, we can conclude that both Models 3 and 6 have better ordering decision, which depend on the length of recovery phases.

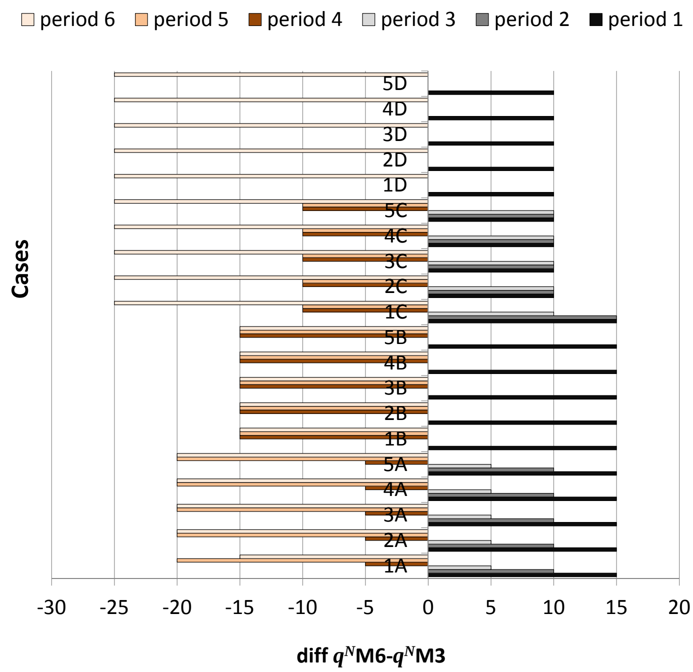


Figure 6.45. The difference of order up-to level between Model 6 and Model 3 in the constant demand model.

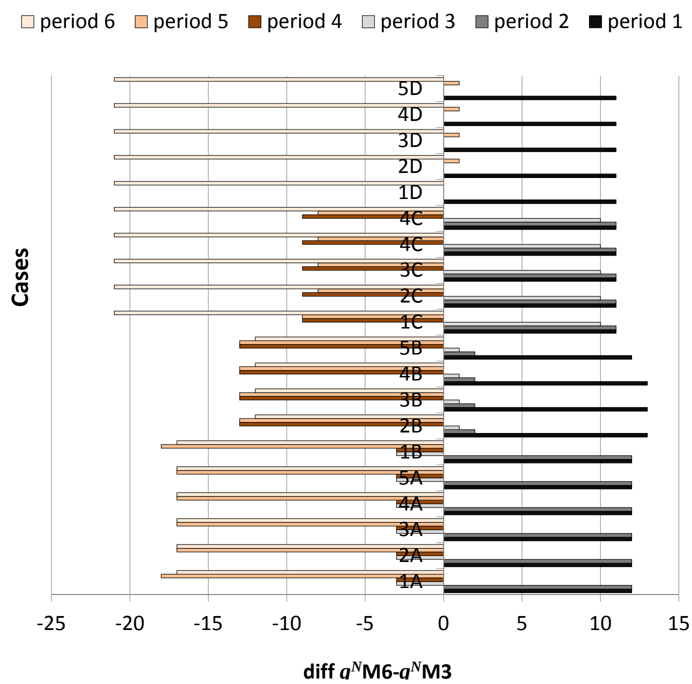


Figure 6.46. The difference of order up-to level between Model 6 and Model 3 in the stochastic demand model.

The comparison on the long-run average cost between Model 6 and Model 3

In this section, the difference of the long-run average costs between Model 6 and Model 3 is examined to compare which model to perform better. The value of the difference of long-run average costs in Model 6 and Model 3 is define as $gM6 - gM3$. If the value of $gM6 - gM3$ is positive ($gM6 - gM3 > 0$), then we can say that Model 3 performs better than Model 6 and vice versa if the value of $gM6 - gM3$ is negative ($gM6 - gM3 < 0$).

From figure 6.47, in both the constant and stochastic demand models, we can see that the values of $gM6 - gM3$ are positive in all α cases, especially in scenarios A, B and C. However, there is no difference in scenario D. Therefore, we can conclude that Model 3 performs better than Model 6. Consistent with the assumption made on the value of information in Model 6 and Model 3, assessable information on the disruption length as soon as the disruption occurs at the offshore supplier is more valuable than just having information on the length of recovery phases.

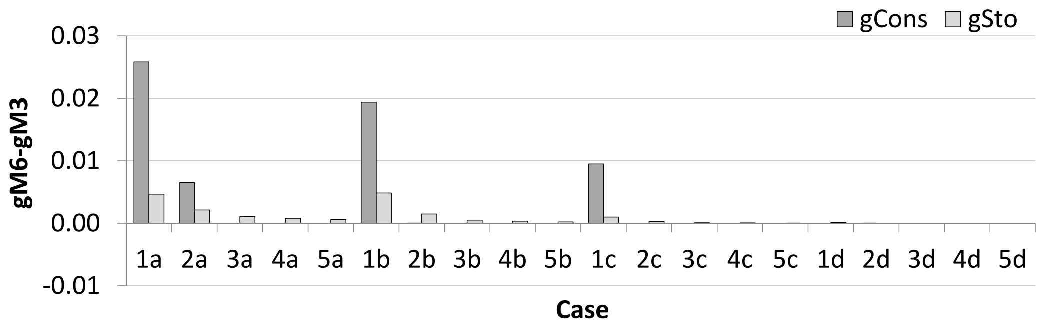


Figure 6.47. The difference of long-run average cost between Model 6 and Model 3 in each case.

The comparison on the performance of the fill rate and the average inventory level between Model 6 and Model 3

In this section, the performance of Model 6 as compared to Model 3 is analysed by examining the differences of fill rate and average inventory level values between Model 6 and Model 3 (i.e., FillRateM6-FillRateM3 and AvgLvlM6-AvgLvlM3) and the differences are illustrated in figures 6.48 and 6.49. We assume that the higher fill rate and the lower average inventory level are the better. If the differences of fill rate and average inventory level between Model 6 and Model 3 are negative and positive, respectively, then, Model 6 performs better than Model 3.

In the difference of fill rate analysis, from figure 6.48, we can see that in most cases, the difference of the fill rate values between Model 6 and Model 3 are negative. The negative values show that Model 3 performs better that Model 6, especially in scenario A (i.e., equal length of recovery in each phase). Information about the disruption length before the occurrence of the disruption, as in Model 3 is more valuable for the firm than just knowing the length of recovery phases, as in Model 6. As we expected, information in Model 3 can help the firm to increase the fill rate values in some α cases and recovery length scenarios.

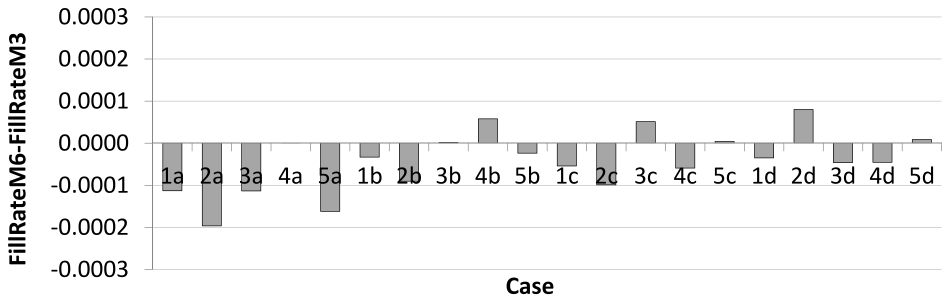


Figure 6.48. The difference of fill rate values between Model 6 and Model 3 in each case.

In the difference of average inventory level analysis, from figure 6.49, the difference values are negative in scenario A and positive in scenario D for most α cases. In scenario A (i.e., constant length in each recovery phase), the finding shows the firm will carry less stock in the inventory if there is information about the length of the current recovery phase, as in Model 6, rather than just having information about the length of disruption as soon as the disruption occurs, as in Model 3. However, additional information about the disruption length before the disruption has occurred (as in Model 3) is more valuable in scenario D (i.e., the scenario where the recovery plan takes longer period during the corrective action than the period during the assessment on the disruption event and the initialisation of normal operation upon the completion of the recovery process). Under this scenario, the firm will carry less inventory in Model 3 than in Model 6. From the findings, we can say that Model

6 performs better under the condition of equal length in each recovery phase (scenario A) and Model 3 performs better under the condition of structured recovery plan (scenario D). We can conclude that the performance of the average inventory level depends more on the process of the recovery plan (i.e., the recovery length scenarios) than the information on the length of disruption and recovery phases.

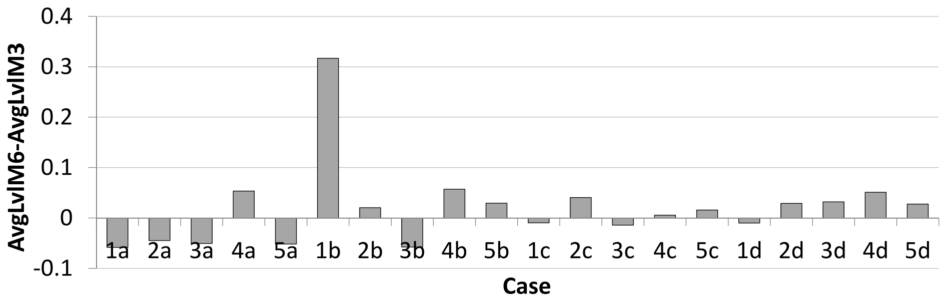


Figure 6.49. The difference of average inventory level values between Model 6 and Model 3 in each case.

The comparison on the fill rate and the average inventory level between Model 6 and Model 3 are also examined by conducting a statistical inference test of two samples *t*-test (assuming unequal variance) for all cases. The null hypothesis in the fill rate and the average inventory level *t*-tests state that there are no difference between the average of fill rate and the average of average inventory level in Model 6 and the average of fill rate and the average of average inventory level in Model 3. These tests have been conducted at a significance level of 0.05 for all cases. Table 6.18 tabulates *p-values* in each case. From this table, in both the fill rate and the average inventory level tests, most of *p-values* > 0.05, thus we fail to reject the null hypothesis (except in case 2*a* in the fill rate *t*-test and 1*b* in the average inventory level *t*-test). Therefore, we can infer that there are no difference between the average of fill rate and the average of average inventory level in Model 6 and the average of average inventory level in Model 3.

Table 6.18. The values of *p-values* in *t*-tests in each case of the fill rate and the average inventory level.

Case	*p-values*	
	P_2	I_A
1a	0.2210	0.0672
2a	0.0310	0.2642
3a	0.2028	0.2241
4a	0.9925	0.1511
5a	0.0795	0.2273
1b	0.6910	0.0000
2b	0.2408	0.6406
3b	0.9815	0.1395
4b	0.4866	0.1982
5b	0.7900	0.3450
1c	0.5202	0.7516
2c	0.2617	0.2987
3c	0.5315	0.7122
4c	0.4820	0.9083
5c	0.9621	0.6706
1d	0.6302	0.7133
2d	0.3581	0.4456
3d	0.6530	0.4440
4d	0.6597	0.2138
5d	0.9255	0.4614

Discussion

A comparison between Model 6 and Model 3 in four different parameters, namely the quantity ordered from the onshore supplier when the offshore supplier is down, the long-run average cost, the fill rate and the average inventory level have showed that the policy in Model 3 is better than Model 6 (showed in the difference of the cost). However, the policy in Model 6 sometimes performs better than Model 3, depending on the pattern of length of recovery phases and the number of recovery length. From the findings, we can conclude that advance information about the disruption length and information about the length of recovery phases are useful for the firm in making plan its purchase from expensive onshore supplier in the event of supply disruption at the offshore supplier.

6.3.9 Conclusion

In Model 6, we addressed how the firm's ordering policies can be affected by the risk of disruptive supply event at the offshore supplier and the planning of recovery phases during the recovery process at this supplier. From the analyses of Model 6, with information about the length of recovery phases, on the observation of the firm, we demonstrate how the properties and the performance of the optimal ordering policy depend on the values of the transition probabilities of the Markov chain model of recovery length to the offshore supplier. From the findings, during the recovery process, we discover that the expected length of recovery phases have more impacts on the optimal order quantity from the onshore supplier, the minimum cost and the long-run average cost than the risk of supply disruption. During the recovery process, the firm will order fewer items from the onshore supplier if the offshore supplier has a structured and detail recovery plan, rather than making the plan based on the current state of the disruption. A comparison on the optimal policy shows that Model 3 is better than Model 6. However, the policy in Model 6 is better for the firm after a certain period of supply disruption. In a long supply disruption which may need a long recovery period, it is more useful for the firm to have information about the length of recovery phases. Therefore, we can conclude that, both information about lengths of supply disruption and recovery phases are useful for the firm in making better plan for its purchase during the recovery process to the offshore supplier.

6.4 Phased Recovery Model with Incomplete Disruption Information

Model 7 uses a modification of the recovery model in Model 6 which has more limited information about disruptions at the offshore supplier. More specifically, the firm has information on the probability distribution of the duration of each phase of the recovery plan, but does not know in advance how long any phase will last. The firm knows which phase of the recovery plan is being implemented and is able to observe the start and end of each phase.

This means that the firm can update its belief about the likely duration of each phase of the disruption. This is similar to the approach used in Model 4, to update the firm's belief about the duration of the disruption. As previously, the recovery process for Model 7 is modelled as a Markov chain and we investigate how the Markov chain model of the recovery process affects the firm's optimal inventory policy. We examine the impact of the rate of transition between phases of the recovery process on the ordering decision and the expected minimum inventory cost.

The structure of this section is as follows. We describe Model 7 and its assumption in sections 6.4.1 and 6.4.2, followed by the formulation of the ordering decision problem under supply disruption via the DMDP in section 6.4.3. Then, in section 6.4.4, we present the transition probability values used when conducting the numerical experiment. The results and findings are reported in sections 6.4.5, 6.4.6 and 6.4.7. Then, a comparison of Model 7 and Model 4 is discussed in section 6.4.8. Finally, the conclusion for Model 7 is presented in section 6.4.9.

6.4.1 Model Description

The firm seeks to split the order between the onshore supplier (or supplier N) and the offshore supplier (or supplier F), under the assumption of limited information about the phased recovery process at supplier F. During normal operations of supplier F, the firm can order from both suppliers. However, during the recovery process after disruption to supplier F, the firm can only order from supplier N. The length of disruption is unknown when a disruption occurs, but the firm has knowledge of a disruption from the outset of the occurrence of the disruption.

The Markov model of the recovery process at supplier F is as follows. The recovery process is assumed to consist of R phases and it is assumed that the firm knows the probability distributions of the lengths of the phases of the recovery plan. These probability distributions are assumed to have finite support. Let W be the maximum length of a phase of the recovery plan. The firm knows as soon as disruption occurs and is able to observe the recovery plan as it unfolds. This means that at each decision epoch, the firm knows whether or not there is disruption and, if there is, which phase of the recovery plan is being implemented and for how long this phase of the recovery plan has lasted. The state of supplier F is represented by two integer values, (j, k). The first represents the phase of the recovery process with 0 representing normal operations as earlier models (i.e., Models 5 and 6). The second represents the number of periods for which the current phase of the recovery plan has lasted. Following the occurrence of a disruption, supplier F enters state (1,1). It then moves through states (1,2), (1,3) and so on until phase 1 of the recovery plan is complete. At this point, supplier F enters state (2,1). The process continues in this way until phase supplier F of the recovery plan is completed and normal operations are restored (state (0,0)). We assume that the length of a period of normal operation of supplier F follows a geometric distribution. For a better understanding, the transitions between normal operation and states of disruption for supplier F are illustrated in figure 6.50.

In figure 6.50, α represents the probability of supplier F failing due to disruption and $h_j(k)$ represents the probability that the duration of phase j of the recovery plan is k periods, given that it is at least k periods. Whenever the state of supplier F is in state $(0, 0)$, the process either remains in state $(0, 0)$, with probability $1 - \alpha$, or moves to state $(1, 1)$, with probability α. When supplier F is in state (j, k) with j and k both greater than 0, the next period will be the last of phase j with probability $h_j(k)$. Hence, supplier F moves to state $(j + 1, 1)$ when $j < F$ or state $(0, 0)$ when $j = F$ next with probability $h_j(k)$. Otherwise, with probability $1 - h_j(k)$, supplier F moves to state $(j, k + 1)$. The formulation of optimal ordering decision problem is presented in the next section.

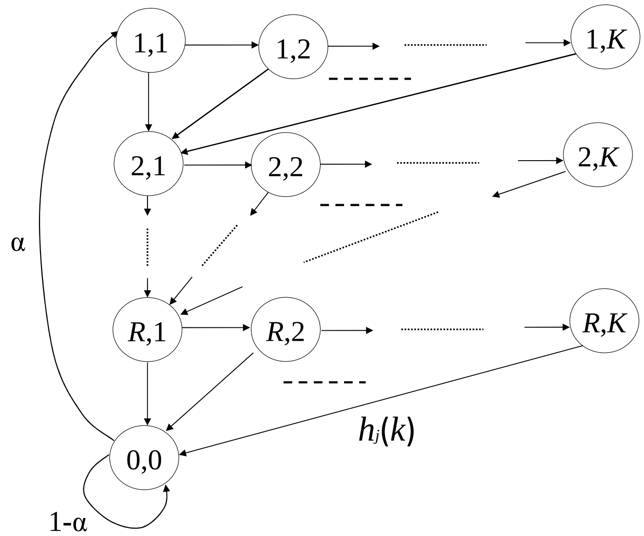


Figure 6.50. The transition structure of the Markov chain model of the recovery process of Model 7

6.4.2 Model Assumptions

The assumptions of Model 7 are as follows:

a. The option of sourcing from supplier F is at risk of disruptions. The order from supplier F arrives either in full or not at all. However, the status of supplier N is perfectly reliable.

b. The length of disruption is unknown when a disruption occurs, but the firm has knowledge of a disruption from the outset of the occurrence of the disruption. Once the disruption has occurred, the firm is able to observe the recovery plan as it unfolds. The firm knows whether or not there is disruption and, if there is, which phase of the recovery plan is being implemented and for how long this phase of the recovery plan has lasted.

c. The recovery process is assumed to consist of R phases, but the firm has no information about the duration of the recovery phases. However, the firm knows the probability distributions of the lengths of the phases of the recovery plan. These probability distributions are assumed to have finite support.

d. The length of a period of normal operation of supplier F follows a geometric distribution.

e. The firm's inventory planning horizon is discrete.

f. Demand is either deterministic or stochastic. The stochastic demand distribution follows the truncated Poisson distribution, $P(d_t) \sim Pois(\lambda, K)$.

g. Customers do not accept backorders, thus the firm encounters lost sales. The firm is charged with a penalty cost for each unit of demand that cannot be satisfied in a period, $PNLTY$.

h. The firm incurs a holding cost for inventory held during period t, $HOLD$.

6.4.3 Model Formulation

In this section, we explain the formulation of the problem analysed in Model 7 as a DMDP model and present the optimality equation.

Components of the DMDP for Model 7

The components of the DMDP for Model 7 are as follows.

Decision Epochs

A decision is taken at the beginning of each period. Let t denote the decision epoch when there are t periods remaining in the planning horizon, $t = 1, 2, \ldots, T$.

States

At each decision epoch, the firm reviews the inventory level, i, and state of supplier F, (j, k) with $j \in \{0, 1, \ldots, F\}$ and $k \in \{0, 1, \ldots, W\}$. The parameters, j and k comprise the state of the process y, such that $y = (i, j, k)$. The state space, Y, of Model 7 is given by:

$$Y = \{(i, j, k) : \; i \in \{0, 1, \cdots, I\}, \; j \in \{0, 1, \cdots, F\}, \; k \in \{0, 1, \cdots, W\}, \; j = 0 \Leftrightarrow k = 0\}$$

Actions

Based on the current state, the firm then decides on the quantity to be ordered from supplier N and supplier F. The action is denoted by $b = (q^N, q^F)$ and the set of admissible actions, $B(y)$ is given by:

$$B(i, R_0) = \left\{(q^F, q^N) : \quad q^F, q^N \geq 0 \quad \& \quad q^F + q^N \leq I - i\right\} \quad \text{for} \quad 0 \leq i < I.$$

$$B(i, R_j) = \left\{(0, q^N) : \quad q^N \in \{0, ..., I - i\}\right\} \quad \text{for} \quad 0 \leq i \leq I.$$

Under the admissible action set of $B(i, j, k)$, the firm can choose either to order from supplier N only or to order from supplier F only or to order from both the suppliers when i is in between 0 and $I - i$. Whilst during recovery of supplier F, under the admissible action set of $B(i, j, k)$, the decision is to place an order for up to $I - i$ items with supplier N only as follows.

Transition probabilities

We model changes in the inventory level and changes in the phases of recovery separately. The transition matrix describing changes in the inventory level depends on the order quantities and is the same as in previous models. See section 4.2.3a for a full description. The transition probabilities describing changes in the state of supplier F follow from figure 6.50 above and are formally presented below.

Let X denote the transition matrix for the Markov model of the state of supplier F. If supplier F is up (i.e., in state(0,0)), the supplier will enter the first period of the first phase of recovery (i.e., state (1,1)) if disruption occurs (with probability α) and will remain up otherwise. Hence:

$$X_{(0,0),(1,1)} = \alpha \quad \text{and} \quad X_{(0,0),(1,1)} = 1 - \alpha.$$

Let L_j be a random variable representing the length of phase j of the recovery plan and let

$h_j(k)$ denote the probability that the length of phase j of the recovery plan is exactly k periods given that it is at least k periods.

$$\begin{aligned} h_j(k) = \frac{Pr(L_j = k)}{Pr(L_j \geq k)} &= \frac{Pr(L_j = k)}{\sum_{\ell=k}^{\infty} Pr(L_j = \ell)} \\ &= \frac{Pr(L_j = k)}{1 - \sum_{\ell=0}^{k-1} Pr(L_j = \ell)} \end{aligned}$$

If supplier F is down and in the kth period of the jth phase of the recovery plan (i.e., in state (j, k)) and this period is not the last of phase j, then the supplier will enter the next period of phase j (i.e., state $(j, k + 1)$). Otherwise, if $j = F$, the supplier will return to normal operations (i.e., state (0,0)) or, if $0 < j < F$, will enter the first phase of the next phase of recovery (i.e. state $(j + 1, 1)$). Hence:

$$\begin{aligned} X_{(j,k),(j,k+1)} &= 1 - h_j(k) \text{ for } 0 < j \leq F, \\ X_{(F,k),(0,0)} &= h_j(k), \text{and} \\ X_{(j,k),(j+1,1)} &= h_j(k) \text{ for } 0 < j < F. \end{aligned}$$

One-step costs:

The one-step cost as a result of action b in state y consists of the ordering cost, $ORDER$, the holding cost, $HOLD$ and the penalty cost, $PNLTY$. In the one-step cost with the stochastic demand, the values of $HOLD$ and $PNLTY$ depend on the random demand during the period. The one-step costs for Model 7 with the constant and stochastic demand settings are based on the same assumptions as the one-step costs for the models in previous chapters. See section 3.2.1 for a detailed explanation.

The one-step cost is denoted by $C_t^y(b)$ and this cost under a constant demand setting is given by:

$$
\begin{aligned}
C_t^y(b) &= ORDER + HOLD + PNLTY \\
&= \sum_{sp\in\{N,F\}} (\delta(q_t^s)c^{sp} + q_t^s v^{sp}) + \; h\Big(\frac{1}{2}(i + \max(i + q_t^N - D_t, 0))\Big) \\
&\quad + \; m\Big(\max(D_t - i - q_t^N, 0)\Big)
\end{aligned}
$$

and under a stochastic demand setting is given by:

$$
\begin{aligned}
C_t^y(b) &= ORDER + \Big(E_d\Big(HOLD + PNLTY\Big)\Big) \\
&= \sum_{sp\in\{N,F\}} (\delta(q_t^s)c^{sp} + q_t^s v^{sp}) + \sum_{d_t=0}^{D} P(D_t = d_t)\Big\{h\Big(\frac{1}{2}(i + \max(i + q_t^N - d_t, 0))\Big) \\
&\quad + \; m\Big(\max(d_t - i - q_t^N, 0)\Big)\Big\}
\end{aligned}
$$

Optimality equation

Let $V_t(y)$ be the minimum cost over the remainder of the planning horizon when the process is in state y at decision epoch t.The optimality equation for Model 7 with constant demand is given by:

$$
\begin{aligned}
V_t(i, j, k) = \min_{b\in B(y)} \Big\{ & \sum_{sp\in\{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + m\Big(\max(D_t - i - q^N, 0)\Big) \\
& + h\Big(\frac{1}{2}(i + \max(i + q^N - D_t, 0))\Big) \\
& + X_{(j,k),(0,0)} V_{t-1}\Big(\max(i + q^N - D_t, 0) + q^F, 0, 0\Big) \\
& + \sum_{z=1}^{F} \sum_{\ell=1}^{W} X_{(j,k),(z,\ell)} V_{t-1}\Big(\max(i + q^N - D_t, 0) + q^F, z, \ell\Big)\Big\}
\end{aligned}
$$

Similarly, the optimality equation for Model 7 with stochastic demand is given by:

$$
\begin{aligned}
V_t(i, j, k) = \min_{b \in B(y)} \Bigg\{ \sum_{sp \in \{N,F\}} (\delta(q^{sp})c^{sp} + q^{sp}v^{sp}) + \sum_{d_t=0}^{\infty} P(D_t = d_t) \Big\{ m\Big(\max(d_t - i - q^N, 0)\Big) \\
+ \Big(\frac{1}{2}(i + \max(i + q^N - d_t, 0))\Big) \\
+ X_{(j,k),(0,0)} V_{t-1}\Big(\max(i + q^N - d_t, 0) + q^F, 0, 0\Big) \\
+ \sum_{z=1}^{F} \sum_{\ell=1}^{W} X_{(j,k),(z,\ell)} V_{t-1}\Big(\max(i + q^N - d_t, 0) + q^F, z, \ell\Big) \Big\} \Bigg\}
\end{aligned}
$$

Using these two equation, we seek to minimise $V_t(y)$ and to find the optimal quantities to be ordered from supplier N and supplier F, depending on the recovery phases and the rates of transition between recovery phases in the transition matrix, X. We are particularly interested in investigating by numerical experiment on how the values in this transition matrix can affect the firm's ordering policy.

6.4.4 Choice of Parameters

The objectives of the numerical study for Model 7 are to analyse how the optimal policy changes with different transition probabilities and to identify better optimal ordering policies between Model 7 and Model 4. The experimental design in this numerical study is similar to Model 5, thus a set of 20 cases based on various combinations of α and recovery phases scenarios are considered in this numerical study. The additional experiment in Model 4 for a comparison of Model 7 and Model 4 also considers the same experimental design as in Model 6. See section 6.3.4 for a detailed explanation. The difference between Model 7 and Model 6 is the values of transition probability due to different design of transition state in the Markov chain process. In Model 7, the firm knows that the recovery process will definitely end at the offshore supplier, but its not aware how long the recovery phase will last. Hence, the transition states for Model 7 depend the observe state on the recovery progress (the number of period for which it has lasted). In Model 7, transition probability values are calculated using formula in section 6.4.3a(iv).

In what follows, we first present results on the effects of the cases on the properties of the ordering decisions, then results relating to the effects on the properties of the costs of policies, and finally the results on the effects of the fill rate and the average inventory under the stochastic demand model analysis. We also make a comparison in these three areas of analyses between Model 7 and Model 4. Note that, the analyses in Model 7 focus on the infinite horizon plan model for both constant and stochastic demand settings.

6.4.5 The Impact of the Transition Probabilities on the Ordering Decision

In this section, we explain how various transition probabilities values in each case can affect the properties of firm's ordering decision. The discussion covers the infinite horizon model under the constant and stochastic demand settings (later known as M7*InfCons* and M7*InfSto*, respectively). The analyses of the constant and stochastic demand models are reported in sections 6.4.5a and 6.4.5b respectively.

The optimal ordering policy of M7*InfCons*

If supplier F is in state u.

The firm will only place the order with supplier N if there is an immediate shortage (i.e., $i < 5$). In this situation, the quantity ordered from supplier N is just enough to meet the immediate shortage (i.e., $5 - i$). This is illustrated in figure 6.51. In this model, an optimal order placed with this supplier is similar to Models 5 and 6 and models in the previous chapters. However, the quantity ordered from supplier F, depends on the supply disruption probability and the pattern of the length of recovery phases scenarios. From figure 6.52, in case 3 ($\alpha = 0.5$), scenario A (i.e., equal length in each recovery phases) are different from scenarios B, C and D. In scenario A, the firm orders a bigger quantity order. Scenarios B, C and D are very similar with only minor differences on the inventory level at which the firm starts to order. In general, the point at which the firm starts to order depends on the pattern of the length of recovery for each phases.

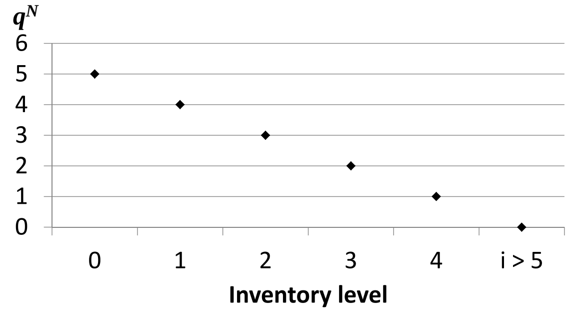


Figure 6.51. M7*InfCons*, state u: The optimal order from supplier N.

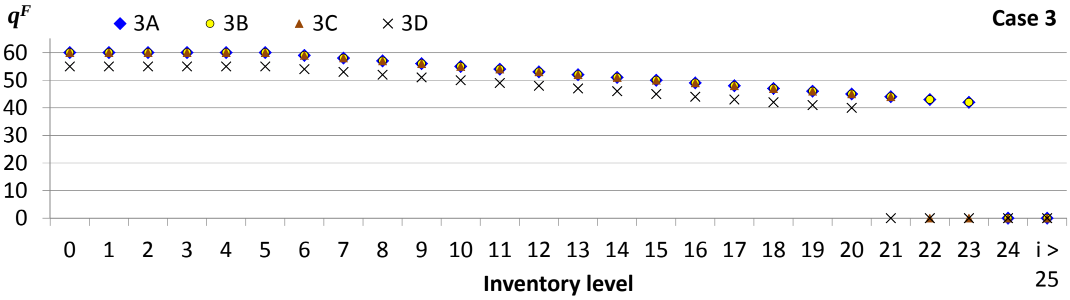


Figure 6.52. M7*InfCons*, state u: The optimal order from supplier F.

We assume the optimal ordering from supplier F as an (s, S) policy for the 20 cases considered and how these 20 cases vary with α and the pattern of the length of recovery scenarios are tabulated in table 6.19. From this table, we see that it is optimal for the firm to increase the quantity ordered from supplier F when α increases (except when $\alpha > 0.7$). As we expected, the firm carries a higher stock of cheaper items from the offshore supplier when the supply disruption probability is decreasing and the chance of the offshore supplier remaining up is getting higher. However, the incentive to order from this supplier decreases when the chance is very high and the probability is too low. The reorder point, s increases when α increases. It means that, the firm keeps lower stock but will not wait until the inventory level is too low before placing order with the offshore supplier when the risk of supply disruption is increasing. The firm keeps a slightly higher stock in scenarios A and B when $\alpha < 0.5$. This suggests that the firm is better able to plan inventory purchases during disruption in scenarios A and B (where the recovery length in each phase is equal or the expected length of phases increasing) compared to other scenario, especially as in scenario D (where the recovery process in this scenario is a structured recovery plan).

Table 6.19. M7*InfCons*: Optimal order from supplier F

Scenario	α				
	1	**2**	**3**	**4**	**5**
A	(9,50)	(19,60)	(23,60)	(24,60)	(24,60)
B	(9,50)	(19,60)	(23,60)	(24,60)	(24,55)
C	(9,50)	(19,55)	(21,60)	(23,60)	(24,55)
D	(9,50)	(18,55)	(20,55)	(22,55)	(23,55)

From figure 6.53, we can see that the order up-to level, S, decreases when the expected normal, π_u, increases. It means that it is optimal for the firm to carry less cheap items from the offshore supplier when the expected normal operating period of this supplier is increasing. The decreases in the order up-to level with an increase in the expected normal time is non-linear.

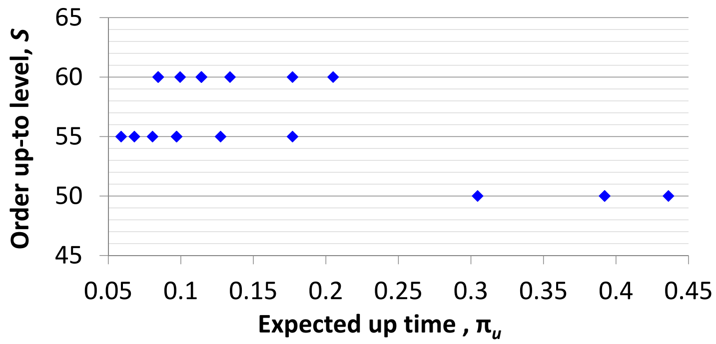


Figure 6.53. M7*InfCons*: The relationship between the order up to level and the expected up time

If supplier F is in state w.

Similar to optimal ordering policy in Model 6, the probability of supply disruption (α) does not affect much the order quantity from supplier N. However, figure 6.54 shows how the quantity ordered from supplier N depends on the phase of the recovery plan in case 1 (i.e.,α = 0.1) and scenario A. The quantity ordered from supplier N decreases as the length of the recovery phases increases. As we expected, the firm will only place order with this supplier if there is an immediate shortage (i.e., $i < 5$). From the finding, we can see that the information on the length of the recovery phases is practical for the firm. For example, in this case, it tells the firm to reduce the quantity ordered from the onshore supplier upon the completion of the recovery phases.

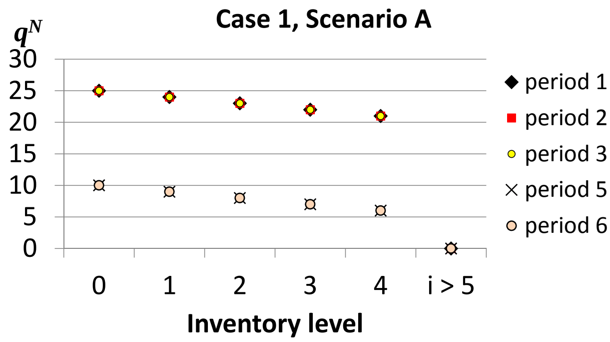


Figure 6.54. M7*InfCons*: The optimal order from supplier N during the recovery phase process for each case.

If we effectively consider optimal ordering from supplier N as an (s, S) policy, the order policy from supplier N for all cases of supply disruption probability, α, and all scenarios of the length of recovery phases are tabulated in table 6.20. This table shows that the supply disruption probability has a minor influence on the order policy. For example, consider the order quantity placed with supplier N when recovery is in period 2. If recovery process is in period 2 and scenario C, the order quantity from supplier N decreases with an increase in α values and, in each α case, the quantity ordered from supplier N varies, depending on the scenario of the length of recovery phases. Overall, lower quantity ordered from this supplier is in scenario D for all α cases.

Table 6.20. M7*InfCons*, state w: Optimal order from supplier N

recovery period, j	scenario	α				
		1	**2**	**3**	**4**	**5**
	A	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
	B	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
	C	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
1	D	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
	A	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
	B	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
	C	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
2	D	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	A	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
	B	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	C	(4,25)	(4,25)	(4,25)	(4,25)	(4,25)
3	D	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	A	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	B	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)
	C	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
4	D	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	A	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)
	B	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)
	C	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
5	D	(4,30)	(4,30)	(4,30)	(4,30)	(4,30)
	A	(4,15)	(4,15)	(4,15)	(4,15)	(4,15)
	B	(4,20)	(4,20)	(4,20)	(4,20)	(4,20)
	C	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)
6	D	(4,10)	(4,10)	(4,10)	(4,10)	(4,10)

The optimal ordering policy of M7*InfSto*

If supplier F is in state u.

An optimal order placed with supplier N, in this model is similar as to the constant demand model. The firm will only place the order with this supplier if the inventory level is very low. In this situation, the quantity ordered from the onshore supplier is just enough to meet the immediate shortage (i.e., $5 - i$). This is illustrated in figure 6.55. However, the optimal order placed with supplier F, varies from case to case, depending on the supply disruption probability and the length of recovery phases scenario. From figure 6.56, in case 2 ($\alpha = 0.3$), all scenarios are very similar with only minor differences on the inventory level at which the firm starts to order. In general, the point at which the firm starts to order depends on the pattern of the length of recovery for each phases.

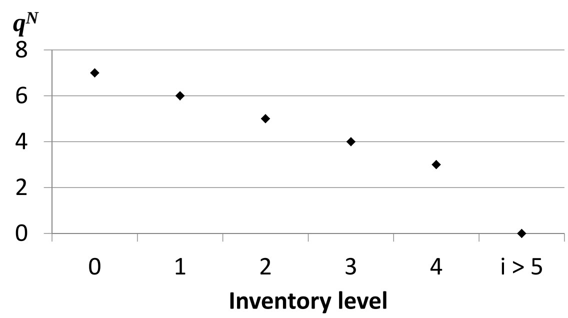


Figure 6.55. M7*InfSto*, state u: The optimal order from supplier N.

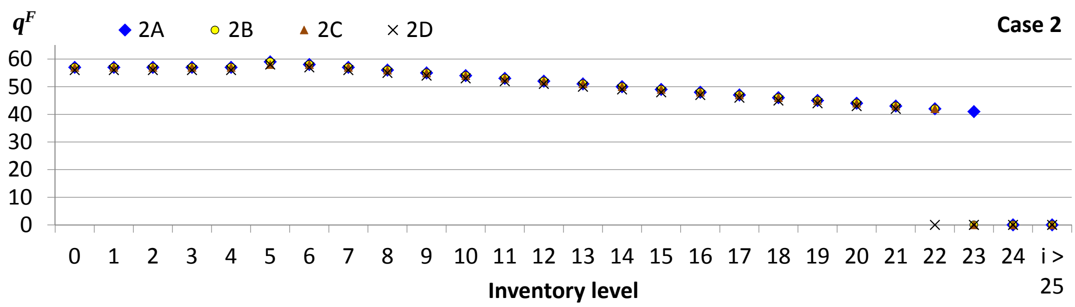


Figure 6.56. M7*InfSto*, state u: The optimal order from supplier F.

Table 6.21 shows how the parameters vary in the 20 cases considered. From this table, we see that the incentive to order from supplier F increases as α increases (except when $\alpha > 0.7$). As we expected, the firm keeps a higher stock of cheap items from the offshore supplier when the chance of the offshore supplier remaining up is getting slim. The policy in scenario of length in recovery phases increasing (scenario B) is similar as to scenario of length in recovery phases decreasing (scenario C). The reorder point in these scenarios are slightly lower than the reorder point in scenario A (i.e., the length in each recovery phase is equal). When $\alpha \geq 0.5$, the reorder point and order up-to level in scenario D are slightly lower than in scenario A. Overall, the firm keeps a slightly lower stock in case D. This suggests that the firm is better able to plan inventory purchases during disruption in scenario A (where the recovery length in each phase is equal) compared to other scenario, especially as in scenario D (a policy with lower reorder point and order up-to level when the risk of disruption increases). The pattern of optimal ordering in this model is the same as in the constant demand model.

Table 6.21. M7*InfCons*: Optimal order from supplier F

α	Scenario				
	1	**2**	**3**	**4**	**5**
A	(14,60)	(23,64)	(26,69)	(27,69)	(28,68)
B	(14,60)	(22,64)	(25,67)	(26,66)	(27,65)
C	(14,60)	(22,64)	(25,66)	(26,66)	(27,65)
D	(14,60)	(21,63)	(24,64)	(25,64)	(26,63)

From figure 6.35, when the normal operation at the offshore supplier is expected to be longer, it is optimal for the firm to carry less cheap items from this supplier. We can see a a non-linear relationship between S and π_u. The pattern of this relationship is similar as in the constant demand model.

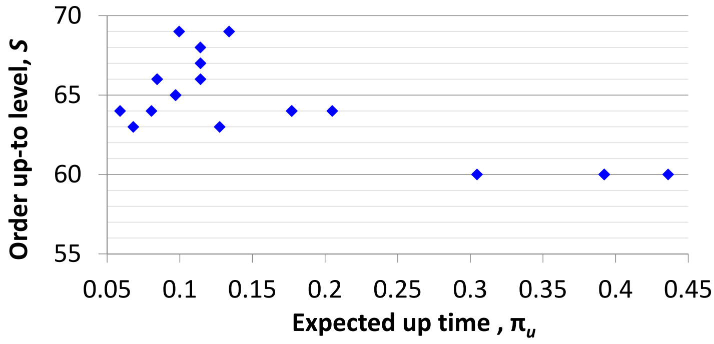


Figure 6.57. M7*InfSto*: The relationship between the order up to level and the expected up time.

If supplier F is in state w.

The order quantity from the onshore supplier depends on the scenario of the length in recovery phases. However, it is not influenced much by the supply disruption probability (α). This is illustrated in figure 6.58. In scenario C and case 3, the quantity ordered from supplier N decreases as the length of the recovery phases increases. As we expected, the firm will only place order if there is an immediate shortage (i.e., $i < 8$). Similar to the constant demand model, information on the length of the recovery phases is meaningful for the firm as it can reduce the quantity ordered from the onshore supplier at the end of recovery process at the offshore supplier.

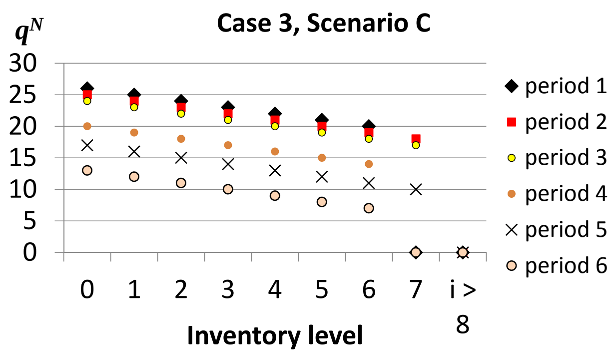


Figure 6.58. M7*InfSto*: The optimal order from supplier N during the recovery phase process for each case.

The optimal ordering policy (i.e., consider as an (s, S) policy) from the onshore supplier during recovery for all scenarios are tabulated in table 6.22. This table shows that the policies are not influence much by the supply disruption probability. However, it does depend on the scenario of the length of recovery phases. In each recovery period, the policy varies, from scenario to scenario. For example, in recovery period 2, the policy in scenario D is better than the policies in other scenarios, where the firm carries less items from expensive onshore supplier and the point at which it places order with this supplier is slightly lower. Perhaps, the firm needs to be more prepared and better able plan its inventory under the situation where the offshore supplier do not has a structured recovery plan.

Discussion

The properties of optimal ordering policy from the onshore and offshore supplier are not much different in both constant and stochastic demand models. During normal operation, the risk of supply disruption and the pattern of the length of recovery process affect the order policy with the offshore supplier than the onshore supplier. On the one hand, the firm places an order with the onshore supplier only if the inventory is very low and the quantity order is used to fill up the immediate shortage. On the other hand, it is optimal to order more from the offshore supplier when the risk of supply disruption increase. The ordering policy from the offshore supplier under a structured recovery plan (i.e., scenario D) is better than policies in other scenarios. With a structured and detail action plan in the recovery process at the offshore supplier, it can help the firm to has better optimal policy in managing its inventory in the event of the disruption.

During crisis operation, the firm only can place an order with the onshore supplier. The order quantity places with this supplier depends on the recovery period in each phase. The firm will carry more items from the supplier when the recovery period increases. However, it is optimal for the firm to decrease the order when recovery process is at the maximum recovery period. As we expected, the firm keeps lower stock from expensive onshore supplier upon the completion of recovery process at the offshore supplier since it has a chance to place an order with cheaper offshore supplier in the next purchase.

Table 6.22. M7*InfSto*: Optimal order from supplier N

recovery period, j	scenario	α 1	2	3	4	5
	A	(6,28)	(6,27)	(6,27)	(6,27)	(6,27)
	B	(7,26)	(7,26)	(7,26)	(7,26)	(7,25)
	C	(6,26)	(6,26)	(6,26)	(6,26)	(6,26)
1	D	(7,24)	(7,24)	(7,24)	(7,24)	(7,24)
	A	(7,25)	(7,25)	(7,25)	(7,25)	(7,25)
	B	(7,22)	(7,22)	(7,22)	(7,22)	(7,22)
	C	(7,25)	(7,25)	(7,25)	(7,25)	(7,25)
2	D	(6,20)	(6,20)	(6,20)	(6,20)	(6,20)
	A	(6,21)	(6,21)	(6,21)	(6,21)	(6,21)
	B	(7,19)	(7,19)	(7,19)	(7,19)	(7,19)
	C	(7,24)	(7,24)	(7,24)	(7,24)	(7,24))
3	D	(6,19)	(6,19)	(6,19)	(6,19)	(6,19)
	A	(7,19)	(7,19)	(7,19)	(7,19)	(7,19)
	B	(7,15)	(7,15)	(7,15)	(7,15)	(7,15)
	C	(6,20)	(6,20)	(6,20)	(6,20)	(6,20)
4	D	(6,18)	(6,18)	(6,18)	(6,18)	(6,18)
	A	(6,14)	(6,14)	(6,14)	(6,14)	(6,14)
	B	(6,14)	(6,14)	(6,14)	(6,14)	(6,14)
	C	(7,17)	(7,17)	(7,17)	(7,17)	(7,17)
5	D	(7,17)	(7,17)	(7,17)	(7,17)	(7,17)
	A	(6,13)	(6,13)	(6,13)	(6,13)	(6,13)
	B	(6,13)	(6,13)	(6,13)	(6,13)	(6,13)
	C	(6,13)	(6,13)	(6,13)	(6,13)	(6,13)
6	D	(6,13)	(6,13)	(6,13)	(6,13)	(6,13)

6.4.6 The Impact of the Transition Probabilities on the Long-run Average Costs

In this section, we explain how various transition probabilities values in each case can affect the properties of the long-run average costs under the infinite-horizon Model 6, which covered the experiment with the constant and stochastic demand settings. From figure 6.59, the pattern of long-run average costs, g, in each α case is the same across the length of recovery phases scenarios in both constant and stochastic demand models. The highest g occurs in scenario A and the lowest g occurs in scenario D. The values of g in scenarios B and C are the same. From the findings, we can see that varied values of g are more influenced by different scenarios in the length of recovery in each phase, but not the supply disruption probability. To reduce the cost, the firm can suggests the offshore supplier to has better structured recovery plan, as in scenario D, rather than start making plan on the recovery based on the observation on the current disruption event, as in scenarios B and C.

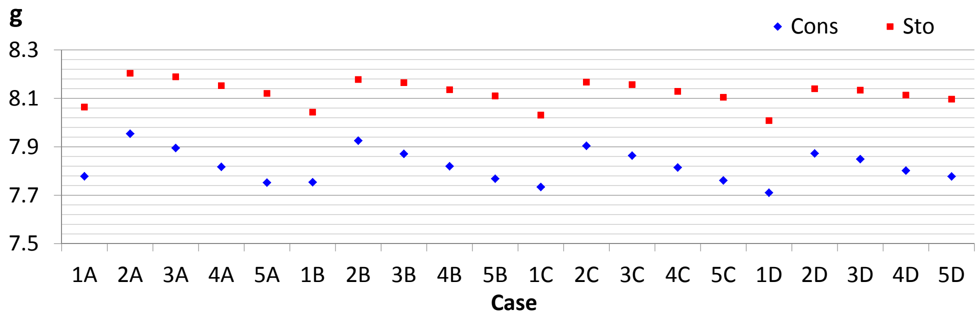
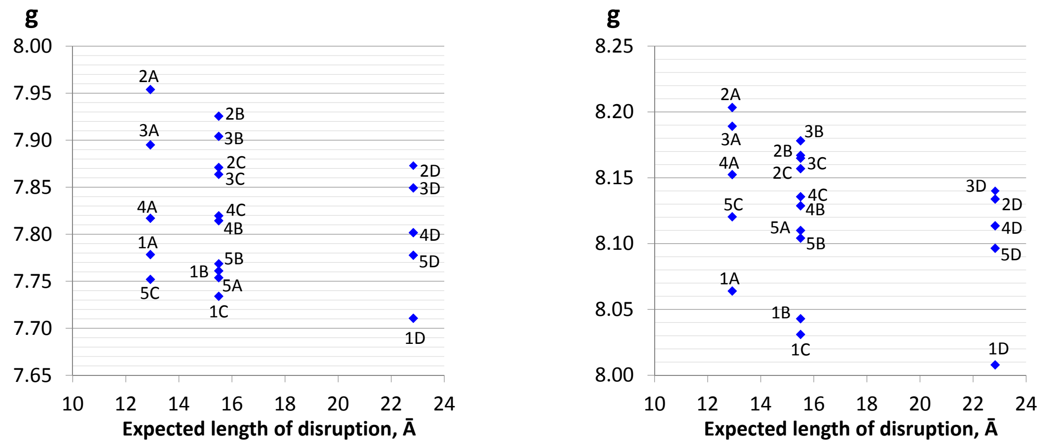


Figure 6.59. M7*Inf*: Optimal long-run average cost in different cases for constant and stochastic demand models

In general, if we consider the relationship by α cases, we can see an approximately linear relationship between the long-run average cost, g, and the expected length of disruption, $\bar{A}$ (see figure 6.60). From figure 6.60a, in the constant demand model, in each α case, the value of g decreases with an increase in $\bar{A}$, except in case 5 ($\alpha = 0.9$). From figure 6.60b, in the stochastic demand model, we can see a similar relationship between the long-run average cost, g, and the expected length of recovery phases, $\bar{A}$, as in the constant demand model. In each α case, g decreases with an increase in $\bar{A}$. In both constant and stochastic demand models, as we expected, the firm faces higher cost when the expected disruption period at the offshore supplier is longer.



(a) M7*InfCons*: g *vs.* $\bar{A}$

(b) M7*InfSto*: g *vs.* $\bar{A}$

Figure 6.60. M7*Inf*: The relationship between the long-run average cost and the expected length of disruption

6.4.7 The Impact of the Transition Probabilities on the Performance of Optimal Policies

In this section, we discuss the performance of the ordering policy under the infinite horizon plan and stochastic demand, focussing on the performance of the fill rate (section 6.4.7) and the average inventory level (section 6.4.7).

Fill rate

From figure 6.61, the rate at which the firm can filled up customer's demand from the existing inventory in all cases are estimated to lie between 99.77% and 99.79% with 95% confidence interval. In general, the fill rate, P_2, decreases when supply disruption probability increases. Nonetheless, the variation in fill rate values from the simulation run in each case is still within the range of good performance for the firm, with the fill rate values in all cases are more that 99.75%. In scenario A (i.e., equal length in each recovery phase), P_2 decreases when $\alpha < 0.5$ but decreases when $\alpha \geq 0.5$. The same pattern of fill rate values occur in scenarios B and C (i.e., the length of recovery phases are increasing or decreasing) across α values. However, in case D, P_2, decreases as the supply disruption probability increases. From the findings, we can conclude that supply disruption probability and various pattern of length of recovery phases can affect the fill rate.

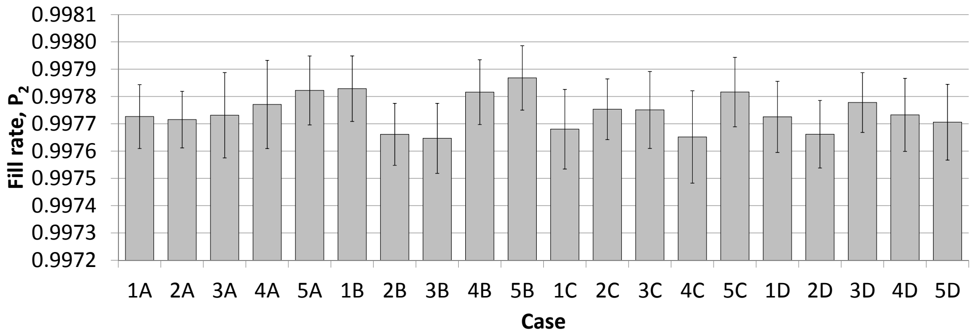
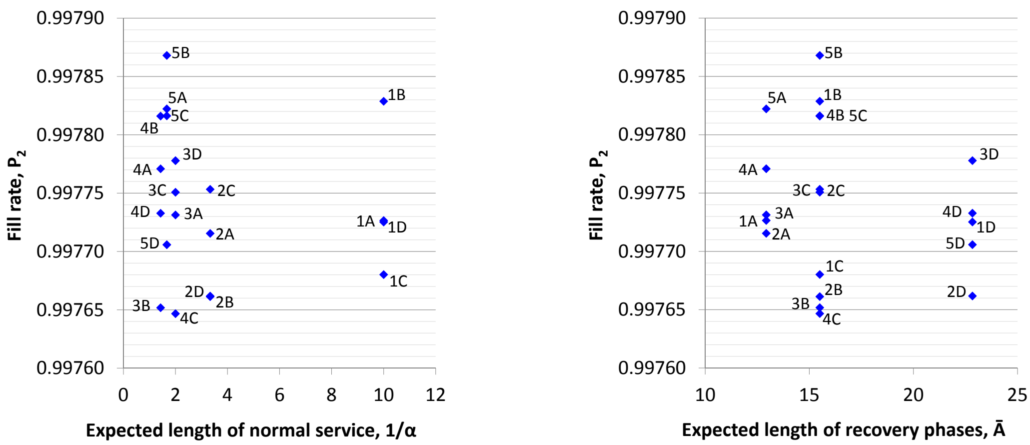


Figure 6.61. M7: Fill rate in each case

If we examine the association between the fill rate and the expected length of normal service and recovery phases at the offshore supplier, from figures6.62a and 6.62b, we can see that there are no relationship between the fill rate values, P_2, and the expected length of normal service, $1/\alpha$, and the expected length of disruption, $\bar{A}$.



(a) P_2 *vs.* $1/\alpha$ (b) P_2 *vs.* $\bar{A}$

Figure 6.62. M7: The relationship between fill rate and the expected lengths of normal service and the recovery phases.

There is also no relationship between the fill rate, P_2, and the expected up time, π_u, as illustrated in figure 6.63.

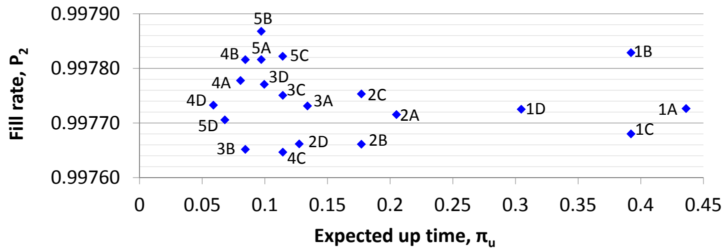


Figure 6.63. M7: The relationship between fill rate and the expected up time

Average inventory level

From figure 6.64, the average inventory level, I_A, in all scenarios are estimated to lie between 31.85 and 38.21 with 95% confidence interval which lie at almost half of the maximum inventory level. In each scenario, I_A increases as α increases when $\alpha < 0.5$ and vice versa when $\alpha \geq 0.5$. As we expected, the firm will carry more stock in inventory if the risk of supply disruption increases.

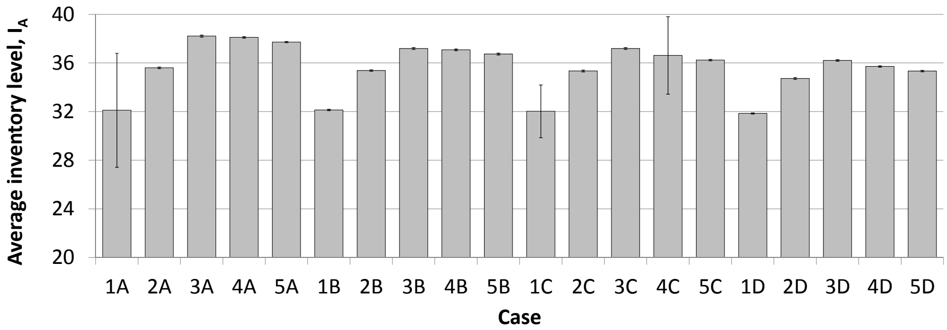
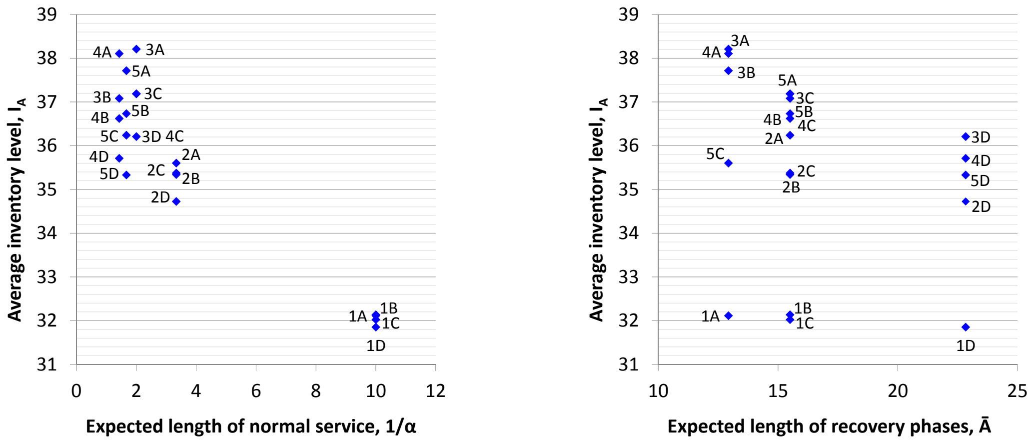


Figure 6.64. M7: Average inventory level in each recovery length scenario for all α cases.

There are negative relationships between the average inventory level, I_A, and the expected length of normal service, $1/\alpha$, and the expected length of recovery phases, $\bar{A}$, as illustrated in figure 6.65. From figure 6.65a, I_A decreases as $1/\alpha$ increases. From figure 6.65b, if we exclude case 1 (i.e., $\alpha = 0.1$) from this plot, I_A also increases as $\bar{A}$ increases.The average inventory level, I_A, also has a negative relationship with the expected up time, π_u. The value of I_A decreases as π_u increases (see figure 6.66).



(a) I_A *vs.* $1/\alpha$ (b) I_A *vs.* $\bar{A}$

Figure 6.65. M7: The relationship between average inventory level and the expected normal service and the expected length of recovery phases

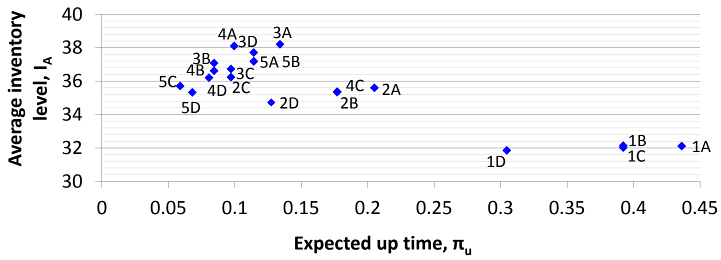


Figure 6.66. M7: The relationship between average inventory level and the expected up time

Discussion

In Model 7 analyses, similar with the analyses of Models 5 and 6, the performance of the firm's ordering policy has been measured with the values of fill rate and average inventory level. Based on the fill rate, the firm still has high capability to satisfy demand from the customer when it chooses to use the optimal ordering policies. The percentage of demand satisfied from stock in hand is still high even though there are high supply disruption probability. In most situations, the firm will keep stock which is half of the maximum inventory level. There are significant evidence show that the expected length of normal service, the expected length of disruption and the expected up time at the offshore supplier can influence the average inventory level, but not the fill rate.

6.4.8 A comparison of Model 7 and Model 4

In the previous section, we mention that Model 7 has been developed in a similar way of Model 4. Both models have been developed with an assumption that the firm has limited information about the length of disruption or the length of recovery phases. The chosen actions on the purchases from the onshore and offshore suppliers are selected according to the firm's belief updated in the ordering process. We expect the optimal policies in Model 7 are likely the same as in Model 4. The similarity can be seen from the formula of the transition state probability. Therefore, in this section, we conduct a comparison of Models 7 and 4 to find the similarity in the optimal ordering policies. The findings are presented in sections 6.4.8a, 6.4.8b and 6.4.8c.

The comparison on the optimal ordering decision between Model 7 and Model 4

This section focuses on the order quantity placed with the onshore supplier, q^N, when the offshore supplier is down. The difference of q^N between Model 7 and Model 4 is examined to find the similarity in optimal ordering decision. The (s, S) policy is used to find the difference q^N between these two models and we refer to the parameter of order up-to level, S, as a reference to check an approximation of the quantity ordered from the onshore supplier between these two models.

From figures 6.67, we can see a positive value of difference of order up-to level between Model 7 and Model 4 (i.e., diffq^NM7-q^NM4) in the second half of the recovery period (i.e., $k > 3$) in each the length of recovery phases for all α cases (except in scenario B). It means that the optimal ordering policy in Model 4 is approximately better than in Model 7. The optimal ordering policy in Model 4 is also approximately better than in Model 7 in the stochastic demand model, as illustrated in figure 6.68. From the findings, we can say that Model 4 is approximately better than Model 7. Perhaps, it is more useful for the firm to update its belief on the disruption length than the belief on the length of recovery phases. We suspect that, the length of recovery phases depends on the length of disruption. It is logical since under limited information, the recovery plan can be updated only after the firm has update information about the disruption length.

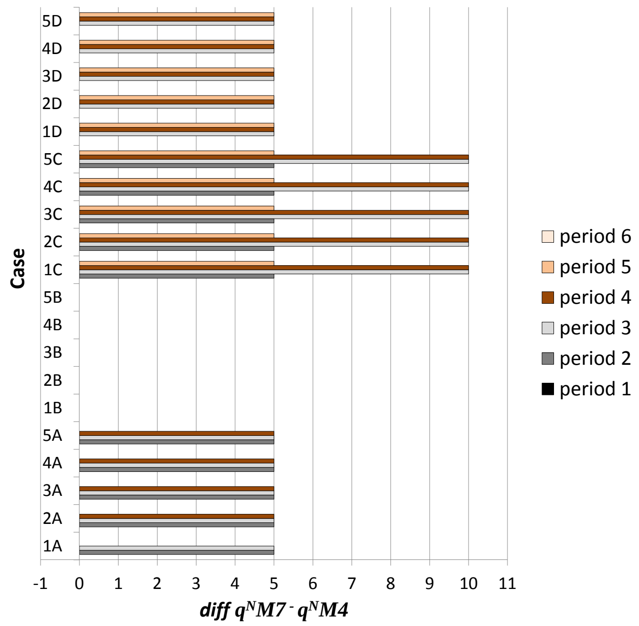


Figure 6.67. The difference of order up-to level between Model 7 and Model 4 in the constant demand model.

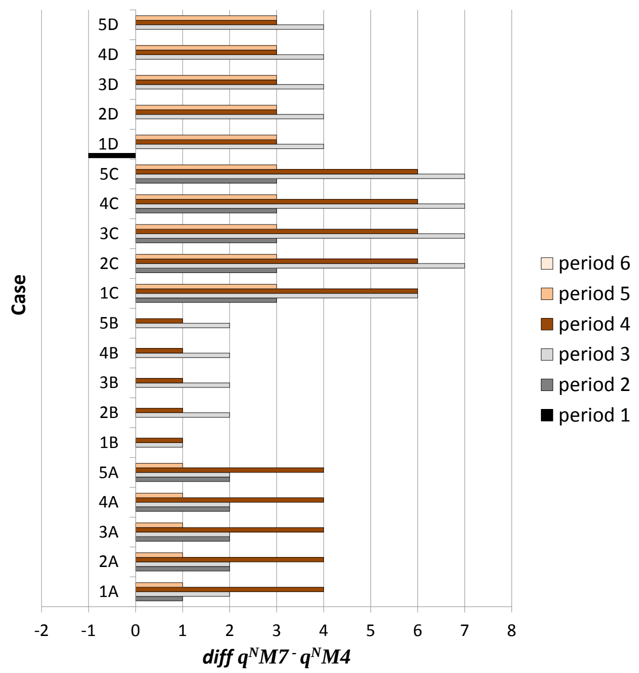


Figure 6.68. The difference of order up-to level between Model 7 and Model 4 in the stochastic demand model.

The comparison on the long-run average cost between Model 7 and Model 4

In this section, the difference of the long-run average costs between Model 7 and Model 4 is examined to find an approximation between these two model. The value of the difference of long-run average costs in Model 7 and Model 4 is define as $gM7 - gM4$.

From figure 6.69, in both the constant and stochastic demand models, we can see that the values of $gM7 - gM4$ are negative in all α cases, especially in scenarios A, C and D. It means that Model 4 is better than Model 7. However, the difference is very small which at most -0.025 ($gM7 - gM4 \leq -0.025$). Therefore, we can conclude that the long-run average cost in both Models 7 and 4 are approximately the same.

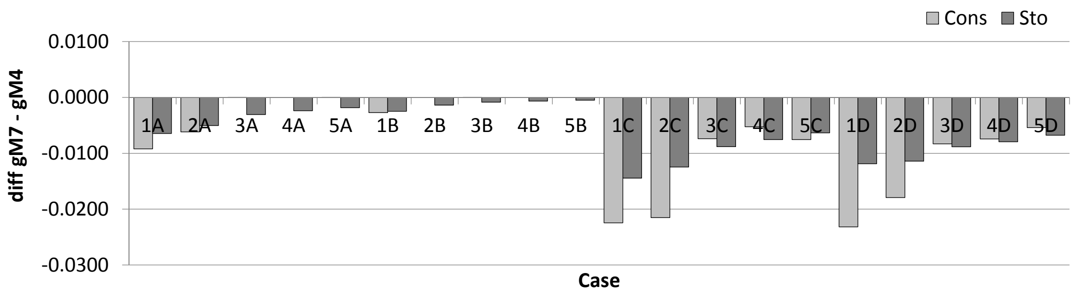


Figure 6.69. The difference of long-run average cost between Model 7 and Model 4 in each case.

The comparison on the performance of the fill rate and the average inventory level between Model 7 and Model 4

In this section, the similarity on the performance of Model 7 as compared to Model 4 is analysed by examining the differences of fill rate and average inventory level values between Model 7 and Model 4 (i.e., FillRateM7-FillRateM4 and AvgLvlM7-AvgLvlM4) and the differences are illustrated in figures 6.70 and 6.71. From figure 6.70, we can see that the difference of the fill rate values between Model 7 and Model 4 are negative or positive. The positive values occur mostly in scenario A, which means that Model 7 is approximately better than Model 4, while the negative values occur mostly in other scenarios, which means that Model 4 slightly better than Model 7. It seem that both information about the disruption length and the length of recovery phases are equally useful for the firm to able better plan its inventory to satisfy demand from its customer.

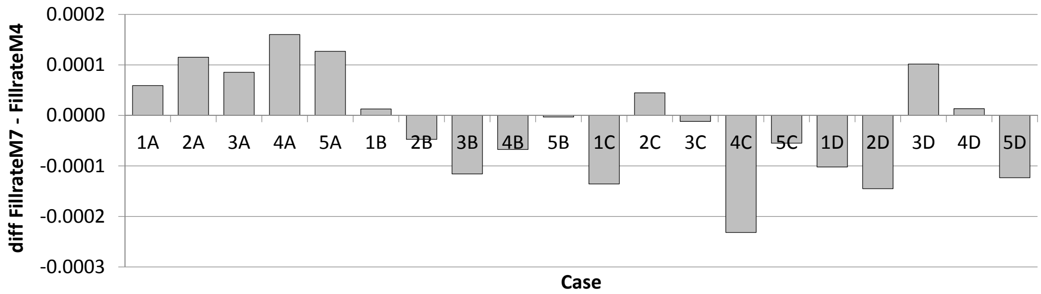


Figure 6.70. The difference of fill rate values between Model 7 and Model 4 in each case.

From figure 6.71, for the average inventory level in Models 7 and 4, the difference values are negative in scenario C and mostly approximately the same in other scenarios, which means that the average inventory level in Model 7 is approximately the same as in Model 4 (except in scenario C). However, under the scenario of the length in recovery phases are decreasing (i.e., scenario C), the firm keeps lower stock in the inventory in Model 7 than in Model 4. From the findings, we can conclude that information about the disruption length (Model 4) and the pattern of the length of recovery phases (Model 7) are equally practical for the firm to carry proportional stock in the inventory only if needed.

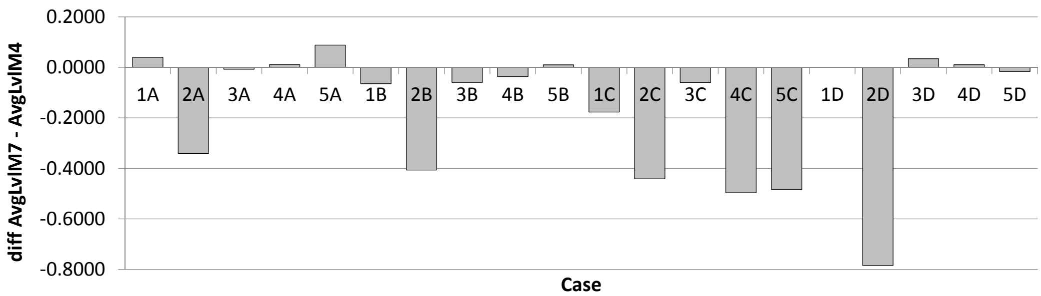


Figure 6.71. The difference of average inventory level values between Model 7 and Model 4 in each case.

The comparison on the fill rate and the average inventory level between Model 7 and Model 4 are also examined by conducting a statistical inference test of two samples t-test (assuming equal variance) for all cases. The null hypothesis in the fill rate and the average inventory level t-tests state that there are no difference between the average of fill rate and the average of average inventory level in Model 7 and the average of fill rate and the average of average inventory level in Model 4. These tests have been conducted at a significance level of 0.05 for all cases. Table 6.23 tabulates *p-values* in each case. From the fill rate test, all of *p-values* > 0.05, thus we fail to reject the null hypothesis. Hence, we can infer that there is no difference between the average of fill rate in Model 7 and the average of fill rate in Model 4. However, we fail to reject the null hypothesis in some cases in the average inventory level *t*-test (case 2a, 2b, 2d and all cases in scenario c except in case 3c). Hence, we can infer that there are differences in some cases of the average of average inventory level in Model 4 and the average of average inventory level in Model 7.

Table 6.23. The values of *p-values* in *t*-tests in each case of the fill rate and the average inventory level.

Case	*p-values*	
	P_2	I_A
1a	0.5042	0.2693
2a	0.1630	0.0000
3a	0.4059	0.8742
4a	0.9925	0.7620
5a	0.1305	0.0060
1b	0.8900	0.0700
2b	0.5905	0.0000
3b	0.1776	0.1439
4b	0.3970	0.2300
5b	0.9760	0.8433
1c	0.1575	0.0000
2c	0.6161	0.0000
3c	0.8955	0.1281
4c	0.0102	0.0000
5c	0.6176	0.0000
1d	0.3286	0.9845
2d	0.1225	0.0000
3d	0.2014	0.4000
4d	0.8661	0.7837
5d	0.1646	0.6548

Discussion

An approximation between Model 7 and Model 4 in four different parameters, namely the quantity ordered from the onshore supplier when the offshore supplier is down, the long-run average cost, the fill rate and the average inventory show that the policy in Model 4 is approximately better than Model 7 (showed in the difference of the quantity ordered from the onshore supplier). Overall, the policies in Model 7 and Model 4, are approximately the same (showed in the difference of costs, the fill rate and the average inventory level). Perhaps, sometimes it is more useful for the firm to update its belief on the disruption length (Model 4) than the belief on the length of recovery phases (Model 7). We suspect that, the length of recovery phases depends on the length of disruption. It is logical since under limited information, the recovery plan can only be updated after the firm has updated its information about the disruption length.

6.4.9 Conclusion

In Model 7, we addressed how the firm's ordering policies can be affected by the risk of disruptive supply event at the offshore supplier and the planning of recovery phases during the recovery process at this supplier. From the analyses of Model 7, without information of the length of recovery phases, on the belief of the firm, we demonstrate how the properties and the performance of the optimal ordering policy depend on the values of the transition probabilities of the Markov chain model of recovery length to the offshore supplier.

From the findings, during the recovery process, we discover that the pattern of expected length of recovery phases has more impact on the optimal order quantity from the onshore supplier, the minimum cost and the long-run average cost than the risk of supply disruption. Similar to Model 6, during the recovery process (the offshore supplier is down), the firm will order fewer items from the onshore supplier if the offshore supplier has a structured and detail recovery plan, rather than making the plan based on the current state of the disruption. A comparison on the optimal policies of Model 7 and Model 4 shows that Model 7 is approximately the same with Model 4.

6.5 Conclusion

In this chapter, we presented the analyses of the ordering policy model with a phased recovery process. Three models have been analysed with different assumptions on the information available on the length of each phase of the recovery process. The first model, which is Model 5 is a basic model of a phased recovery process. This model is used to explore how to employ a quantifiable measurement of recovery in inventory policies. The second and third models are the extension of Model 5, which refer to Model 6 and Model 7. In Model 6, there is advance information of the length of a phase of the recovery that available at the beginning of each phase, while in Model 7 there is no information on the duration of each phase, but has a belief from the firm. Similar to models in the previous chapters, we analysed the policies of ordering from the firm with two non-identical supplier in a simple supply chain setting with an additional information on the risk of disruption to the offshore supplier (these two suppliers are distinguished by their lead-times and ordering unit costs). Model 5 is analysed in four different settings, the model with finite planning horizon and constant demand, the model with finite planning horizon and stochastic demand, the model with infinite planning horizon and constant demand and,the model with infinite planning horizon and stochastic demand. Model 6 and 7 however are analysed in two different settings only, the model with infinite planning horizon and constant demand and, the model with infinite planning horizon and stochastic demand.

The focus in this chapter is to investigate the practicability for the firm to has quantifiable measurement of recovery, which consists of several phases rather that just one phase in its inventory plan. In Model 5, we find that the expected length of each recovery phase can influence the firm's optimal policy, especially on the optimal ordering from the onshore supplier during the crisis operation. The firm will order more from the onshore supplier if the expected length of each recovery phase is lower or the expected length of phases increases as the recovery plan progresses. However, the firm will reduce the order quantity placed with this supplier if the expected length of each recovery phase is higher or the expected length of phases decreases as the recovery plan progresses. We also discover that the pattern of the length of phases in the recovery plan can influence the firm's optimal policies in Model 6 and Model 7. We find that the firm has better optimal policies under the condition where the offshore supplier has a structured and detail plan on its recovery process. This process should consist of an assessment on the disruption, the corrective actions on the impact of the disruption and an initialisation of normal operation upon the completion of recovery plan. However, the firm has able to plan its inventory under the condition where the offshore supplier only update its recovery process according to current state of the disruption.

A comparison of Models 6 and 3 and, Models 7 and 4 have been made to observe the similarity or the advantages of having additional information on the length of recovery phases. Under the condition of the firm with advance information about the disruption length in Model 3 and the length of recovery phases in Model 6, we find that the policies in Model 3 are always better than in Model 4. The comparison of these two model is in the way we had expected. Model 3 has information on the length of disruption as soon as the disruption occurs, while in Model 6, it only knows about the length of the current recovery phase. However, the comparison of Models 4 and 7 resulted in the same equality of the usefulness of having disruption information or phased recovery process.

This chapter has presented the values of phased recovery information for the firm in managing its inventory in the process to recover from the disruption. Overall, we manage to find the necessity of having information about the duration of each recovery phase, which equally important as to have information about the duration of the disruption. Other than having information about the disruption and the phased recovery in mitigating supply disruption, the firm also should consider its dependency to the onshore supplier as only backup option during the crisis operation. Therefore, in the next chapter, we intend to investigate the phenomena of *order pressure* to the onshore supplier in the firm's supply chain.

7. The Scenario of Order Pressure in the Supply Chain

7.1 Introduction

This chapter focuses on the phenomena of *order pressure* that may exist in the supply chain due to supply disruption. An order pressure refers to the situation in which the order delivery sourced from a reliable supplier (i.e., the onshore supplier) has been affected by the disruptive supply event at an unreliable supplier (i.e., the offshore supplier). For example, the order pressure could occurs because many firms are affected by the disruption to their suppliers and all these firms are now trying to source supply from the same supplier. In the previous chapters, we might think disruption to the offshore supplier has no effect to the supply delivery from the onshore supplier. In fact, the firm has placed an exceptionally large order to the onshore supplier to make up the loss of supply from the offshore supplier if the disruption is too long. Verification of this statement is provided in the analysis of Model 4. This action may creates the phenomena of order pressure in the firm's supply chain. This phenomena can be measured by the increases in the inventory costs and the average inventory level and, the decrease in the fill rate. Realising the order pressure could has adverse impact on the firm's inventory planning, our study in this chapter will focus on the existence of the order pressure in the supply chain due to supply disruptions.

The structure of this chapter is as follows. We first present the introduction on a scenario of order pressure in section 7.2. Then, we describe a basic model of order pressure in section 7.3, followed by the formulation of the ordering decision problem with the order pressure scenario under several studies on the availability of disruption and phased recovery information via the DMDP in section 7.4. Then, in section 7.5, we present the test values used when conducting the numerical experiment. The results and findings are reported in sections 7.6, 7.7 and 7.8. Finally, the conclusion for this chapter is presented in section 7.10.

7.2 A Basic Model of Order Pressure

The aim of a basic model of order pressure is to explore the existence of order pressure scenario in inventory policies. In this model, we develop an exploratory model in the presence of order pressure in the firm's inventory plan under various conditions of information on the disruption and the phased recovery processes via the DMDP technique. The DMDP modelling framework of basic order pressure model is similar to the models in Chapter 5 (i.e., Models 3 and 4) and Chapter 6 (i.e., Models 6 and 7), with the main difference being that the existence of order pressure in each model. As in earlier chapters, the focus of the study on the order pressure is on a simple two-echelon supply chain with one firm and two suppliers in a single product setting. The firm implements only the diversification strategy in its disruption mitigation plan and relies on a backup supply from only one supplier during the disruptions (i.e., the backup supply is sourced from the onshore supplier). Note that, there are other several supply disruption mitigation strategies suggested by the researchers in the previous studies. See section 2.3.1 for a full description on the supply disruption mitigation strategies.

The assumption that the disruption event of one supplier temporarily or permanently going out of business is totally independent from other suppliers, this does not hold in all cases. The fact is the event that causes the disruption is likely to affect a number of different suppliers at the same time. These suppliers might have a link with their customers, suppliers, geographical location and trade rules and regulations (Christopher et al., 2011). It is not reasonable in some cases to assume that a disruption will occur for one supplier and that the disruption will not have an effect on other suppliers.

Therefore, in this chapter, let us assume that the onshore supplier may or may not be able to deliver this order in full and on time. This assumption is logical. This supplier maybe does not has enough stock in its inventory due to unexpected high volume order from the firm in a short notice. Due to the onshore supplier failure to comply with the exact replenishment time that has been agreed between the firm and this supplier, the condition has created an order pressure in the firm's supply chain.

The scenario of the order pressure in this model has a similarity with the study by Schmitt and Tomlin (2012). They evaluate the impact of disruption correlation with a pair-wise correlation coefficient to examine the condition of suppliers dependency. The condition of suppliers dependency in their study has been modelled with an assumption that the affected supplier can dominate the available supplier in certain degree of supplier disruption correlation. Their study and our study about the order pressure are linked. In our study, the disrupted order from the offshore supplier has affected the order delivery process from the onshore supplier, thus shows that there is a dependency between the onshore supplier and the offshore supplier.

Two variants of model are analysed in this model, namely Variant f and Variant p. Under the analysis of Variant f, we assume that the order pressure has no effect in the chain of supply during the disruption at the offshore supplier and the onshore supplier can deliver every order in full and on time. While under the Variant p analysis, we assume that the order pressure has affected the chain of supply and the onshore supplier only delivers a proportion of the order immediately and delivers the remainder later.

We believe that the scenario of order pressure has an added-value of indication on the remaining length of a disruption. This indication maybe beneficial for the firm to better able plan its inventory. From a strategic decision-making perspective, we hope that this study can help decision makers to be caution against the order pressure as an added-value of information that capture the status of their business during the disruption. This information can be very useful for business that needs a component that complex or difficult to be made in producing a product. Examples of business such as an airplane maker (i.e., Airbus) a high technology weapon maker (i.e., The SIPRI) and a pharmaceutical company (i.e, Bayer Healthcare). If there is a disruptive event for those companies' supplier, this event may creates the order pressure due to difficulty in finding a backup supply. Verification of this statement is provided in the analyses of the Variants p and f.

7.3 Model Description

Recall that the firm gets the supplies from two non-identical suppliers, which are the onshore supplier (or supplier N) and the offshore supplier (or supplier F). These two supplier are distinguished by its ordering costs and lead-times. supplier N is always reliable and supplier F is at risk of disruption. During normal operations of supplier F (or this supplier is up), the firm can order from both suppliers. However, during a disruption at supplier F (or this supplier is down), the firm can only placed an order with supplier N. When supplier F is in the down state, supplier N may or may not faces the order pressure. For a better understanding, figure 7.1 describes the sequence of order pressure event during t, that is interval between the point with t periods to go in the planning horizon and the point with $t-1$ periods to go.

From figure 7.1, let q^{NNow} denote the quantity to order from supplier N that arrives with lead time L^N and q^{NAft} denote the quantity to order from supplier N that arrives with lead time L^F. In Variant f, when there is no order pressure, the firm will receive q^{NNow} items instantly when the order is placed at the beginning of the period and arrive in full, thus $q^{NNow} = q^N$. While in Variant p, when there is the order pressure, the firm will receive half of the items immediately when the order is placed at the beginning of the period and the other half of items arrive before the end of the period, thus $q^{NNow} = \frac{q^N}{2}$ and $q^{NAft} = q^N - q^{NNow}$.

In this study, the scenario of order pressure is examined under the conditions of information about supply disruption (i.e., Models 3 and 4) and the phased recovery process (i.e., Models 6 and 7). Therefore, the Markov process in Variants f and p follow the Markov process as in the Markov model for Models 3, 4, 6 and 7. See section 5.2.1 for the Markov model with full information at start of disruption (Model 3), section 5.3.1 for the Markov model with partial information on the length of disruption (Model 4), section 6.3.1 for the Markov model with with additional information about the length of recovery phases (Model 6) and section 6.4.1 for the Markov model with incomplete information about the length of recovery phases (Model 7). Variants f and p for each model later known as Variant f of

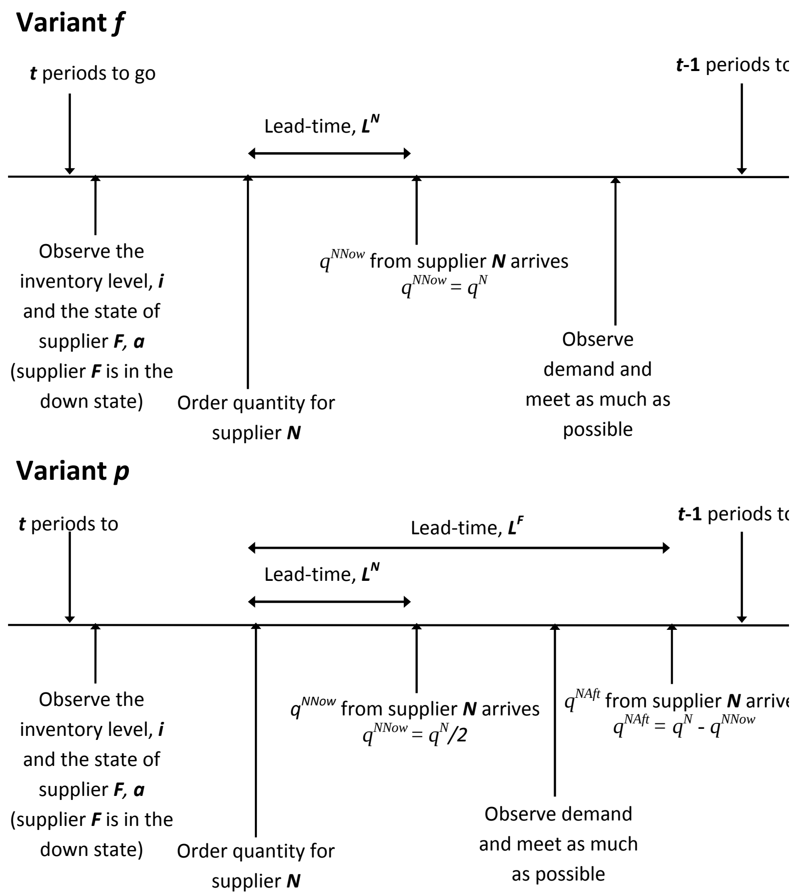


Figure 7.1. The firm's ordering process

Models 3, 4, 6 and 7 and Variant p of Models 3, 4, 6 and 7.

7.4 Model Formulation

In this section, we explain the formulation of the problem analysed in Variants p and f as the discrete Markov decision process model (DMDP) and present optimality equations.

The DMPD models for Variants f and p of Models 3, 4, 6 and 7 use the same DPMDP models for Models 3, 4, 6 and 7. Please refer to sections 5.2.3, 5.3.3, 6.3.3 and 6.4.3 for full descriptions on the DMDP components of the DMDP for Models 3, 4, 6 and 7, respectively.

7.5 Choice of Parameters Values

In this section, we present the transition probability values used for the numerical analysis. Our objective are to analyse how the optimal policy under the order pressure scenario changes with different transition probabilities and to examine how the existence of order pressure can influence the optimal policy.

In this numerical study, the experimental design between Variants f and p of Models 3, 4, 6 and 7 and Models 3, 4, 6 and 7 are identical. Hence, for the experiment, we use the same scenarios as in Models 3, 4, 6 and 7. Full descriptions on the experiment in Models 3, 4, 6 and 7 have been explained in sections 5.2.4, 5.3.4, 6.2.4 and 6.3.4, respectively. In this experiment, we just consider the models under the infinite horizon plan and the constant demand settings.

In what follows, we first results on the effect of order pressure for the cases on the properties of the ordering decision, then results of effect on the costs of policies and finally the results of the effects on the properties of the average inventory. To examine how the order pressure influence these two properties of optimal policies and one property of policies' performance, we make a comparison of the optimal policies in Variant f of models and Variant p of models.

7.6 The Impact of the Order Pressure on the Ordering Policy

In this section, we examine the existence of the order pressure in the firm's supply chain by comparing reorder point, s, and order up-to level, S, of the offshore supplier between Variant f of each model (i.e., sVar$f$ and SVar$f$) and Variant p of each model (i.e, sVar$p$ and SVar$p$). To do this, we observe the differences of s and S between Variant f and Variant p of each model. The differences of the reorder point, s, and order up-to level, S, between Variant f of each model and Variant p of each model is positif for all cases. This is illustrated in figures 7.2, 7.3, 7.4, 7.5, 7.6, 7.7, 7.8 and 7.9.

From figures 7.2, 7.3, 7.4 and 7.5, in each model,we can see that the reorder point of Variant f of models is always higher than Variant p of models. For example, from figure 7.10, in Variant f of Model 3, sVar$f$ of Model 3 values are always higher than sVar$f$ of Model 3 values. In each disruption period case (i.e., cases A, B, C and D), sVar$f$ - sVar$p$ decreases when α increases if $\alpha < 0.5$ and increases if $\alpha \geq 0.5$ for all disruption period cases. From the equal average and variance disruption length scenarios, the order pressure in case B is higher than in case A (i.e., equal average disruption length scenario) and the order pressure in cases C is higher than in case D (i.e., equal variance disruption length scenario). The same pattern of sVar$f$ - sVar$p$ occurs in Model 4, as illustrated in figure 7.3. From the findings, we can conclude that the firm needs to be caution in cases B and C (i.e., the offshore supplier has higher risk to be down for 3 or 5 periods) due to higher risk of the occurrence of the oder pressure.

The difference of cost between Variant f of Models 6 and 7 and, Variant p of Models 6 and 7 shown in figures 7.4 and 7.5. From these figures, the pattern of sVar$f$ - sVar$p$ varies for each recovery phases period scenario (i.e., scenarios A, B, C and D) when supply disruption probability (α) increases. In addition, the order pressure in scenario A is higher than in other scenarios. From the findings, we can see that there is higher risk of order pressure to be exist in the supply chain under the condition that the offshore supplier was observed to has equal period for each phase for its recovery planning (i.e., scenario A), compare to the conditions with other recovery plan (i.e., scenarios B, C and D). We can conclude that the firm needs to be more caution during the remaining of disruption periods if the offshore supplier has implemented a simple plan during for recovery (i.e., the process of recovery is implemented with equal length in each phase).

There is not much different in the difference of order up-to level between Variant f and Variant p of each model, as shown in figures 7.6, 7.7, 7.8 and 7.9. From the findings, the firm will increase the quantity ordered from the offshore supplier by at most 10 or 5 units when the supply chain is under pressure. We can conclude that, the existence of the order pressure will increase the reorder point and the order up-to level of the offshore supplier.

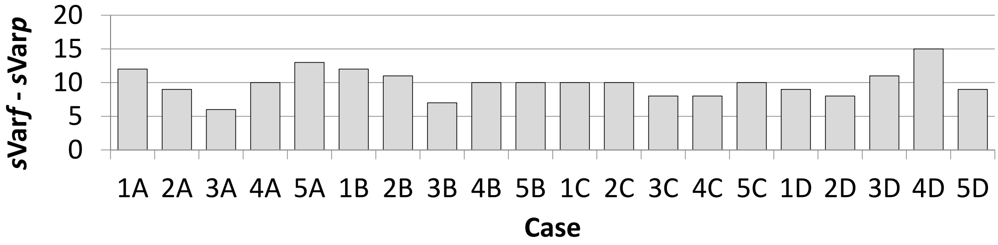


Figure 7.2. The difference of reorder point in each case of Variants p of Model 3 and Variants f of Model 3

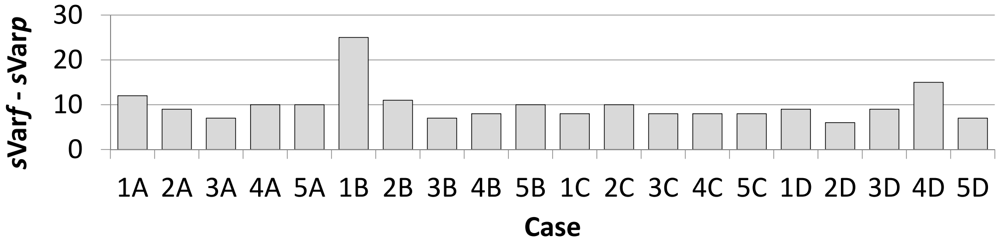


Figure 7.3. The difference of reorder point in each case of Variants p of Model 4 and Variants f of Model 4

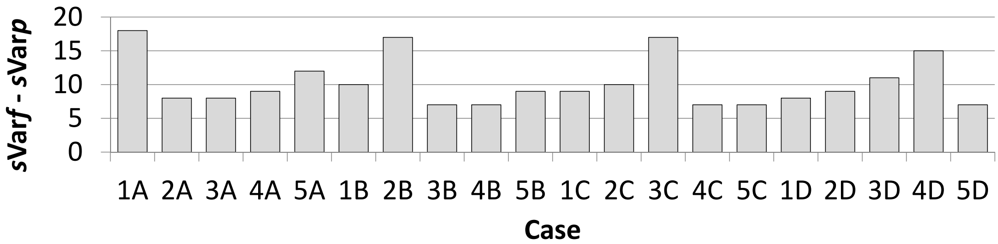


Figure 7.4. The difference of reorder point in each case of Variants p of Model 6 and Variants f of Model 6

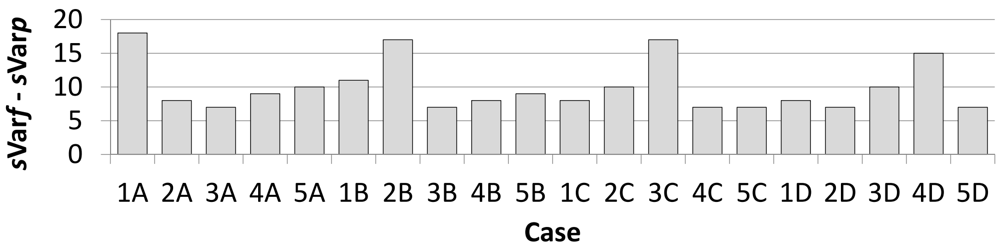


Figure 7.5. The difference of reorder point in each case of Variants p of Model 7 and Variants f of Model 7

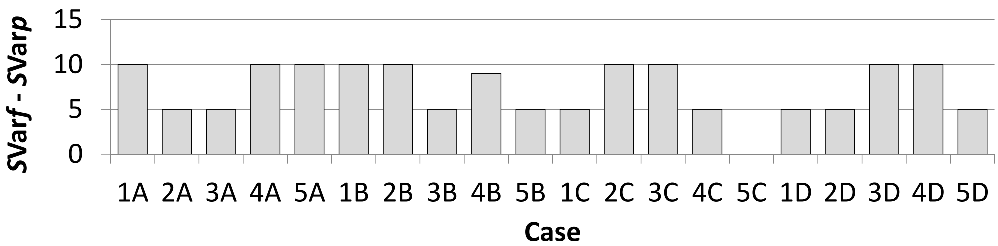


Figure 7.6. The difference of order up-to level in each case of Variants p of Model 3 and Variants f of Model 3

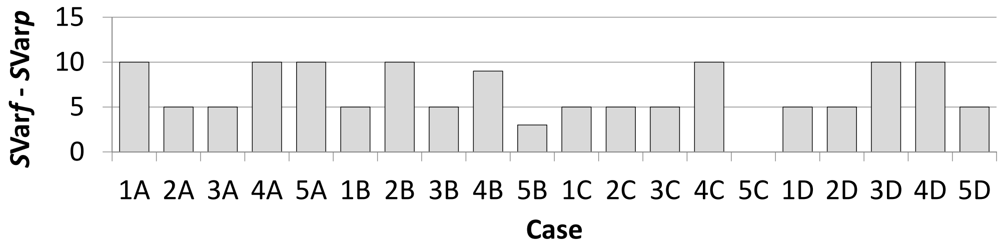


Figure 7.7. The difference of order up-to level in each case of Variants p of Model 4 and Variants f of Model 4

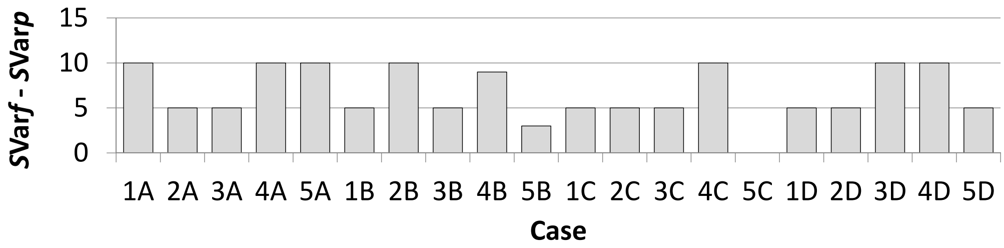


Figure 7.8. The difference of order up-to level in each case of Variants p of Model 6 and Variants f of Model 6

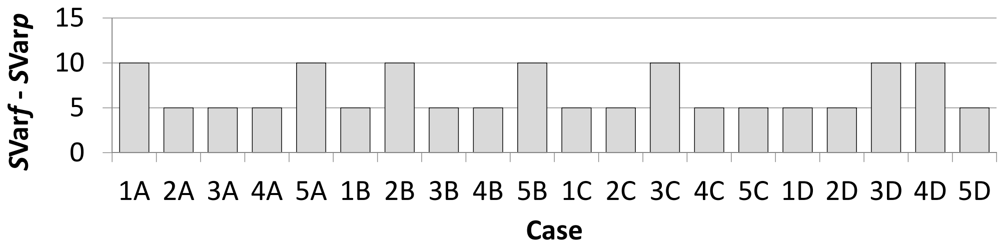


Figure 7.9. The difference of order up-to level in each case of Variants p of Model 7 and Variants f of Model 7

7.7 The Impact of the Order Pressure on the Optimal Policies Costs

In this section, we examine the existence of the order pressure in the firm's supply chain by comparing the long-run average cost, g, between Variants f of models (later known as gVar$f$) and Variants p of models (later known as gVar$p$). We observe the values of the difference of g between Variants f and p of each model (i.e., gVar$f$ - gVar$p$). If the value of gVar$f$ - gVar$p$ is positive (gVar$f$ - gVar$p$ > 0), then we can say that the long-run average cost can be affected by the existence of the order pressure. This is illustrated in figures 7.10, 7.11, 7.12 and 7.13.

From figure 7.10, we can see that gVar$f$ of Model 3 is always higher than gVar$p$ of

Model 3 (i.e., gVar$f$ - gVar$p > 0$) . In each disruption period case (i.e., cases A, B, C and D), gVar$f$ - gVar$p$ increases when supply disruption probability(or α) increases. From the equal average and variance disruption length scenarios, the order pressure in case B is higher than in case A (i.e., equal average disruption length scenario) and the order pressure in cases C is higher than in case D (i.e., equal variance disruption length scenario). The same pattern of gVar$f$ - gVar$p$ occurs in Model 4, as illustrated in figure 7.11. From the findings, we can conclude that the firm needs to be caution in cases B and C (i.e., the offshore supplier has higher risk to be down for 3 or 5 periods) due to higher risk of the occurrence of the oder pressure.

The difference of cost between Variant f of Models 6 and 7 and, Variant p of Models 6 and 7 shown in figures 7.12 and 7.13. From these figures, In each recovery phases period case, gVar$f$ - gVar$p$ decreases when supply disruption probability (α) increases. From various pattern of recovery phases period perspective (i.e., scenario A, B, C and D), the order pressure in scenario D is higher than in other scenarios. From the findings, we can see that there is higher risk of order pressure to be existed in the supply chain under the condition that the offshore supplier was observed to has structured recovery plan (i.e., scenario D), compare to the conditions without proper recovery plan (i.e., scenarios A, B and C). Therefore, we can conclude that the firm needs to be more caution during the remaining of disruption periods to the offshore supplier, even though this supplier has implemented structured plan for recovery.

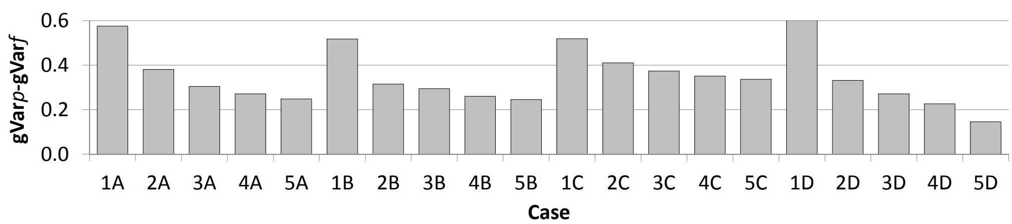


Figure 7.10. The difference of costs in each case of Variants p of Model 3 and Variants f of Model 3

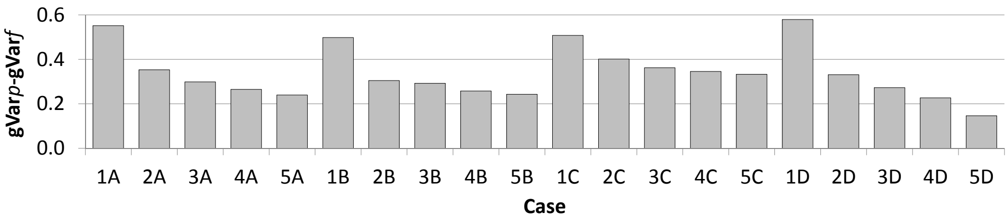


Figure 7.11. The difference of costs in each case of Variants p of Model 4 and Variants f of Model 4

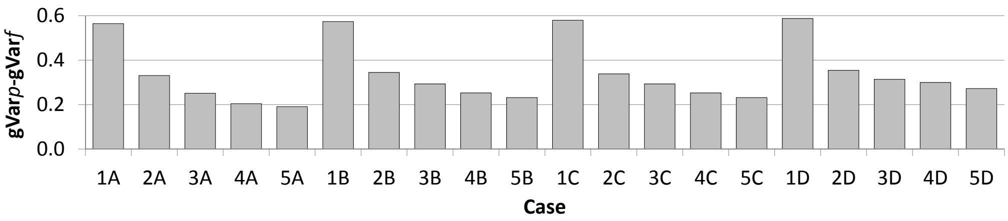


Figure 7.12. The difference of costs in each case of Variants p of Model 6 and Variants f of Model 6

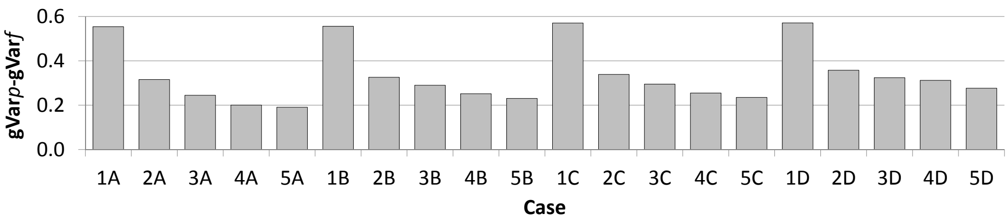


Figure 7.13. The difference of costs in each case of Variants p of Model 7 and Variants f of Model 7

7.8 The Impact of the Order Pressure on the Average Inventory Level

In this section, the performance of Variants f of models as compare to Variants p of models is analysed by examining the difference of average inventory level values in Variants p of models and Variants f of models (i.e., AvglvlVar$p$ - AvglvlVar$f$). The existence of order pressure in the supply chain can be observed from higher average inventory level, thus if AvglvlVar$p$ - AvglvlVar$f > 0$, then there is the order pressure in the firm's supply chain.

From figures 7.14, 7.15, 7.16 and 7.17, we can see that the values of AvglvlVar$p$ - AvglvlVar$f$ are positive (i.e., AvglvlVar$p$ - AvglvlVar$f > 0$) for all cases. The average inventory level in Variant p of models, compared to the average inventory level in Variant f of models, are always higher. Therefore, we can conclude that the existence of the order pressure in the supply chain has a negative effect on the performance of the average inventory level. The firm has to keep more stock in its inventory when there is the order pressure in the supply chain.

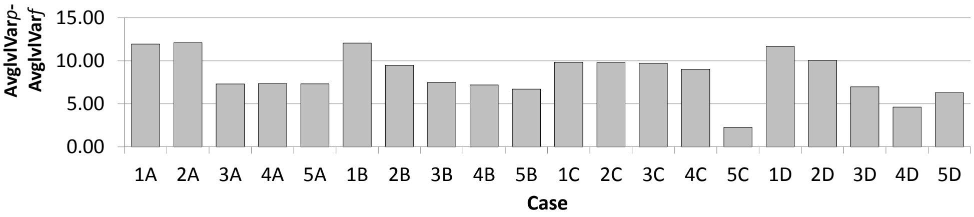


Figure 7.14. The difference of average inventory level values in each case of Variants p of Model 3 and Variants f of Model 3

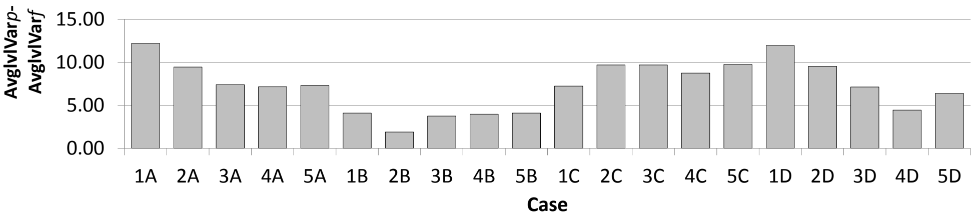


Figure 7.15. The difference of average inventory level values in each case of Variants p of Model 4 and Variants f of Model 4

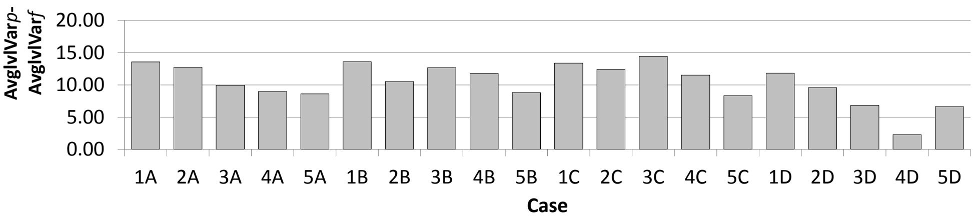


Figure 7.16. The difference of average inventory level values in each case of Variants p of Model 6 and Variants f of Model 6

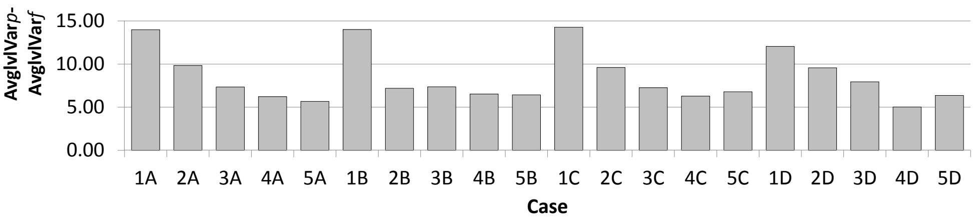


Figure 7.17. The difference of average inventory level values in each case of Variants p of Model 7 and Variants f of Model 7

7.9 Discussion

The variants p and f of models in this chapter represented as a simple model where the onshore supplier delivers 50% of an order immediately and 50% after a delay of one period. From the findings, we showed that the existence of order pressure can affects the optimal ordering policy. One issue with this (which is a common problem in yield uncertainty models) is that the firm could just double (or at least increase) its order quantity in the knowledge that it will get the items it wants at the start of the period and the only penalty is having to accept extra at the end. In this case, it would be possible to look at alternative models of order pressure.

7.10 Conclusion

In this chapter, we presented the analyses of the ordering policy model with a phenomena of order pressure. This model is used to explore how the order pressure can effect the inventory policies. The model has been analysed with different conditions of disruption information and phased recovery process information, as in Chapters 5 and 6. Overall, we manage to find the necessity of having order pressure information. We found out that the order pressure has increased the reorder point, the order up-to level, the long-run average cost and the average inventory level.

8. Thesis Conclusion and Recommendation

This chapter briefly summarises the research that has been conducted in this thesis. Then, limitations of theoretical and numerical study are identified. Finally, possible directions for future research are suggested.

8.1 Summary and Contribution

This thesis investigates the effectiveness of considering a dual sourcing strategy in managing supply disruption in the supply chain, by examining the cost-effectiveness of disruption information and phased recovery process in the inventory management. Phased recovery process has not been sufficiently studied, where researcher just only focusing on minimising the risk of disruption, and ignoring the fact that the disruption risk planning supposedly covers the process of disruptions discovery and recovery. However, such vast lack of information on the disruption discovery and recovery has ignored the possibility that the firm might be able to observe the remaining of the disruption period. In fact, the process of phased recovery can be used as an added-value of information that captures the status of a business during the disruption. Verification of this statement is provided in the thesis.

This thesis also investigates the scenario of supplier dependency, by observing the order pressure phenomena that might be exist in the process of order delivery from a reliable supplier. The additional increments in inventory costs and stocks are examined to identify the existence of the order pressure. Little research has been done on this phenomena, where researcher ignored the possibility that the order pressure has existed in the supply chain at some significant degree of supply disruption. Verification of this statement is also provided in the thesis. Research in this thesis focused on a firm who implements a strategy to diversify the component order quantities from more than one supplier to deal with the risk of supply disruption. These suppliers are distinguished by its reliability, unit prices and lead-times. The following assumptions are made; one supplier is always reliable with shorter lead-time but sells higher units cost (i.e., the onshore supplier) and one supplier is unreliable (i.e., usually exposed to disruption risk) with longer lead-time but sells lower unit cost (i.e., the offshore supplier). In this case, it is important for a firm to decide how they could best devise and implement a dual-sourcing strategy that will avoid overspending on the onshore supplier and/or underestimating the supply disruption risk for the offshore supplier. In this thesis, the uncertain state of the offshore supplier is addressed as a Markov process and the discrete Markov decision process (DMDP) modelling framework has been developed to illustrate problems that related to dual-sourcing strategy. Subsequently, a Value Iteration (VI) algorithm was used to compute the optimal policy value for the DMDP model. The steps of the VI algorithm is implemented in Java programming language.

The nine DMDP models presented in this thesis were used to investigate three strategic issues in the firm's dual-sourcing planning: (1) on the value of supply disruption information, (2) on the value of phased recovery process information and, (3) the phenomena of order pressure. The values of optimal policies costs were discovered and the optimal ordering quantities from the suppliers were determined through the analysis of these nine DMDP models. In some cases, the performances of the optimal policies were checked, by examining the values of the fill rate and the average inventory level. These two values were derived from a simulation method. Some managerial insights on how the firm should respond to those three strategic issues according to those four values were also provided in the models analyses. The experiment results and findings of each of the nine models are briefly discussed below.

8.1.1 The Ordering Policies

Chapter 4 presented and analysed two basic models which are the routine ordering model (Model 1) and the crisis ordering model (Model 2). Model 1 is a preliminary work that has been developed with minimal restrictions. This model is purposely been designed in that way with the intention of examining the policies of ordering for the firm with two non-identical suppliers in a simple supply chain setting. Model 2 introduces a simple model to explore the effect of supply disruption on the ordering process in a similar supply chain setting. This model was analysed in different 25 cases generated from a combination of supply disruption probability and disruption recovery probability values.

In Models 1 and 2, the costs related to the ordering process are the dominant factor when making decision on sourcing from the onshore and offshore suppliers. Since the onshore supplier sells the items/semi-items with a higher cost when there is no disruption in the supply chain, the firm would never sole-source from it even though the delivery period is shorter than for the offshore supplier. The finding showed that the order from the onshore supplier only needed as a backup supply to satisfy the immediate shortage in the firm's inventory. Under the crisis operation, the firm needs to trade off the costs versus the state of the offshore supplier. One interesting discovery in Model 2 is on the properties of the optimal ordering from the offshore supplier. If the estimated chance of disruption recovery by the offshore supplier is low, the firm should try to keep a higher volume of stock from this supplier to avoid the high shortage cost during disruptions. However, if the chance of disruption recovery by the offshore supplier is too low, then it seems that it is not possible to do this.

8.1.2 Value of Supply Disruption Information

Chapter 5 extended the works in Chapter 4. This chapter consider the visibility of disruption information in the event of supply disruption particularly to the disruption processes. Two models were analysed in this chapter, which are Model 3 and Model 4. These two models differ in the disruption information available when disruption occurs. In Model 3, it assumed that the length of the disruption is known to the firm, while in Model 4, it assumed that the firm only knows the probability distribution of the length of disruption. Findings of these two models determine how much inventory the firm would need to fully protect itself against any supply disruption according to the severity (length) of the disruption. These two models were analysed with the combination of various supply disruption probability values and disruption period scenarios.

In Model 3, with the exact number of periods of disruption at the offshore supplier, the firm increases the quantity order from the offshore if the risk of disruption and the expected disruption periods are increased. Only when the offshore supplier is down, the firm will place an order with the onshore supplier if the inventory level is critically low, but the firm will start carry more items from this supplier when the expected disruption periods increases. In Model 4, having no disruption information but having a trust on the firm, no matter how long the disruption is, the risk of disruption just has a small effect to the firm's ordering process. Only when the offshore supplier is down, the firm believe that the offshore supplier will recover from the disruption after a long disruption period. Hence, it is optimal for the firm to carry less expensive items from the onshore supplier upon the completion of disruption process.

The similarity between Models 3 and 4 and Model 2 are verified. The assumption that has been made in Models 3 and 4 on the additional of disruption information can improve the firm's ordering policy to be accurate, as indicated by lower costs and average inventory levels and, higher fill rates.

8.1.3 Value of Phased Recovery Process Information

In Chapter 6, the practicability for the firm to have a quantifiable measurement of recovery, which consists of several phases rather than just one phase in its inventory plan, was explored and analysed in Model 5, Model 6 and Model 7. Model 5 is an exploratory model with recovery assessment that uses a basic model of a phased recovery process, which is similar to Model 2 in Chapter 4. Model 6 and Model 7 are the extension of Model 5. These two models focus on the information available on the length of each phase of the process, in a similar way to Model 3 and Model 4 in Chapter 5. The phase of a recovery plan could provide a good indication of the remaining length of a disruption. In Model 6, there is advance information of the length of a phase of the recovery that available at the beginning of each phase, while in Model 7 there is no information on the duration of each phase, but has trust from the firm.

The results showed that the optimal ordering from the onshore supplier during the crisis operation (i.e., the offshore supplier is down) is highly dependent to the strategy of recovery plan for the offshore supplier and information about the expected length of each recovery phase. From the findings, the firm will carry more items from the onshore supplier if the expected length of each recovery phase is lower or the expected length of phases increases as the recovery plan progresses. In addition, the firm also has inventory when the offshore supplier does not has organisational recovery plan. These three models were analysed with the combination of various supply disruption probability values and phased recovery length scenarios.

A comparison of Models 3 and 6 and, Models 4 and 7 have been made to observe the similarity or the advantages of having additional information on the length of recovery phases. From the findings, Model 3 is always better than Model 4, as indicated by lower costs and average inventory levels and, higher fill rates, as we had expected. Model 3 has information on the length of disruption as soon as the disruption occurs, while in Model 6, it only knows about the length of the current recovery phase. However, the comparison of Models 4 and 7 resulted in equality of the usefulness of having disruption information or phased recovery process.

8.1.4 The Phenomena of Order Pressure

The research in Chapter 7 studied the phenomena of order pressure that might be existed at the onshore supplier due to supply disruption at the offshore supplier. Two variants of Models 3, 4, 6 and 7 above, named Variant f and Variant p were analysed, with the following assumptions. In Variant f, there is no order pressure in the chain of supply and the onshore supplier can deliver every order in full and on time. While in Variant p, there is order pressure in the chain of supply and, when the offshore supplier is down, the onshore supplier only delivers a proportion on time and delivers the remainder later.

From the findings, a comparison of Variants f and p showed the order process exists in the supply chain when the offshore supplier is down, as indicated by the increases in the inventory costs and the average inventory level and, the decrease in the fill rate, with respect to different levels of information about the disruption length (i.e, Models 3 and 4) and the length of recovery phases (i.e, Models 6 and 7).

8.2 Limitations of the Research

The findings from this thesis has shown a potential of improvement in managing supply disruption by having detail information about the disruption periods, the phased recovery process and the scenario of order pressure. However, there are some limitations in our approach and there are as follows.

One limitation of this research is that the decision variable in the DMDP model here, which are the order quantities from the suppliers are constrained only by the maximum storage capacity of the firm but are otherwise arbitrary functions of the inventory level and, in the case of finite horizons, the number of periods remaining in the planning horizon. In spite of that, the research conducted for the inventory models with non-identical suppliers does suggest the dynamic inventory policies, which characterised by one or two target inventory positions with an aim to minimise the inventory cost (such as a base-stock policy or dual-index policy). Ordering strategies with this type of simple structure have the practical advantage of being easier to implement, but imposing such a strategy would complicate the DMDP model. We believe that our approach is sufficient to capture managerial insights of dual sourcing strategies as one aspect of the supply disruption mitigation plan.

The second limitation is on the chosen input values of variable ordering cost of the onshore supplier per item, v^U, for the numerical experiment under the scenario of crisis operation to the offshore supplier, with an assumption that the onshore supplier sells the items with the same price in both normal and crisis operation. In fact, the research on the supplier-buyer contract with supply disruption does suggest that an additional cost will be imposed on an unexpected additional order from the buyer during crisis operation at a certain degree of disruption process. Ordering strategies with this type of pricing structure have the practical advantage in describing the inventory cost related problems, but unit ordering cost of the onshore supplier is not a dominant factor in the DMDP models here when dealing with various Markov disruption and recovery processes. Therefore, we believe that the experiments with one unit cost of the onshore supplier is still sufficient to capture managerial insights of dual sourcing strategies as one aspect of the supply disruption mitigation plan.

Another limitation of this research is that the phased recovery models (Models 6 and 7) presented here are not robust. Two-dimensional state of the offshore supplier that represented the number of phases, j, and the length of each phase, k (i.e., (j,k)) has been mapped to a single dimensional representation, thus it is difficult to distinguish the number of phases and the phases period from the results. Alternative solution methodologies might need to be considered in order to have better three-dimensional state space, which can represent one state of inventory level and two state of the offshore supplier here. Nonetheless, the two-dimensional state space in the DMDP models here still work to measure the recovery plan that charts the path of the disrupted supplier back to normal operations. Verification on this statement has been provided in Chapter 6.

One last limitation in this thesis is the order pressure model presented here is a very simple model. Variant f of models that introduced here has a similarity with the DMDP models when the offshore supplier is in the down state. While Variant p of models one could think as a yield problem, a form of supply uncertainty where quantity produced and received differs from the quantity ordered in a random way. Alternative solution methodologies might need to be considered in order to have better description on the scenario of order pressure.

Nevertheless, the findings from the order pressure model here have shown the order pressure has influenced on the optimal ordering policy, higher inventory level and cost.

In this thesis, the DMDP models that has been developed to describe the disruption processes in the firm's supply chain, presented maybe do not related to a specific industry or product market due to many assumptions had to be made regarding the nature of the model. In this thesis, developing mathematical supply disruption inventory model is a big challenge for us. The complexities of inventory model and the structure of supply chain network make our work impossible to accommodate a great number of parameters and variables in the DMDP modelling framework and there are always limitations in the mathematical model. Sometime, we need to impose several strict assumptions with the idea to avoid any complexity in finding the optimal ordering policy. However, all the aforementioned limitations can be improved in several ways.

8.3 Further of Research

The review of the literature and the limitation of this thesis have highlighted avenues for future research. Several works have been identified and are described below.

From the first limitation, it would be interesting to study supply disruption problems under the dual-index policy. There is quite a number of research that study this policy extensively, but has not been studied under the supply disruption scenario, to the best of our knowledge. Scheller-Wolf et al. (2003); Veeraraghavan and Scheller-Wolf (2008) study a dual-index policy with two target levels that performs close to optimality. Basically, the dual-index policy represents a policy that tracks inventory over regular and expedited lead-times based on two target levels of inventory positions. In every period, if the expedited inventory position is below expedited order-up-to target level, it is brought back to this level by placing an expedited order. After the expediting order is made, regular orders are placed, restoring the regular inventory position to its regular target level (Veeraraghavan and Scheller-Wolf, 2008).

From the second limitation, we could consider higher unit cost of the onshore supplier and penalty cost.

From the third limitation, another interesting future research is on an alternative methodology in presenting the two-dimensional state of the offshore supplier in Chapter 6. We could consider a two-level Markov decision process model (later known as two-level MDP model). The states in the two-level MDP model are formed by the status of both the upper and the lower levels. The upper and lower levels states are modelled by the Markov jump process and the deterministic functions, respectively. The transition states can be caused by the changes in status of each level. Actions in this model relies on the difference in the time scale of the two levels. A long-run average or sum from the lower level is used to make decisions at the upper level state. The two element of the upper and lower levels in two-level MDP model make the two-dimensional state of the offshore supplier here possible to be distinguished for better mapping of the phased recovery process.

From the last limitation, another possible future research is on other ways to represent the order pressure scenario in Chapter 7. Direction future research in this chapter is by altering the modelling assumption on the existence of order pressure in the supply chain. We could assume that the order pressure does exists when the supply chain is under stress due to disruption. The condition that the supply chain is under stress is assumed to follow the known probability distributions. For instance, uniform or binomial case. Such an example, we could include some uncertainty over how the onshore supplier behaves when under pressure. If the order quantity is q, the onshore supplier will provide x items at the start of the period and $q - x$ items at the end with probability $1/(q + 1)$ for $0 \leq x \leq q$. In addition, we also could add another variable of response rate which can be used to detect the responsive time for the onshore supplier to deliver the order.

8.4 Conclusion of the Thesis

This thesis has demonstrated the importance of considering information about disruption and phased recovery processes when managing supply disruption and of understanding the phenomena of order pressure that may exist in the supply chain during disruption. This research has highlighted a number of avenues for future research in topics relating to disruption recovery and order pressure. Information on the phased recovery process is useful for a firm to update the status of its disrupted supplier and to monitor the process of disruption. Moreover, the order pressure scenario can be an additional information to identify the effectiveness of having backup suppliers, which is a key issue in the context of reducing cost spending on backup suppliers. Furthermore, a firm also be able to plan a better inventory control system. This thesis straighten out on how this could be achieved by examining how a firm can employ a monitoring system that include both disruption and phased recovery process information.

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